MAGNETISM AND ELECTRICITY

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VOLUME IV.

THE HIGHER TEXT-BOOK
OF
MAGNETISM & ELECTRICITY

BY

R.obert WALLACE STEWART, D.Sc. Lond.

AUTHOR OF "THE HIGHER TEXT-BOOK OF HEAT," "A TEXT-BOOK OF LIGHT," ETC.

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PREFACE.

THE basis of this text-book is the author's *Text-Book of Magnetism and Electricity.* In view of the rapid advance which has been made in electrical theory in the last few years, it was felt that the older text-book was inadequate; accordingly the whole work has been recast, and a very considerable quantity of new matter has been inserted.

The book is now written up to the standard necessary for final degree work, and it includes the essential theory of those subjects which the advance of electrical practice has brought into prominence.

The treatment throughout is of an elementary character. The author has not, however, hesitated to make use of the notation and first principles of the Calculus. Students of Physics cannot realise too early that a knowledge of the rudiments of Differential and Integral Calculus is an essential part of their mental equipment.

The author and publishers are indebted to Professor J. J. Thomson, F.R.S., and the Oxford University Press for permission to use several diagrams from *Recent Researches in Electricity* [which are inserted in this book as Figures 79-84, 86-88, 97-100, 293-298], and to Professor Burch, F.R.S., and the Royal Society for permission to reproduce

the diagram of the Capillary Electrometer (Fig. 219). Our thanks are also due to the following firms who have kindly supplied us with blocks from which to print illustrations of apparatus which they manufacture:—Messrs. Kelvin and White (Glasgow), Messrs. Nalder Bros. (Westminster), The General Electric Company (Queen Victoria Street), Messrs. R. W. Paul (Holborn), Messrs. Johnson and Phillips (Old Charlton, Kent), and The British Thomson Houston Company (Rugby).

The author is also indebted to Mr. O. W. Griffith, B.Sc., the Hartley University College, for help in the revision of proofs and in the preparation of Chapter XXXIII.

NOTE TO SECOND EDITION.

This Edition contains an extra chapter on the Electron Theory of Matter and Radio-activity written by Dr. J. Satterly, Assistant Professor of Physics at Toronto University.

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ELECTRICITY AND MAGNETISM.

PART I.

ELECTROSTATICS.

CHAPTER I.

ELECTRIFICATION.

1. Introductory. If a piece of sealing wax be lightly rubbed with a piece of flannel it acquires the peculiar property of being able to attract light bodies, such as cork dust, fine shreds of paper, pieces of pith, and other similar objects. Many other substances beside sealing wax exhibit this property, and Thales of Miletus, who lived about 600 B.C., mentions amber as one of these. It is probable that amber and jet were the only substances known by the ancients to possess this property, and it was not until the year 1600 A.D. that Dr. Gilbert, physician to Queen Elizabeth, showed by experiment that many other substances are similarly affected by friction. Such substances he called *electrics*, from the Greek word for amber (*ēlektron*), and since his time the name "electricity" has been applied to the agent to which these and other electrical phenomena are due. An electric, when excited so as to be capable of exercising this property of attraction, is said to be *electrified* or *charged with electricity.* The question as to the real nature of this agent, electricity, need not here be discussed, for we shall be chiefly concerned with simple experimental facts; but the student cannot too soon learn that, unlike heat, light, and sound, electricity is not in

itself a form of energy; nor is it a form of matter in the ordinary meaning of the term. In the past, men of science were led to consider electricity as a subtle material fluid, and the nomenclature of the subject is based on the fluid theories then framed. The nomenclature is still, in some measure, retained, but the theories have been abandoned in their original terms to reappear in a new bearing but in a very similar form. Remembering this, the student will be prepared to read of the "flow" of electricity without assuming that any material fluid theory is thereby endorsed, and he will ultimately find that the mode of thought and language acquired is readily adapted to the more modern views of the science.

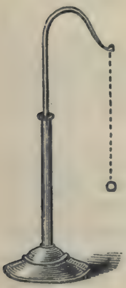


Fig. 1.

2. Attraction of Neutral Objects by Electrified Bodies. This simple phenomenon derives importance from the fact, already mentioned, of its being the starting-point of electrical science. It supplies the simplest method of testing whether a body is electrified or not; and until we are able to employ more satisfactory methods we shall consider this a sufficient test of electrification.

If a rod of sealing wax be lightly rubbed with warm dry flannel or fur, and then held near a few pith balls or other light objects, it will be found to attract them strongly. A rod of ebonite rubbed with fur, or a glass rod or tube rubbed with silk, will be found to exercise a similar attraction. These substances are therefore said to be electrified and, by experimenting in a similar way, it will be found that many substances can be electrified by friction with a suitable rubber. In practice, instead of employing a number of light objects to indicate the attraction of an electrified body, it is more convenient to suspend a single pith ball by a thread from a suitable stand (Fig. 1), and to test for the electrification of any body by bringing it near the suspended ball.

This arrangement is very sensitive, and can be employed

to detect very feeble electrification. It is, in fact, a simple form of *electroscope* or instrument for indicating whether a body is electrified or not. Other convenient forms of this instrument may be readily devised by the student; the earliest form was that employed by Dr. Gilbert, and consisted (Fig. 2) of a stiff straw freely balanced on a needle point.

Let us now consider whether this property of attraction exhibited by *electrified* bodies is exercised generally and on

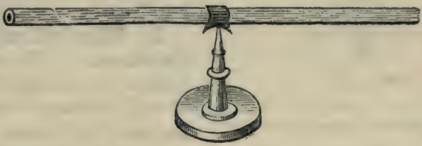


Fig. 2.

all substances, or whether it is confined to very light substances. On consideration, we at once conclude that in all probability the property is general, but that light substances are chosen to exhibit it, because, being light, they are easily put in motion by the attracting force. To verify this then, it is evident that if the body to be attracted be not light it must be suspended or pivoted in some way so as to be free to move. For example, take a lath or rod of wood and suspend it horizontally by a thread, or balance it on the bottom of a small Florence flask. On holding an excited glass tube near one end of the rod it will be seen that the latter is attracted, and rotates so as to approach the tube.

We must further determine whether this attraction is mutual—that is, whether the electrified body, in addition to attracting other bodies, is itself attracted by these bodies. Newton's third law of motion at once tells us that this must be the case, and we can verify the fact by suspending the electrified body (Fig. 3) in a wire stirrup attached to a *silk* thread. When thus suspended it will be found that it is attracted by any body brought near

it, just as the rod of wood mentioned in a previous experiment was attracted by the electrified body.

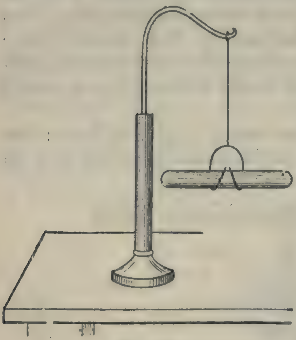


Fig. 3.

So far, then, we see that some substances on being rubbed by others acquire the property of attracting *neutral* bodies or bodies in their normal condition, and that this attraction is general* and mutual.

3. Electrification by Friction. If *any* two bodies be rubbed together it will be found that, in the majority of cases, neither of the bodies will, under ordinary conditions, exhibit the property referred to above. In a few cases, however, it may be found that one of the bodies is distinctly electrified while the other is not; and in one or two cases, it is possible that both bodies may exhibit feeble electrification. If we choose one of the substances showing distinct electrification, and test it by rubbing it with others, we shall find that its power of attracting light substance varies with the substance used as a rubber—that is, the electrification produced on a given substance varies with the rubber producing it, and it is found that if the rubber be exactly similar physically to the substance rubbed, then no electrification whatever is produced. It can also be shown that the electrification is confined to the external surface, for if the body be made in the form of a tube it will be found that the electrification produced on it, under given conditions, is the same as on a solid rod of the same diameter, and is quite independent of the thickness of the walls of the tube.

On experiment it will also be noticed that only that portion of the surface of the body which is subjected to friction becomes electrified; for example, if one end of a

* See Art. 11.

rod of sealing wax be rubbed with flannel, only that end, and no other portion of the surface of the rod, will be excited. This shows that, in the cases here considered, the electrification produced on one portion of the surface does not spread over the whole surface, but appears to be confined to the area on which it is produced.

Again, if the substance be excited so as to exhibit distinct electrification, and then tested from time to time, it will be found that it gradually loses its electrification, and in a few minutes returns to its original *neutral* condition. This loss of electrification will be found to depend on external circumstances; for example, if the air is moist the rate of loss is very rapid, but if dry, as on a bright frosty day, the body may remain electrified for hours. The same fact is emphasised by holding a freshly excited body in a current of steam or dipping it into water; it at once loses its electrification, and returns to the neutral state. These experiments show that when *dry* air is in contact with an electrified surface it tends to prevent loss of electrification, but that the presence of moisture* tends to accelerate the loss. Similar experiments would show that all dry gases tend to prevent loss of electrification, but that liquids behave in various ways; some, such as water, mercury, dilute acids, and solutions of metallic salts, etc., admit of instant loss of electrification, while others, including oils and various organic compounds, more or less completely prevent this loss. Similarly, if the excited body be drawn through the hand it will be found to have almost entirely lost its electrification, thus proving that contact with the hand or person of the operator results in loss of electrification.

The action of solids in this relation is less easily determined by direct experiment. There is evidently no difficulty in securing intimate contact between an electrified surface and a gas or liquid, but with a solid good contact can be obtained at only a few points, and the loss of electrification produced by such contact would depend more upon the action of the intervening air-film than upon that

* Dry aqueous vapour is *itself* an insulator; the loss of electrification resulting from the presence of moisture is due to the *conducting* film of liquid deposited on the surface of the conductor.

of the solid. With one solid, however, the electrified body itself, there is no difficulty, for there is already intimate contact between the electrified portion of the surface and the adjacent portions of the same surface. Hence, it is evident that if the substance of a body be such as to admit of loss of electrification from an electrified surface with which it is in contact, then it will be impossible, without special precautions, to exhibit the electrification of that substance. For if any portion of the surface be electrified by friction it immediately loses its electrification through adjacent portions and the hand of the experimenter, and thus electrification is lost as quickly as it is produced. Similarly, if a body can be electrified, it must evidently be composed of a substance which does not admit of loss of electrification. We are thus able to divide all substances, solid and fluid, into two classes—one class composed of substances which, when placed in contact with an electrified surface, admit of loss of electrification, the other composed of substances which, under similar circumstances, prevent this loss. The substances in the former class are called *conductors*, those in the latter *insulators*.

We are now able to understand why electrification is not always evident when any two substances are rubbed together. It is not because none is produced, but because its exhibition depends upon whether the substances employed are insulators or conductors. If both are conductors, then neither appears to be electrified; if one is an insulator and the other a conductor, then the former becomes electrified, the latter remaining neutral; if both are insulators, then both may exhibit electrification.

That electrification is produced on a conductor may be made evident by *insulating* it—that is, mounting it on a handle or stand of insulating substance, and then rubbing it with some suitable substance. The escape of the electrification is prevented by the insulator, and the conductor becomes electrified, but it will be found that the electrification appears over its whole surface, although only a small portion of that surface may have been directly excited by friction. This is evidently a result to be expected from the nature of the substance, and corresponds to the fact

mentioned above, that the electrification on the surface of an insulator is confined to the area directly subjected to friction.

Thus if *any* two dissimilar bodies be ***first insulated*** and then rubbed together, it may be shown that both are electrified. Further, if two insulated *conductors* be put in contact it can be shown that, without any friction taking place, both become electrified, but the electrification developed is extremely feeble. This indicates that the true cause of electrification in the cases considered above is *contact of dissimilar substances* and not friction; but in the case of insulators, intimate contact at nearly every point on the surface is necessary, for the electrification cannot, as in the case of a conductor, spread over the surface from one or two points of contact. Friction secures this intimate contact, and helps the transfer of electrification along the surface. It is thus evident that the degree of electrification produced in any given case cannot depend on the mechanical work done in friction but rather on the nature of the substances considered.

We may now summarise by stating that the contact of any two dissimilar substances tends to produce electrification of both, and that the degree of electrification produced depends only on the nature of the substances and the conditions of contact.

4. Conductors and Insulators. The general meaning of the terms insulator and conductor has been given in the preceding article. From the explanation there given we know that the contact of a conductor with an electrified body causes loss of electrification, while contact with an insulator results in no such loss. It must, however, be remembered that this is only generally true. The terms conductor and insulator are comparative terms, there being no perfect conducting or insulating substance. An insulator may be considered as a very bad conductor, or a conductor as an extremely bad insulator; and, as might be expected, there are many substances, such as wood, ivory, stone, paper, etc., which occupy an intermediate position, being neither good conductors nor good insulators. Hence, in practice, all substances admit of some loss of electrification

—that is, all substances are conductors,—but if the loss is inappreciable then, for all practical purposes, the substance is an insulator or *non-conductor*; if, however, the loss is readily appreciable by experiment, the substance is classed as a conductor in the usual meaning of the term. Further, the conducting power of any given substance often varies with its physical condition; for example, glass at ordinary temperatures is a typical insulator, but if its temperature be raised, its conducting power increases, and at temperatures just below a red heat it is a fairly good conductor. Similarly the conducting power of the different forms of carbon is very different; the diamond acts as an insulator, while the dense carbon obtained from gas retorts is a good conductor. The properties of insulators and conductors may be illustrated by a great many simple experiments. In 1729, Stephen Gray, who first directed attention to the phenomena of conduction and insulation, found that a cork inserted into the end of an electrified glass tube exhibited attraction for light bodies. Similarly a rod of wood stuck into the cork was found to be electrified, and even a pith-ball pendulum attached to this rod by a long hemp-rope exhibited slight electrification. If, however, the ball be suspended by a *dry* silk thread the ball does not become electrified, showing dry silk to be an insulator, but if the thread be moistened it at once acts as a conductor, and the ball becomes charged.

These experiments are readily repeated, any simple form of electroscope being employed to detect the electrification; and it may be easily shown that electrification is readily conducted from one conductor to another through considerable lengths of conducting substance. For example, if a moist cotton thread or a metal wire several yards long be surported on silk loops and connected at each end with an insulated conductor, it will be found that if one of the conductors be directly electrified the other also becomes electrified, and can be made to affect any electroscopic* arrangement near it.

* In these experiments it is best to employ the gold-leaf electroscope described in Art. 12.

If an insulated electrified conductor be placed in contact with another insulated but non-electrified conductor, it will be found that the former at once transfers a portion of its electrification to the latter; that is, an electrified conductor readily communicates its electrification to any other conductor brought into contact with it. If, however, an electrified insulator be brought into contact with a conductor, little or no transfer of electrification takes place. At the point or points of contact direct transfer may take place, but the portion of the surface from which this transfer takes place is confined to the actual points of contact, and the electrification of the rest of the surface is unaffected.

Similarly, if an electrified conductor be brought into contact with an insulator, transfer of electrification takes place only at the points of contact. These facts emphasise the distinctive properties of insulators and conductors, and correspond to those given in Art. 3 in connection with the electrification of conductors and insulators. This transfer of electrification from conductor to conductor supplies the most convenient test of the conducting power of any given substance. For example, let a conductor be insulated and electrified; if not further interfered with, it will retain its charge until it is gradually dissipated by leakage through the insulator and the surrounding air, but if put in direct contact with the earth it at once loses its charge, thus showing that the earth acts as a conductor, and that after sharing its charge with so large a conductor as the earth, any charged body is practically discharged.

Similarly, if the electrified body be touched with the finger, it instantly loses its electrification by transfer to the conducting body of the experimenter and the other conductors with which he is connected. Under ordinary circumstances, the experimenter communicates with the earth by conducting substances, and therefore in the case considered the electrified body shares its charge with the earth, and all conducting bodies on its surface, and thus has practically no charge left. If before touching the electrified body the experimeter insulates himself from the earth by standing on a stool with glass (insulating) legs, it will be found that although the body loses the greater portion

of its electrification by transfer to the experimenter, both still exhibit feeble electrification.

Hence, if an insulated electrified body be touched with any conducting substance, in direct or indirect contact with the earth, it at once loses its electrification. On the other hand, if touched with an insulator it retains its charge or loses it very gradually, according as the substance tested is a good or bad insulator.

We can now understand why it is necessary to insulate any body which we wish to electrify. If the body is a conductor this is effected by attaching it to a stem or handle of some insulating substance, such as glass or ebonite; if an insulator, any portion of it may be considered as already insulated from the other portions, and may therefore be electrified. The phenomenon of transfer of electrification from conductor to conductor is applied in the use of the *proof plane*. This simple instrument consists of a small disc of metal, or of cardboard covered with tinfoil, attached to an insulating handle, such as an ebonite penholder. It is used for testing the electrification of any body. When placed in contact with the body it receives by transfer a small charge, which may be removed with it, and used as an indicator of the nature and magnitude of the original charge.

It is somewhat difficult to give a correct list of conductors and insulators in the order of their conducting powers, for different specimens of the same substance vary greatly in this respect. The following table, however, gives a general idea of the relative conducting or insulating powers of the substance referred to. Taken as a table of conductors in the order of their conducting power, the list should be read downwards; as a table of insulators arranged in the order of their insulating power, the list should be read upwards.

	Silver.	
	Copper.	
	Other metals.	
	Gas Coke.	
	Charcoal.	
Bad Insulators.	Graphite.	Good Conductors.
	Acids.	
	Metallic Salts.	
	Water.	
	The Body.	

Partial Insulators.	Linen. Cotton. Wood. Stone. Marble. Paper. Ivory.	Partial Conductors.
Good Insulators.	Oils. Porcelain. Wool. Silk. Resin. Sulphur. Guttapercha. Shellac. Sealing Wax. Ebonite. Paraffin. Glass. Dry Air.	Bad Conductors.

5. The Two Electrical States. If an ebonite rod be electrified by means of a fur rubber and held near a few pith balls the latter will, as we know, be attracted by the rod. If, however, we watch the behaviour of any one of the attracted balls we shall find that there is something further to explain. The ball is attracted up to the rod; but directly it touches the latter it seems to cling to it for an instant, and is then cast off to be again attracted as before. It will also be found that under the circumstances of this experiment the rod loses its electrification much more quickly than if it were merely exposed to the air. The pith ball acts as a conductor, and directly it comes in contact with the electrified rod it becomes slightly electrified by transfer of electrification at the point of contact. When it is cast off by the rod it carries this electrification with it, and finally gives it up to the earth through the table or other conducting stand to which it returns. It thus becomes neutral, and is again attracted by the rod, and the process described is repeated. We thus see that the pith balls act the part of carriers, and convey the electrification of the rod to the earth, thus causing it to lose its electrification more rapidly than it otherwise would.

The fact that the ball after becoming electrified is repelled from the rod is, however, still unexplained. If, instead of experimenting with loose pith balls lying on the table, we suspend one by means of a dry silk thread, we shall find, on holding the electrified rod near it, that the ball is at once attracted, but after contact it is quickly repelled; and if the rod be again brought near, the ball, instead of being attracted, is strongly repelled. Similarly, if two pith balls be suspended from the same stand, they will be seen to repel one another after both have become electrified by contact with the ebonite rod. From these experiments we come to the conclusion that bodies charged with the *same* electrification repel one another, and the repulsion of the pith ball from the rod is an instance of this general rule. If the same experiment be repeated with an excited glass rod exactly similar results will be obtained. If, however, the suspended pith ball be charged by contact with an electrified ebonite rod, and an excited glass rod be then brought near, it will be found that the charged ball is strongly attracted instead of being repelled, as it is by the ebonite rod. Similarly, if the pith ball be charged by contact with an electrified glass rod, and an excited ebonite rod be then brought near it, the ball will be seen to be attracted, although repelled by the glass rod. Further, if an ebonite rod be excited and suspended on a wire stirrup, as shown in Fig. 3, it will be found to be distinctly repelled by another similarly excited ebonite rod. Also, if the experiment be repeated with glass rods excited with silk, sticks of sealing wax excited with flannel, or with any two *similar* substances similarly excited, the same result will be obtained; that is, the two similarly excited rods will exert mutual repulsion. If, however, we take a glass rod excited by silk, and an ebonite rod excited by fur, and suspend either in the stirrup, it will be found to be *attracted* by the other. It is not, however, necessary to employ rods of different substances to exhibit attraction instead of repulsion, neither is it necessary to employ rods of the same substance to exhibit repulsion. Two pieces of the *same* substance may, by the use of appropriate rubbers, be electrified to exhibit mutual attraction, and rods of

different substances may be so excited as to exhibit mutual repulsion; for example, glass rubbed with silk is attracted by glass rubbed with fur, and sealing wax rubbed with flannel repels glass rubbed with fur. So far, all this is empirical; but experiment shows that if any two electrified substances attract or repel the same electrified body, then they invariably repel one another, but if one attracts while the other repels the given body, then they invariably attract one another. From these observations it has been inferred that *there are two opposite kinds of electrification, and that bodies similarly electrified repel one another, while bodies oppositely electrified attract one another.* Further, if a given substance be electrified by a given rubber it will be found that, if proper precautions be taken to ensure insulation, the rubber and the substance rubbed *always* exhibit mutual attraction—that is, *they are oppositely electrified.* For example, if a rod of ebonite be rubbed with a piece of fur mounted on a glass or ebonite handle, the rod, when suspended in the wire stirrup, will be found to be attracted by the fur. Hence we see that if any two substances are rubbed together they become oppositely electrified; and the nature of the electrification developed on either depends not only on the nature of its own material, but also on that of the other. Symmer and Du Fay independently discovered and investigated these phenomena, and, in explanation of them, the latter suggested that there might be two kinds of electricity which exercised mutual attraction but self-repulsion. The electricity which appears on glass when rubbed with silk he called *vitreous* electricity, and that which appears on sealing wax and other resinous substances when rubbed with wool or fur he called *resinous* electricity.

These terms are misleading, for they are founded on the erroneous assumption that only one kind of electrification can be produced on a given substance; they have, therefore, been discarded in favour of the terms *positive* and *negative*, introduced by Franklin. A rod of glass excited by a silk rubber is said to be *positively* electrified, and any resinous substance excited by a fur or woollen rubber is said to be

negatively electrified.* This is also expressed by saying that a positively electrified body is *charged* with *positive electricity,* and a negatively electrified body with *negative electricity.*

The student must, however, remember that the use of these terms does not necessarily imply that there are two different and opposite kinds of electricity. They arose out of the various attempts to frame a theory of electrification in accordance with observed facts, and they are still convenient for the purposes of speaking and writing.

Several theories have been developed to explain the phenomena of electrification. Symmer advanced the *two-fluid* theory, which assumed that there were two kinds of material but imponderable fluids corresponding to the two states of electrification. By this theory neutral unelectrified bodies were supposed to contain equal quantities of the two fluids which neutralised each other when combined in equal quantities but which gave rise to two states of electrification when one or other of the fluids was in excess. The phenomena of attraction and repulsion were explained by assuming the fluids to be mutually attractive but self repelling.

Franklin proposed the *one-fluid* theory as a simplification of Symmer's theory. This theory supposes one "electric" fluid to have a normal neutral distribution in all bodies and that the two states of electrification result when this normal distribution is disturbed, so as to produce excess or defect of the normal amount in the electrified body.

These theories have now been abandoned, and it is now beyond doubt that electricity is not a material fluid. Any explanation of the modern theories of electrification is out of place at this stage, but it may be well to state here that electrification may be considered as a phenomenon of the insulating medium surrounding the "charged" or "electrified" body. By this theory the charge itself is a surface effect appearing at the surface of separation of

* That is, it is agreed to call one kind of electrification positive, and the other negative; but there is nothing but general convention to decide which shall be considered positive, and which negative.

the medium and the material of the body, and is associated with the state of strain in the medium. This state of strain can be set up only in an insulating medium, not in a conducting medium; hence the surface of separation between an insulating medium and a charged conductor is the surface of separation between a strained and an unstrained medium, and this surface is the seat of the electrification which is spoken of as the charge on the conductor.

From a number of experiments it is possible to draw up a list such that when any substance in the list is rubbed with another lower down on the list, the former becomes positively, and the latter negatively electrified. The list given below is arranged on this principle, but it must be remembered that the order of the list might be different for *different specimens* of the *same* substances.

Fur	Silk
Wool	The Hand
Ivory	Metals
Wood	Caoutchouc
Shellac	Amber
Resin	Sulphur
Sealing Wax	Ebonite
Glass	Guttapercha
Cotton	Collodion
Paper	

The relative positions of substances standing close together on the list are somewhat uncertain. When substances far apart on the list (for example, fur and ebonite) are rubbed together, each becomes strongly electrified, but when substances close together (for example, sulphur and ebonite) are dealt with, the electrification produced is very feeble, and it is therefore difficult to decide as to its nature for either of the substances.

6. Electrical Attraction and Repulsion. The force of attraction between two oppositely electrified bodies, or the force of repulsion between two similarly electrified bodies, is found to depend on three things—the degree of electrification of each of the bodies, their distance apart, and the nature of the intervening medium. The more strongly electrified the bodies are, the greater is the force of

attraction or repulsion exerted by them. This condition may be more definitely stated by considering an electrified body as charged with electricity, and saying that the magnitude of the force depends upon the quantity of electricity with which each of the electrified bodies is charged—that is, upon what are called their *charges.* This way of dealing with the question evidently involves the necessity for some method of measuring quantity of electricity; but, assuming for the present that this is possible, let us consider *how* the force between two charged bodies varies with the magnitude of their charges. Imagine two small positively charged bodies separated by any definite distance, and suppose the charge on *one* to be doubled, then evidently the force of repulsion between them will also be doubled. Similarly, if the charge on the other body be doubled, then the magnitude of the force is evidently again doubled—that is, by doubling the charge on *each* of the bodies the force is quadrupled. Similarly, if one charge be doubled while the other is trebled, the magnitude of the repelling force will be increased to six times its original value. This result can be expressed generally by saying that *the force exerted between two charged bodies varies as the product of their charges.*

Considering the second condition on which the magnitude of the force depends, it is found *by experiment* (Art. 8) that if the distance between two charged bodies be doubled then the force is reduced to one-fourth of its original value, that is, *the force exerted between two charged bodies varies inversely as the square of the distance between them.* This law is rigorously true only when the charges are supposed to be concentrated at points. This involves a physical impossibility, but the law may be taken to be very approximately true when the dimensions of the charged body are small compared with the distance between them.

Experiment shows that the laws stated above hold for any given medium. That is, in any medium the force exerted between two charged bodies varies directly as the product of their charges, and inversely as the square of

the distance between them; but the magnitude of the force, under given conditions, varies with the nature of the medium. These laws may be concisely expressed by

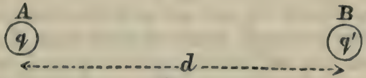


Fig. 4.

means of a formula. Let two small spheres, A and B (Fig. 4), be charged respectively with quantities of electricity denoted by q and q' and let the distance between their centres be denoted by d, then, as explained above, we have—

$$f \propto \frac{qq'}{d^2}$$

where f denotes the force exerted between the charged spheres. That is,

$$f = \frac{1}{K} \frac{qq'}{d^2}$$

where K is a constant depending in magnitude upon the nature of the medium surrounding A and B. For any given medium K has a definite constant value for all values of q, q' and d, but this value is different for different media. For example, if in one medium the force exerted between two charged bodies is twice as great as it is under the same conditions in another medium, then the value of K for the first medium would be half as great as its value for the second. To determine the value of K for any given medium it would evidently be necessary to know the value of f corresponding to known values of q, q' and d. The values of f and d can be determined in known units, but those of q and q' cannot be so determined until the unit in which quantity of electricity is to be measured is defined.

The unit of quantity of electricity adopted in electrostatics assumes that in the relation,

$$f = \frac{1}{K} \frac{qq'}{d^2},$$

K has unit value for air. Hence, **in air** we have $f = qq'/d^2$, and if $q = q'$ this reduces to $f = q^2/d^2$, so that if f and d are each given unit value q also takes unit value. That is, *unit quantity of electricity is that quantity which, when placed in air at unit distance from another equal similar quantity, repels it with unit force.* Hence, if unit quantity of electricity be placed at unit distance from a given charge, the magnitude of the force which it experiences is a measure of the quantity of electricity constituting the given charge. The full significance of this definition should be carefully noted. If two equal quantities of electricity are placed as point charges at unit distance apart *in air* the force of repulsion between them will have a definite value. By increasing or decreasing the quantities of electricity the force of repulsion can be adjusted to unit value, and when this adjustment is exact each quantity of electricity is the unit quantity defined above. In the C. G. S. system of units this unit of quantity of electricity is evidently that quantity which when placed in air at a distance of *one centimetre* from a similar and equal quantity repels it with a force of *one dyne.* This quantity is the C. G. S. electrostatic unit of quantity of electricity, and in the electrostatic system of electrical units based on it $K = 1$ for air.

A charge of q units of positive electricity is denoted by q, or $+q$, and an equal charge of negative electricity is denoted by $-q$. With this notation, the force of repulsion exerted between two *similar* and equal charges is evidently given by $f = q^2 / d^2$ whether the charges be positive or negative, but for two dissimilar equal charges $f = -q^2/d^2$, that is, when f is positive the force is one of repulsion, but when negative it is one of attraction.

It is found by experiment that if any number of conductors whose charges are all of the same sign are made to touch and are then separated, the sum of their charges is unaltered, though the individual charges are usually altered. But if some of the conductors are positively and some negatively charged before contact, they are all similarly

charged after contact, and the sum of their charges after contact is the *algebraic* sum of their charges before contact, positive and negative charges being considered of opposite signs. If two equal conducting spheres are put in contact, their charges after contact are equal.

7. Field of Electrical Force. Consider a charged body surrounded by an unlimited insulating medium. If another similarly charged body be brought near the first it will experience a force of repulsion varying in magnitude with its position, the amounts of the charges, and the nature of the insulating medium. The whole space surrounding the charged body in which this electrical force is exerted is called the *field of force* due to the charge on that body. The *intensity of the field* at any point, or the magnitude of *the electric force at any point in the field*, is determined by the force which a *unit positive charge* would experience if placed at that point. In the case of a point-charge the direction of this force is along the line joining the point considered to the position of the charge.

In any field of force the direction of the force at any point may be indicated by drawing a short line at that point, and in this way the field can be mapped out by a series of continuous lines or curves, each line indicating the direction of the force at the points through which it passes. These lines are called *lines of force*, a line of force being a line such that its direction at any point through which it passes gives the direction of the electric force at that point. Thus, in the field due to a single charged sphere, the lines of force are radial straight lines, diverging from the centre of the sphere, and the direction of the force for all points lying on any given line is therefore the same, but the magnitude of the force diminishes as the square of the distance from the centre of the sphere increases. If, however, the field is due to two or more charged bodies, then to determine the direction of the electric force at any point in the field it is necessary to determine the magnitude and direction of the force at that point due to each charge individually, and then to take the resultant of the several forces thus

determined. The direction and magnitude of this resultant determines the electric force at the given point due to the several charges considered. In this case it is also possible to map out the field by lines of force, such that the resultant direction of the electric force at any point is determined by the direction of the line of force passing through that point. In general, however, the lines will not be straight but curved, showing that the direction of the force varies from point to point along any given line, the direction at any point being given by the tangent to the line at that point. Hence, a line of force may be defined generally as a curve such that the direction of the electric force at any point through which it passes is given by the tangent to the curve at that point. The figures given below show the lines of force drawn in this way for three simple cases. Fig. 5. gives the lines of force for the simple field round a positively charged sphere. The direction of the lines is determined conventionally by the direction in which a positive charge would be urged and is therefore, in this case, outwards from the positively charged sphere. Fig. 6 shows the lines of force in the field round two equal similar charges and Fig. 7 gives the field round two equal and opposite charges. By a conventional arrangement the lines of force may be made to indicate, not only the direction, but also the magnitude of the electric force at any point in the field. Theoretically,

Fig. 5.

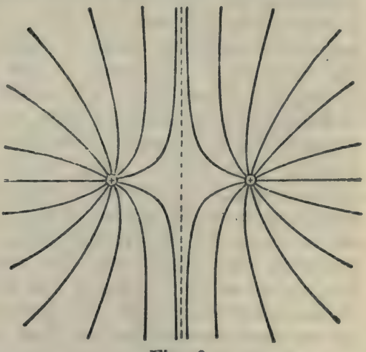


Fig. 6.

the number of lines that can be drawn in any field is infinite, for at all points in the field the electric force has a definite direction. It is, however, agreed to indicate the

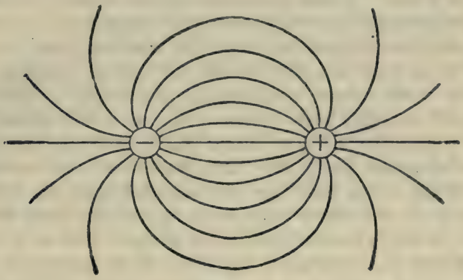


Fig. 7.

intensity of the field by the number of lines drawn according to the following convention. Let the intensity at any point of a given field be denoted by F, then the number of lines of force per unit area at that point crossing a plane at right angles to the line of force passing through the point shall also be denoted by F—that is, the number of lines of force per unit of area at the point is a measure of the intensity of the field at the point considered. Hence, when we speak of the number of lines of force passing through a given area, we refer to the *flow of force* across that area.

From what has been said of lines of force it will be evident that no two lines of force can intersect, for intersection would imply that at the point of intersection there are two directions for the resultant force and this is impossible. If then we imagine a small closed curve to be drawn on the surface of a charged body, and lines of force to be drawn through every point of this curve out into the medium, these lines will form a tubular surface enclosing a portion of the medium. The tube thus formed has been called a tube of force. If we further suppose the whole

surface of the charged body to be marked out by a network of closed curves, and a tube of force based on each mesh of the network, then the whole medium in which the field of force exists will be divided up into contiguous tubes of force. These tubes are not endless. In this case they are assumed to start from the surface of the charged body and they extend out into the medium, and the outer end of each tube is found at the surface of contact of the medium with some conducting substance. Thus if a positively charged body be hung up by a silk thread in an empty room, the tubes of force starting from the surface of the charged body are terminated by the conducting walls of the room, and the medium between the body and the walls of the room may be mapped out into tubes of force. It will also be found that at each end of a tube of force where the insulating medium is in contact with the surface of a conductor electrification exists. The charge or quantity of electricity at one end of a tube is equal and opposite to the charge at the other end. This is another way of saying that the medium in the tube is in a state of strain, and the surface effect associated with this strain, at each end of the tube, is equal in magnitude, but opposite in sense. The energy of the electrification appearing at each end of the tube lies in the strained medium which fills the tube. The nature of the strain in the medium is such as would be produced by a tension along the length of the tube, and a pressure at right angles to its length. Faraday expressed this by saying that the tubes of force tend to contract and to repel each other, that is the strain in the medium is such as would be produced by each tube tending to contract, and at the same time to get thicker. It will be evident from this that corresponding to the charge or electrification on any body there is always an equal and opposite charge on the surface of the conductor or conductors at which the tubes of force coming from the surface of the charged body terminate. The force of attraction or repulsion between charged bodies is due to this state of stress in the intervening medium. If it be remembered in studying Figs. 6 and 7, that the tubes of force, that is, the portions of the medium between the

lines of force tend to contract and repel each other, it will be evident why attraction obtains in the case of Fig. 7, and repulsion in the case of Fig. 6.

If the surface of a charged body be mapped out in such a way that the area from which each tube starts possesses unit quantity of electricity, or unit charge, then the tubes into which the medium is divided are called **unit tubes.** The number of unit tubes starting from any charged body will thus be numerically the same as the measure of its charge. The direction of any tube is by convention the same as a line of force, that is, a tube of force is said to start from a positively electrified surface and to terminate at a negatively electrified surface. It is also evident from the convention of Art. 7 that the electric force at any point in the length of a very narrow tube of force varies inversely as the area of cross section of the tube at that point,—that is, as the tube widens the electric force decreases.

This conception of lines and tubes of force, due to Faraday, is very helpful in the study of electrification. It helps to associate the phenomena with the state of the insulating medium, and to realise with some definiteness the nature of the strain in the medium. It gives also a fuller insight into the distribution of positive and negative charges in any particular case of electrification, and emphasises the fact that the existence of a charge on any conductor implies the existence of a complementary equal and opposite charge on some other conductor or conductors separated from the first by the insulating medium in which the field of force due to the separation exists.

8. The Torsion Balance. One of the earliest attempts at quantitative experiment was that made by Coulomb, by means of the Torsion Balance, which he invented for the purpose of comparing the forces of repulsion exerted between two similarly charged bodies under different conditions of distance and charge. He also employed it to make corresponding magnetic measurements, as described under that head. The instrument (Fig. 8) consists essentially of a fine silver wire, *w*, suspended within a glass case from a brass head, B, and carrying at its lower extremity

a light lever, *ab*, made of shellac, or some light insulating substance. This lever moves in a horizontal plane, and is furnished at one extremity, *b*, with a gilt pith ball. The brass suspension head, B, fits into a brass tube, in which it can be easily rotated, and carries a pointer which moves over a brass disc, D, on which is cut a circular scale divided into degrees. This arrangement for the suspension and control of the wire is called the *Torsion head.* On that portion of the glass case which encloses the lever a scale, *ss*, is etched round its circumference at the level of the horizontal axis of the lever, and through an aperture in the cover a second gilt pith-ball, *c*, attached to the end of a glass rod, *g*, can be introduced in the position shown in the figure. To absorb moisture and thus improve the insulation a small vessel, V, containing calcium chloride, or pumice stone soaked in sulphuric acid, is placed in the bottom of the case. The principle of the action of the instrument consists in balancing the force of repulsion between the two charged balls, *b* and *c*, against the torsion of the wire. The torsion head is adjusted until the balls *b* and *c*, are lightly in contact. The pointer of the torsion head should then be at the zero of its scale, and the ball *b* should be opposite the zero of the scale *ss*. The ball *c* is now removed and charged; on being replaced it shares its charge with *b*, and mutual repulsion ensues. As a result, *b* is repelled until the moment of the force due to the torsion on the wire balances that due to the force of repulsion between the balls. As the instrument is used to make comparative measurements of electrical force, it is evidently essential that the law connecting the angle of torsion with

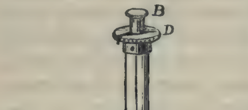


Fig. 8.

the force producing it should be known. The law is known from experiment, and it is found that the angle of torsion is directly proportional to the couple producing it; that is, for example, the couple necessary to twist a given wire through 20° is twice as great as that necessary to twist it through 10°. Hence, if in one experiment the force of repulsion between the charged balls is sufficient to repel *b* through 10°, and in a second experiment it is sufficient to repel it through 20°, then the force due to torsion is twice as great in the second case as in the first. But before we can compare the charges on the balls in this way, we must eliminate the effect due to the change of distance between the balls. This can be done by adjusting the torsion head so that the distance between the balls shall be the same in each experiment. Thus in the example given above the distance between the balls in the second experiment is twice as great as in the first experiment, but by turning the torsion head so as to force *b* nearer to *c*, the ball *b* may be brought back to its position in the first experiment at 10° from *c*. Let it be necessary in order to effect this adjustment to turn the torsion head through 70°. The twist on the *wire* is now 70° due to the torsion head above, and 10° due to the deflection of the lever below, that is 80° in all. In the first experiment the torsion head was untouched, and the twist on the wire was due only to the 10° deflection of the lever. Hence at the *same* distance apart the force of repulsion between the balls is eight times as great in the second case as in the first—that is, the charge on each ball is $\sqrt{8}$, or $2\sqrt{2}$ times* as great in the second case. Similarly from the data of the second experiment the law of inverse squares can be verified. Assuming what is approximately true only for small angles, that the linear distance between the balls is proportional to their angular distance, we see that by halving the distance between the balls, the force of repulsion is increased to four times its original value. For when the deflection is

* From the formula, $f = \frac{q^2}{d^2}$, f is evidently proportional to q^2, that is, q is proportional to $\sqrt{f}$, d being constant. The balls *b* and *c* are here supposed to possess equal charges, each denoted by q.

20° the twist on the wire is also 20°, but when the deflection is reduced to 10°, the twist on the wire has to be increased to 80°. Similarly in one of Coulomb's experiments the angle of deflection after charging the balls was found to be 36°, the corresponding twist on the wire being also 36°. To reduce this deflection to 18° it was found necessary to turn the torsion head through 126°, thus producing a twist of 144° (126°+18°) on the wire. That is, by halving the distance between the charges, the force exerted between them was increased fourfold, as shown by the increase of the balancing twist on the wire from 36° to 144°. Coulomb also investigated the force of attraction between two oppositely charged balls, by comparing the relative amounts of torsion required to keep them at different distances apart.

The method of calculation adopted above gives approximately correct results only when the angle of *deflection** is small— say less than 10°. For large angles, a more correct method (see Ex. I. 5) should be employed; but owing to the difficulty of ensuring good insulation, the disturbing effect of the size of the balls, and various other defects, the experimental results obtained with the Torsion Balance are very rough, and it is therefore of no practical value to adopt rigorously exact mathematical methods. Further, owing to the advance of electrical science, the methods of measurement have greatly changed, and the Torsion Balance is now but seldom used and is of little more than historical interest.

9. Energy of Electrification. An electrified body is capable of attracting and repelling other electrified bodies, and is thus capable of doing work. This implies that the electrification of any body is associated with a gain of energy, and hence before a body can be electrified work must be done. The mechanical work done during friction does not furnish the energy of electrification, for, as we shall see later, it has no direct bearing on the quantity of electricity produced. Electricity itself, whatever be its

* The law of torsion is true for all angles, large or small, within the elastic limits.

nature, is not energy. Hence we have to consider how this energy is produced and in what it consists. It has already been stated that when electrification is produced by the contact or friction of two dissimilar substances, these substances become oppositely electrified. Hence, during contact, the substances attract one another, and the work that has to be done against this force of attraction before the substances can be separated is the equivalent of their energy of electrification, and as has been stated (Art. 7) this energy exists in the medium as energy of strain.

10. CALCULATIONS.

THE force exerted in air between two small bodies having charges q and q', and separated by a distance d, is given by $f = \frac{qq'}{d^2}$. 1. Art. 6. If q and q' are expressed in terms of the unit defined in Art. 6, and d in centimetres, then f is given in *dynes*. The direction of the force is along the line joining the two bodies.

The intensity at any point of the field due to any charge or system of charges is determined in magnitude and direction by the force which a unit positive charge would experience if placed at that point. Thus at a point at a distance r from a charge q the intensity of field due to that charge is given by—

$$\frac{q \times 1}{r^2} \text{ or } \frac{q}{r^2}.$$

EXAMPLES I.

1. Two small spheres are charged respectively with 6 units of positive and 4 units of negative electricity. Find the force between them when they are placed 12 cms. apart.

Here $q = 6$ units of quantity

$q' = -4$ „ „ „

$d = 12$ cms.

$$\therefore \ f = \frac{-24}{(12)^2} = -\frac{1}{6} \textit{ dyne.}$$

That is, either sphere *attracts* the other along the line joining them with a force of $\frac{1}{6}$ *dyne.*

2. A small brass ball is charged with 12 units of positive electricity, and is then placed in contact with an equal and similar brass ball. Find the force exerted between the balls when they have been separated by 9 centimetres.

On contact the two balls, *being equal*, will share the charge of 12 units equally between them, and will therefore each have a charge of 6 units.

Hence, we have

$$f = \frac{36}{(9)^2} = \frac{4}{9} \textit{ dyne.}$$

That is, either ball repels the other along the line joining them with a force of $\frac{4}{9}$ *dyne.*

3. A brass ball is charged with 100 units of positive electricity. Find the intensity of the field due to this charge at a distance of 50 cms. from the ball.

The intensity of the field at the point considered is measured by the force which a positive unit of electricity would experience if placed at that point. Here, therefore, we have

$$f = \frac{100 \times 1}{50^2} = \frac{1}{25} \textit{ dyne.}$$

That is, the intensity of the field at a point 50 cms. from the charge is $\frac{1}{25}$ *dyne*, in a direction away from the charge.

4. In an experiment with the torsion balance the lever was deflected through 10°. Find the torsion necessary to reduce this deflection to 5°, supposing the charges on the balls to remain constant.

If the deflection be reduced from 10° to 5°, that is, halved, the force exerted between the charged balls will be quadrupled, and therefore the torsion on the wire must be quadrupled. Hence if the *torsion head* be turned through T° before the deflection is reduced to 5°, the twist on the wire will be $(T+5)°$, and we must have

$$T+5 = 4 \times 10,$$
$$\text{or } T = 40 - 5 = 35$$

5. In an experiment with the torsion balance the lever is deflected through 90°. Find the torsion necessary to reduce this deflection to 60°.

[In this example the deflections are too large to admit of the application of the method of the preceding example, and the following more exact method must be adopted.]

Let the balls each be supposed to carry a charge q. Then from Fig. 9 we have

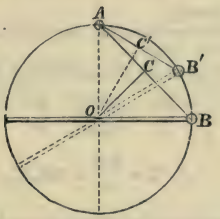


Fig. 9.

$$f_1 = \frac{q^2}{(AB)^2}$$

where f_1 denotes the force of repulsion between the balls when the

deflection is 90°. The moment of this force round $O = f_1 \times OC = q^2 \frac{OC}{(AB)^2}$.

Similarly, if f_2 denote the force exerted between the balls when the deflection is 60°, we have the moment of f_2 round $O = q^2 \frac{OC'}{(AB')^2}$.

Now these moments are in each case balanced by the moment due to the torsion on the wire, and as the moment of torsion is proportional to the angle of torsion we have—

$$90 : T :: q^2 \frac{OC}{(AB)^2} : q^2 \frac{OC'}{(AB')^2},$$

where T denotes the twist *on the wire* when the deflection is 60°.

$$\therefore\ 90 : T :: \frac{OC}{(AB)^2} : \frac{OC'}{(AB')^2},$$

or

$$\frac{90}{T} = \frac{OC}{OC'} \times \left(\frac{AB'}{AB}\right)^2.$$

And from the geometry of the figure—

$$\frac{OC}{OC'} = \frac{\sqrt{2}}{\sqrt{3}} \text{ and } \frac{AB'}{AB} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

$$\therefore \frac{OC}{OC'} \cdot \left(\frac{AB'}{AB}\right)^2 = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{1}{2} = \frac{1}{\sqrt{6}}.$$

$$\therefore\ \frac{90}{T} = \frac{1}{\sqrt{6}} \text{ or } T = 90\sqrt{6}.$$

That is, T = 220.5° nearly.

To produce this twist *on the wire*, the torsion head must be turned through 220.5° − 60° = 160.5°.

[If this question be worked by the method of Ex. 4 we have

$$\frac{90}{T} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

$$\therefore\ T = \frac{810}{4} = 202.5°.$$

That is, the torsion head must be turned through 202.5° − 60° = 142.5°].

6. A small brass sphere is charged with 30 units of positive electricity, and is then made to touch another equal sphere having a charge of 10 units of negative electricity. Find the force exerted between the charged spheres when they are separated by a distance of 10 cms.

7. Two charges of $+10$ and -10 units are placed at two corners of an equilateral triangle of 10 cms. side. Find the magnitude and direction of the resultant force acting on a charge of $+10$ units placed at the third corner of the triangle.

8. Three charges of $+10$, $+20$, and -10 units are placed at three corners of a square of 1 metre side. Find the intensity of the field at the fourth corner due to these three charges.

9. Two equal positive charges of 10 units each are placed at the extremities of the diameter of a circle of 5 cms. radius. Find the intensity of the field due to these charges at the middle point of the semicircular arc joining the charges.

10. Assuming the balls on the lever and rod of a torsion balance to be equal, compare the charges given to the latter when the deflections produced are respectively 10° and 15°.

11. In an experiment with the torsion balance a deflection of 5° is produced. Explain in what direction and how far the torsion head must be turned to increase this deflection to 15°.

12. Six equal charges are placed at the corners of the base of a hexagonal pyramid. If the slant edge of the pyramid is equal to the diagonal of its base, find the intensity of the field at the apex due to the charges at the base.

CHAPTER II.

ELECTROSTATIC INDUCTION.

11. Electrostatic Induction. If a conductor be placed in an electric field the state of strain in the medium in which the field exists does not extend into the substance of the conductor, and the surface of the medium in contact with the conductor is therefore, as explained in Art. 7, a seat of electrification.

The nature of this electrification is evident from a study of the lines of force diagram of the case. Let C, Fig. 10,

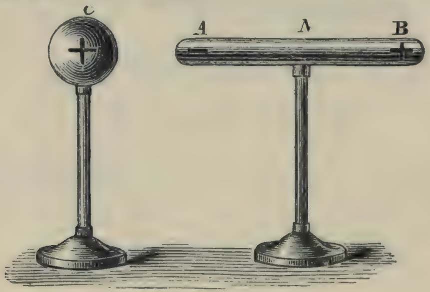


Fig. 10.

denote a positively charged sphere. If surrounded by a uniform medium the lines of force are as shown in Fig. 5. If, however, a conductor be placed as at A B, then the distribution of strain in the field is indicated by Fig. 11. The strain in the space occupied by A B is annihilated, and in the rest of the medium the redistribution of the strain

gives the field here indicated. The nature of the electrification on A B is also readily deduced from the diagram. Where tubes of forces starting from C terminate on the conductor at the end marked A, there will be negative electrification. Where new tubes of force start from the end B of the conductor and spread out into the medium to terminate on an infinitely distant conductor* there will be positive electrification. Also, since the conductor A B is initially in a neutral state the positive electrification at B must be equal in magnitude to the negative electrification at A, that is, the number of unit tubes terminating at A will be equal to the number starting from B. It is also evident from the figure that the state of strain in the medium between A B and C causes attraction between C and the conductor A B. These phenomena of the electrification of A B and the attraction between C and A B constitute what is known as **induction.**

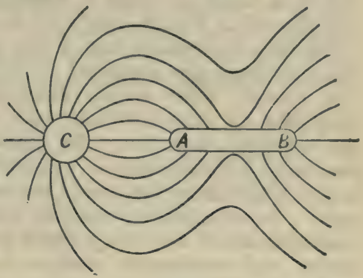


Fig. 11.

If a piece of non-conducting or insulating material is placed in the electric field at A B the state of strain in the material differs from that in the surrounding medium and the induction exhibited depends therefore on the nature of the material.

It will be seen that induction considered in this way is really a question of the redistribution of strain and strain energy in the medium round C consequent upon the annihilation of strain in the space occupied by the conductor A B. The result of this redistribution is that negative electrification is developed on A B at A, and positive electrification

* The diagram is drawn on the assumption that all conductors except A B are at an infinite distance from C.

at B. The discussion of induction problems on the basis of strain redistribution is too complicated for general use and it is usual to adopt a conventional method of treatment based on the old two fluid theory.

By this convention two kinds of electricity—positive and negative—are supposed to exist in every substance. The electric force in an electric field urges positive electricity in one direction and negative electricity in the opposite direction. In insulating media the displacement of electricity is opposed by the stress set up in the medium by the strain accompanying the displacement. In conducting material no strain is produced by displacement of electricity, there is therefore no opposing stress, and continuous electric displacement or flow of electricity is possible.

It follows from this that, in an electric field due to a positive charge, positive electricity may be said to be repelled from the charge and negative electricity attracted towards it.

Hence, leaving the question of strain and stress in the medium out of consideration it may be said in simple terms that in the case considered, the positive electricity on C acts *inductively* on A B, attracting negative electricity to the end A, and repelling positive to the end B. As before, however, the student must be warned that this is merely a convenient way of expressing the facts ; it is a somewhat misleading way, for the function of the medium, the most important factor of the phenomenon, is liable to be lost sight of, but for the purposes of speaking and writing it is concise and convenient.

Let us again consider the inductive action of a charged conductor C on two small conductors A, B (Fig. 12), connected by a wire or thread of conducting substance. A and B, together with the connecting wire, form one conductor, and therefore, as explained above, under the inductive action of the charge on C, negative electricity is attracted to A and positive repelled to B. While this distribution of electricity obtains, the connection between A and B may be cut by an insulated pair of scissors, and the conductors A and B will retain their *induced charges*.

Similarly, if B be removed and A connected by a wire to the earth, negative electricity is attracted to A, and positive repelled to earth, and, when the wire is removed A is left negatively charged. The same result may be obtained by touching A with the finger instead of connecting it to earth by means of a wire—the positive electricity is then

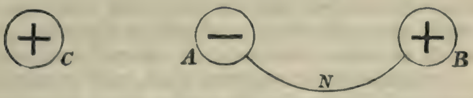


Fig. 12.

repelled through the body to the earth, and, on removing the finger, A is left negatively charged as before. Hence, when a body is charged in this way—*by induction*—the sign of its charge is opposite to that of the inducing charge. If, in the case illustrated in Fig. 12, the conductor B be touched with the finger for an instant, the positive charge repelled into it by C is further repelled to the earth. Hence, on removing the finger, B is left neutral, and A negatively charged as at first. If, however, C be now removed, this negative charge on A is freed from the attraction of the charge on C; that is, it ceases to be "bound," and is free to distribute itself over the whole conductor formed by A, B, and the connecting wire. Further, if instead of touching B the finger be placed upon A, the positive charge originally repelled to B will now be repelled through the body of the experimenter to the earth as the most distant conductor, and the negative charge in A, being bound by the attraction of the charge in C, will not be disturbed, so that on removing the finger the result is similar to that obtained on touching B. Hence, we may say, generally, that if an insulated conductor, or system of connected conductors, be subjected to induction, electrical separation results, and a charge of sign opposite to that of the inducing charge is attracted by the latter to the nearer portions of the system, while a charge of the same sign is repelled to the more distant portions. Also, if *any* point of the system

be connected to earth, there is a redistribution. A repelled charge, larger in general than before, passes to earth and an attracted or bound charge numerically equal to the repelled one is left. If the earth connection is broken and the inducing charge then removed, the bound charge becomes "free" and arranges itself differently.

If an insulated conductor be placed in an electric field due to several charges, each of the latter acts inductively on it, and the total effect produced is the sum of these individual actions. Similarly, if several insulated conductors be placed in an electrical field, each is acted on inductively, not only by the charges to which the field is due, but also by the induced charges on the other conductors; that is the distribution of any one conductor is determined by the distribution—initial and induced—on all other conductors in the field.

12. The Gold-leaf Electroscope. A simple form of this instrument is shown in Fig. 13. It consists essentially of two rectangular strips of thin gold leaf (*g*, *g*) attached by their upper edges to a short T-piece of brass, which forms the lower extremity of a thin brass rod, *b*. This rod passes through the stopper S of the glass case G, and carries a small circular brass disc, D, at its upper extremity. This disc is sometimes called the *cap* of the instrument. The glass case which is fixed on the base board of the instrument serves to protect the leaves and also to insulate the brass rod *b*. In order to improve this insulation the stopper S is stuffed with shellac or some good insulating substance, and the outer surface of the glass is lightly varnished with good shellac varnish. This prevents the formation of a film of moisture on the outer surface of the case, and the interior is kept dry by means of

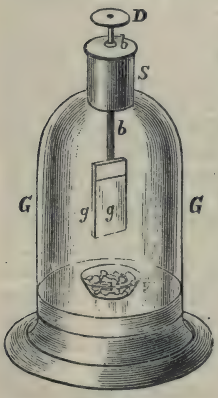


Fig. 13.

calcium chloride, or pumice stone soaked in sulphuric acid, contained in a small vessel, V. The principle of action of the instrument depends upon the fact that two similarly charged bodies repel one another. If a charge is given to the cap of the electroscope it communicates a portion to the gold leaves, which thus become similarly charged and repel each other. The leaves being light this repulsion is indicated by a divergence of the leaves. For example, if the brass cap be lightly beaten with a piece of fur, the electrification of the brass is at once indicated by a slight but gradually increasing divergence of the leaves.

By means of this convenient instrument, the phenomena described in the preceding article are readily illustrated. Thus, if a charged ebonite rod be brought near the cap of an electroscope, the leaves at once diverge, showing that induction is taking place, the negative charge on the rod being supposed to attract positive electricity to the cap and to repel negative to the leaves. If, while the leaves still diverge, the cap be touched with the finger, they at once collapse, showing that the negative charge originally repelled to the leaves now passes to earth, the attracted positive remaining "bound" on the cap. If the charged rod be now removed, this bound charge spreads to the leaves, and they will be seen to again diverge, but this time with positive instead of negative electricity. The electroscope is now *charged inductively* with a charge of opposite sign to that on the ebonite rod. If it be required to charge the leaves with a charge of the same sign, it cannot be readily done by bringing the charged rod into direct contact with the cap; for the rod, being an insulator, will not readily communicate its charge to the cap, and the only effect of the contact is to intensify the induction by the close proximity of the inducing charge.* One method of obtaining the desired result is to charge any insulated conductor by the inductive action of the rod, and then to charge the electroscope by the induction of the charge on this conductor.

* If this experiment be tried, it will be found that the electroscope becomes slightly charged by induction, owing to the escape of some of the repelled charge from the thin edges of the gold leaves. The rod must not be too strongly excited, or the leaves may be torn off.

The nature of the charge on any body can be readily determined by means of the electroscope. Suppose the instrument to be positively charged, say by the induction of an excited ebonite rod. If a negatively charged body—for example, the ebonite rod just used—be now brought near the electroscope, negative electricity is repelled to the leaves, and partially neutralises their positive charge. As a result, the leaves partially collapse, and if the charged body be brought nearer the cap, they may be made to totally collapse, and finally again diverge, with excess of the repelled negative charge. Similarly, if a positively charged body, such as an excited glass rod, be brought near, a positive charge is repelled to the leaves and increases the positive charge they already possess, that is, the divergence of the leaves is increased. Hence, if an electroscope be charged with electricity of a given sign, the approach of a similarly charged body causes increased divergence of the leaves, but the approach of an oppositely charged body causes first a diminished divergence, then total collapse and finally a re-divergence of the leaves. Unless this second divergence takes place the latter test is not conclusive, for if a *neutral conducting* body be brought near a charged electroscope, the leaves show a very sensible decrease of divergence; for example, if the hand be held over the cap of a charged electroscope, the divergent leaves at once partially collapse. This is due to the fact that the neutral body is acted on inductively by the charge in the electroscope, and the attraction of the nearer induced charge withdraws some of the inducing charge from the leaves to the cap of the instrument, and thus causes a partial collapse. This effect of a neutral body will evidently be more or less marked, according as the body is a good or bad conductor; for example, with a perfect insulator no such effect would be observed, for no electrical displacement would result from the inductive action of the charge in the electroscope, but the greater the conductivity of the body the more complete is this displacement, and the more marked is the effect on the electroscope. Hence, this effect indicates a rough method of testing the relative conductivity of different substances, but it is essential that the test for each

substance be made under exactly similar conditions. The relative conductivity of different substances may also be exhibited by means of the following experiment, illustrative of induction. A metal can, C (Fig. 14), is insulated from the earth by means of a block of paraffin, I, and connected by a wire, *ab*, with the cap of an electroscope, E. On bringing a charged body near C, the leaves of the electroscope at once diverge, showing that the repelled charge passes from C, through *ab*, to E. By varying the material of the con-

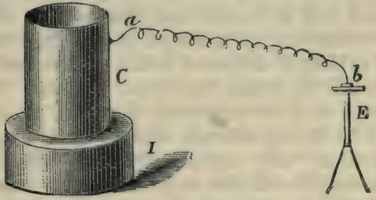


Fig. 14.

nection *ab*, it will be found that the divergence of the leaves varies with the conductivity of the material; for example, with metal wire the divergence is immediate and distinct, with *dry* silk thread no divergence is obtained, with dry cotton thread the effect is not quite so marked as with wire, with a wet thread (silk or cotton) the divergence is quite distinct, and with no connection other than the intervening air no divergence is obtained. These results show that air and dry silk are insulators, metals and water good conductors, and cotton a partial conductor. Perhaps the readiest test of the conductivity of any given substance is afforded by the result of touching the cap of a charged electroscope with the substance in question. If that substance be a conductor, the instrument is at once discharged and the leaves collapse; if a perfect insulator, then no effect is produced; but if a partial conductor (or insulator), then the electroscope is gradually discharged, and the leaves slowly collapse, the relative rapidity of collapse giving a rough indication of the relative conductivity of the substances.

Being now able by means of the electroscope to determine the nature of any given charge, we can proceed to verify the statement made in the preceding article with regard to the distribution of electricity on a conductor subjected to the inductive action of a neighbouring charge. For example, in the case illustrated by Fig. 10 a negative charge is supposed to be collected at the end A of the conductor A B and a positive charge repelled to the end B. If a *proof-plane* (Art. 4) be placed in contact with A B at A it should on removal possess a negative charge; on testing it at the electroscope it will be found that this is so. Similarly, if the charge on the end B be tested it will be found to be positive. Further, if different points on the surface of AB be tested, it will be found that about one half of the conductor, from A to N, is negatively charged, while the other half, from N to B, is positively charged, but that the degree of electrification very rapidly decreases from the ends towards the middle of the conductor. At the boundary line between the two charges the electrification is *nil*, and this line is therefore sometimes called the *neutral line*. Again, if A B be touched at *any* point, C still remaining in position, testing as before will show that the distribution of the negative charge is much the same as before, but that the surface originally charged with positive electricity is now neutral. If, however, C be removed, then the whole surface of A B will be found to be negatively electified.

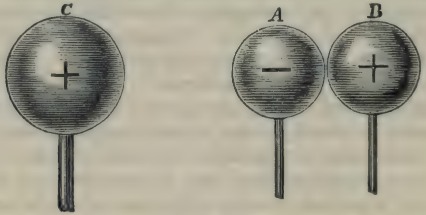


Fig. 15.

The same results are more conveniently illustrated by the following experiments. Two insulated brass spheres, A and B (Fig. 15), are placed in contact, and a positively charged

conductor, C, is brought near them. Under the influence of this charge a negative charge is induced in A and a positive charge in B. If the spheres be now separated, each retains its induced charge, and, on testing by the electroscope, the nature of the charges will be found to be as first stated. If either conductor is touched before separation is effected, A will still be found to be negatively charged, but B will be neutral. If, after touching either conductor, C is removed before separation, then both A and B will be found to be negatively charged.

13. Field of Induction. The electrical field (Art. 7) surrounding any charged body is sometimes called the field of induction due to the charge on the body. The lines of force in the field indicate at any point the direction in which the electric force due to the charge tends to effect electrical displacement; that is, they indicate the direction of the inductive action, and for this reason are sometimes called *lines of induction.* In the same way we may speak of tubes of induction instead of tubes of force. It must be understood, however, that when an electrical field is considered as a field of force, the *stress* in the field is under consideration, but when considered as a field of induction the *strain* in the field is dealt with. In ordinary media, such as air, stress and strain are in the same direction, that is, force and induction have the same direction and tubes of force therefore coincide with tubes of induction. Strictly speaking, however, tubes of force have to do only with the electric force, and the associated stress in the field, while tubes of induction have to do with the strain, and therefore with the electrical charges in the field. Thus the property referred to above that a tube of force in air has equal and opposite charges at its extremities is properly the property of a tube of induction, and is true for tubes of force only because in air they coincide with tubes of induction.

Let us imagine a charged insulated conductor to be placed on a table in the middle of a room. Each surrounding object is subjected to the inductive action of this charge; if it be a conductor, then electrical displacement takes place without producing any strain, and

therefore without calling up any opposing stress, in the substance; if an insulator, then slight electrical displacement takes place, until the stress produced by the strain thus induced balances the displacing electric force, and equilibrium obtains. As results, therefore, we find, induced *charges* on the conductors and a *strain* in the insulators. If a conductor be insulated, then the repelled charge on the side remote from the inducing charge transmits the inductive action into the space beyond; if earth-connected, then the repelled charge goes to earth, *and the inductive action does not travel further.* Thus, in the case considered, the field of induction is bounded partly by the earth-connected walls of the room, and partly by the earth-connected objects which intercept the inductive action before it reaches the walls.*

If there be any gaps in these walls, or if any portion of them be made of insulating material, or of insulated conducting material, then these areas do not act as portions of the bounding surface, and the space beyond them may, if the gaps are wide, become part of the field of induction. Thus, all conducting objects within the room possess induced charges, and the air and all other insulating substances in the field are in a state of strain. In fact the field may be briefly defined as all portions of space in which a state of strain associated with the inducing charge is found, that is, all portions of the space that lies between the inducing charge and its complementary charge of opposite sign. This definition evidently excludes the interior of all conducting bodies in the field, for a strain cannot be produced in such bodies; and this will be the case whether the body be solid or hollow, for in the latter case, whatever the cavity may contain, the strain cannot be transmitted to it through the substance of the surrounding conductor.

Further, if a charged body be surrounded by an earth-connected conductor made of wire gauze, this conductor will limit the field of induction as effectually as a completely continuous conductor, for the tubes of force will all tend

* The portions of those walls, which are, as it were in the shadow of these objects, possess no induced charge.

to terminate on the oppositely charged conducting surface and not on the gaps; that is, the inductive action will be concentrated on this surface, and no strain will be transmitted through the gaps to the external medium. This result is evident if we remember that a line of force may be defined as the direction along which a positive unit of electricity will travel, for, if we imagine a positive unit to be repelled by the inducing charge along any one of the lines of force in the field, we can understand that, as it approaches the negative charge on the inner side of the conductor, it will experience attraction tending to draw it up to this surface and preventing it from passing out through any of the neighbouring gaps. This, of course, is true only when the wires of the gauze are sufficiently closely set. Similarly, the space within a conducting surface, perforated with a number of small gaps, is effectually protected from inductive action from without.

Some of these facts are readily illustrated by simple experiments. For example, if a plate of brass, held in the hand, be placed between a charged body and an electroscope, the latter will exhibit no signs of inductive action, showing that the earth-connected brass plate limits the field in the direction of the electroscope. If, however, the plate be insulated, then its interposition will not interfere with the inductive action of the charge on the electroscope. Similarly, a plate of glass will have little effect, unless its thickness is considerable compared with the distance between the charged body and the electroscope. In this case the result will be modified by the change of medium; that is, if induction takes place more easily through glass than through air, then the effect will be increased by the interposition of the glass. In this connection it is well to notice why the inductive effect of a charged conductor on any other conductor increases as the distance between the two diminishes. It is not because induction does not take place as well through a great thickness of the intervening medium as through a small thickness, but because the portion of the field subtending the surface of the conductor subjected to induction rapidly increases as this distance decreases.

Moreover, as in the case of the non-continuous conductor considered above, some of the lines of force which would otherwise pass outside the conductor are gathered up so as to terminate on the surface bearing the attracted induced charge, and this effect also increases as the distance between the inducing and induced charges diminishes. A study of Fig. 11 will make this plain. The displacement of the tubes of force shown between C and A B will become less and less marked the greater the distance between the two bodies, and when the distance is very great the distribution of the tubes of force or tubes of induction practically becomes that shown in Fig. 5. It should be noticed that as A B approaches C it not only intercepts a greater number of tubes, but more and more of the external tubes bend round so as to terminate at the end A of A B.

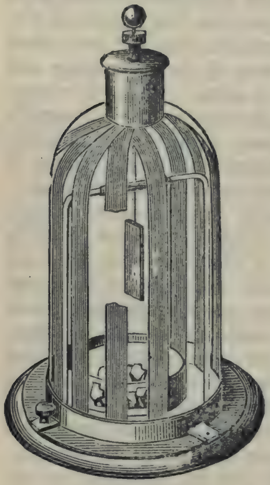


Fig. 16.

As an example of the effect of a conducting enclosure, the behaviour of an electroscope placed in a tin can may be noticed. An electroscope so placed is quite unaffected by charges external to the can, and this will be found to be the case even when the can is perforated with a large number of holes—in fact an enclosure of perforated sheet zinc or wire netting, or a glass case covered with strips of tinfoil, is quite as effective a screen as a conducting enclosure with continuous walls. Professor W. E. Ayrton has taken advantage of this fact in the construction of his improved form of Gold-leaf Electroscope. This instrument is shown in Fig. 16. Special precautions indicated in the figure are taken to ensure good insulation of the leaves, and the case

is covered with tinfoil, leaving a few narrow gaps through which the leaves may be observed. The leaves are thus protected from all inductive action from without (not directed on the cap of the instrument), and the case being earth-connected limits the field of the leaves when they are charged, and thus prevents their divergence being affected by inductive action on external neutral bodies. It is also important to notice that the interior of a hollow conductor is screened, not only from the inductive action of neighbouring charges on other conductors, but also from the inductive action of its own charge, if it possess one. Faraday illustrated this in a very striking manner. He constructed a wooden cube of twelve feet edge, covered it with tinfoil, insulated it, and went inside, taking his most delicate electroscopes with him. The cube was then strongly charged, but the electroscopes were not in the least affected. This result may perhaps be explained thus: consider an insulated conducting body inside a positively charged hollow conductor; the negative electricity in the former may be considered to be equally attracted in all directions by the surrounding positive charge, and the resultant force acting on it is therefore zero. Similarly, the positive may be considered to be repelled equally in all directions, and is likewise in equilibrium. Hence the body shows no induced charge. If, however, it be connected to earth, the line of connection is one along which the positive electricity may pass to earth, leaving the negative electricity bound by the external charge.

These effects are readily illustrated by the following experiment. A metal can is placed on a block of paraffin to insulate it and charged positively, say by the inductive action of an excited ebonite rod. A smaller can, held by an ebonite handle, is now placed inside this one, touched with the finger, and then removed. On testing, it will be found to be negatively charged.

14. Faraday's Ice-pail Experiments. If a charged conductor be suspended by an insulating thread inside an insulated hollow conducting vessel which completely surrounds it (Fig. 17), the *total* induced charges will be found on this vessel, the attracted charge being on its inner

surface and the repelled charge on its outer surface. Assuming the vessel to be neutral to start with, it is evident that these two charges must be equal to one another, for no charge has been communicated to the vessel; and, therefore, if the induced charges are allowed to again unite, that is, if the inducing charge is removed the vessel will again become neutral, thus showing that the induced charges are equal and opposite to each other. This tells us nothing of the actual magnitude of these charges—we simply know that they are equal to one another and of opposite sign. It is important, however, to test experimentally whether there is any relation between the magnitude of the inducing charge and that of the induced charges, and, if so, to determine the nature of this relation. With this object in view, Faraday performed the following experiment, which has now become historical.

Fig. 17.

An ice-pail, I (Fig. 18), is placed on an insulating stand and connected by the wire, w, with the cap of the

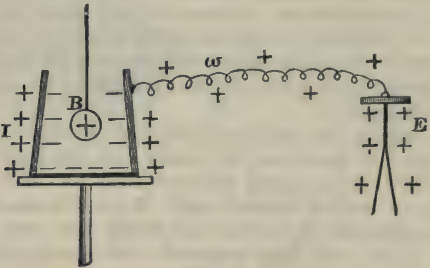


Fig. 18.

electroscope, E. A positively charged body, B, suspended by a dry silk thread, is now slowly lowered into the pail. As B gradually descends, the inner surface of I comes more and more under its inductive action, and a gradually

increasing negative charge is attracted to this surface, while a corresponding positive charge is repelled to the outer surface, and thence to E. As a result of this action, the gold leaves of the electroscope begin to diverge as B is about to enter I, and this divergence slowly increases as B is lowered into the pail. When, however, B is well inside I, as shown in the figure, practically the whole* of its inductive action is concentrated on the inner side of that vessel, and the negative charge there found is consequently the total induced charge of that sign, while the positive charge repelled to the outer surface and to the electroscope represents the total induced positive charge. Hence, when this result is obtained, further lowering of B into I will not affect the induced charges, and consequently the divergence of the leaves of the electroscope remains unchanged. If now B be allowed to *touch* the inner surface of I, and the leaves of the electroscope be carefully watched, it will be found that they are not in the least disturbed by the contact. Further, if B be now removed, it will be found to be completely discharged. The significance of these results is important. When B touches the inner surface of I the positive charge on the former combines with the negative induced charge on the latter, and, since B is completely discharged and the external charge on I and E remains quite undisturbed, it is evident that these two opposite charges must exactly neutralise each other, that is, the negative induced charge is exactly equal in magnitude to the positive inducing charge.

Hence, we may state generally, that the induced charges are equal to one another, but of opposite sign, and the *total* magnitude of either of the induced charges is numerically equal to that of the inducing charge. The first part of this statement applies to any case of induction; that is, whenever a body is subjected to induction, the induced charges are equal and opposite. In the second part, however, the significance of the word *total* must be noted—the magnitude

* If the pail is not too wide compared with its depth, very few of the tubes of force emanating from B pass out through the opening at the top. They all, or nearly all, terminate on the negatively charged inner surface of the vessel.

of the induced charges on any body is equal to that of the inducing charge only when the whole inductive action of the charge is concentrated on that body.

15. Illustrative Experiments. Let a number of metal cans of different sizes be arranged, as shown in Fig. 19, as

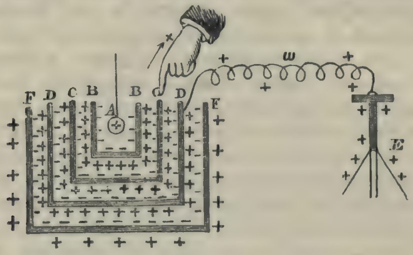


Fig. 19.

a nest of conductors insulated from one another, and let an electroscope, E, be connected to any one of them by a wire, *w*. If a charged body, A, be now lowered into the inner can, B, the leaves of the electroscope at once diverge, no matter with which of the conductors it is connected. This illustrates an effect which is sometimes called *successive induction*. Suppose the charge on A to be positive; then it acts inductively on the inner can, attracting a negative charge to its inner surface, and repelling a positive charge to its outer surface; this repelled charge in turn acts inductively on the next can, with a result indicated in the figure, and by this successive action induced charges are separated on each conductor. It will be noticed that the inductive action on any one conductor is due to the *algebraic sum* of the charges in its interior; for example, in the case of the conductor C, there are three *equal* (Art. 14) charges in its interior, and the effective charge, that is, the algebraic sum of the three, may be represented by that on A, or by that on the outer surface of the first can. Hence the induced charges on the second can are equal to those on the first can, and each numerically equal to the charge on A. Similarly, the

induced charges on any one of the conductors are each numerically equal to that on A.

If the electroscope be connected to any one of the conductors, and another *interior* to that one be touched with the finger, it will be found that the electroscope is not affected when A is lowered into the inner can, thus showing that the earth-connected conductor limits the field of induction. This result is evident from Fig. 19; when C is touched the algebraic sum of the charges inside D is zero, owing to the fact that the odd positive charge on C has escaped to earth. If, however, a conductor external to that connected with the electroscope, for example, F, be touched, then the inductive action of A is at once indicated by a divergence of the leaves of the electroscope.

The fact that the inductive action on any hollow conductor is due to the algebraic sum of the charges in its interior has an important application.

In Art. 5 we have seen that when electrification is produced by friction, the rubber and the substance rubbed become oppositely electrified. We are now able to show by experiment that the charges so produced are not only opposite in sign, but *equal in magnitude.* Two metal cans A and B, are arranged as shown in Fig. 20, and insulated from each other. The inner, A, is lined with fur, and an ebonite rod, R, fits closely into it. The outer can, B, is connected with an electroscope. On rotating R, electrical separation takes place, the fur becoming positively and the rod negatively electrified. If these two charges are unequal, then the excess of the greater will act inductively on B, and the leaves of the electroscope will diverge. Experiment, however, shows that no such divergence takes place, thus proving that the algebraic sum of the charges in the interior of B is zero, that is, the charges produced on the

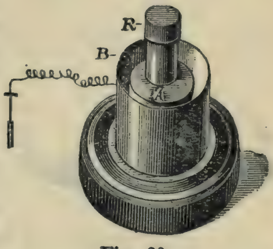


Fig. 20.

rubber and the body rubbed are equal in magnitude, but of opposite sign. If, however, R be removed, it carries the negative charge with it, and the positive charge in A, acting inductively on B, causes an immediate divergence of the leaves. On replacing R, the leaves at once collapse, showing directly that the charge on A is equal and opposite to that on R.

16. The Electrophorus. It is often desirable to be able to charge a conductor with electricity of a given sign. In Art. 4 we have seen that an insulator charged by friction does not readily yield up its charge to a conductor placed in contact with it, and cannot therefore be conveniently used for this purpose, but in Art. 11 it is explained how inductive action may be applied to effect the charging more satisfactorily than by direct contact. Thus, although a metal can is not appreciably charged by touching or rubbing it with a charged ebonite rod, it can be easily charged by induction. The rod is placed in the interior of the can and the latter touched with the finger; as a result, the can at once receives a charge equal in magnitude but opposite in sign to that on the rod, and this is effected *without diminishing the charge on the rod*, so that, theoretically, the latter may be used to charge any number of cans in the same way. In practice, however, the charge on the rod gradually escapes owing to defective insulation, but under favourable circumstances a large number of charges could be obtained without again exciting the rod. This is the principle of the electrophorus. As usually constructed the instrument consists, as shown in Fig. 21, of a disc of ebonite,* E, and a plate of brass, B, of slightly smaller diameter, attached to an insulating glass handle. The ebonite is sometimes fixed

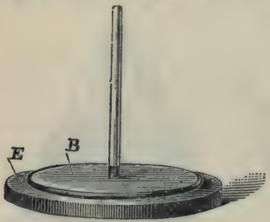


Fig. 21.

* *Theoretically* any insulating substance would answer the purpose, but ebonite gives the best results. Cakes of sulphur, sealing-wax, and other resinous substances, are sometimes used.

in a brass base called the sole, but this is not an essential part of the instrument, and is frequently represented by a coating of tinfoil on the under surface of the ebonite or by the table on which the disc rests. The action of the instrument is indicated in Fig. 22. The ebonite is excited by rubbing it with fur, and a strong negative charge is thus developed on its surface. The brass plate is now laid

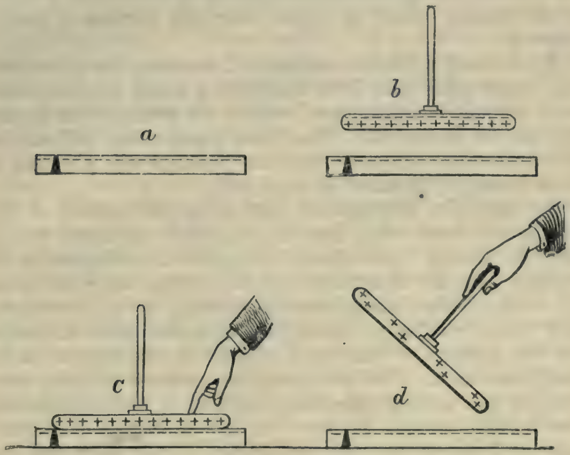


Fig. 22.

on the surface of the ebonite, and the charge on the latter, acting inductively on the plate, attracts positive electricity to its lower surface and repels negative to its upper surface. The brass is now touched with the finger and the repelled negative goes to earth, leaving the positive charge on the plate bound by the negative on the ebonite. If the brass plate be now removed, this positive charge distributes itself over its surface, and may be communicated by contact to the body which it is desired to charge, or applied to any other purpose.

It should be noticed that when the brass plate is laid on the ebonite, it does not receive a charge by conduction, but is acted on *inductively*. This is due to the fact that neither surface being truly plane the points of actual contact are comparatively few, and the total area of contact is very small. Hence, only an inappreciable fraction of the charge on the disc can be directly communicated to the brass, for the ebonite, being an insulator, gives up only that portion of its charge which is spread over the surface of actual contact.

To do away with the necessity of touching the plate with the finger each time it is charged, a brass pin connected with the sole is sometimes let into the ebonite disc in the way shown in Fig. 22. The brass plate, when laid on the ebonite, makes contact* with the point of the pin, and is thus connected to earth as required.

In the brief explanation just given the action of the *sole* is not dealt with, but the complete explanation will now be readily understood. Suppose the ebonite to be excited, and let the charge on its surface be represented by -20. This charge acts inductively in all directions, and there will be an equal and opposite induced charge on the surface of the earth-connected conductors in its neighbourhood. The sole being the nearest of these conductors will possess the greater portion of this charge, and the remainder will be, say, on the ceiling of the room. Let $+16$ denote the induced charge on the sole and $+4$ that on the ceiling. On placing the *insulated* brass plate on the ebonite neither of these charges will be disturbed, but on connecting it to earth it is put in connection with the sole, and the total inductive action of the charge on the ebonite is now concentrated on these two conductors. Hence the four units originally appearing on the ceiling are now transferred to the brass plate, and this conductor being now nearer than the sole, the greater portion of the induced charge will lie on it. Hence the distribution of the induced charge may now be represented by, say,

* Actual contact is not essential; the repelled negative from the brass will readily spark across to the point of the pin if the distance is not too great. See Art. 17.

+15 on the brass plate and +5 on the sole, that is, 11 units have passed from the latter to the former, and these combined with the four originally found on the ceiling make up the 15 it now possesses. On removing the plate the 15 units are removed with it, and the induced charge on the sole gradually increases until, when the plate is some distance away, the original distribution of 16 on the sole and 4 on the ceiling again obtains. When the plate rests on the ebonite the 15 units are distributed over its *lower* surface; if it be slowly raised vertically, the charge gradually spreads over the upper surface, and when removed beyond the influence of the negative charge both surfaces are equally charged.

Hence, when the plate of the instrument is touched a positive charge flows *out* of the sole, and when the charged plate is removed a positive charge flows *into* the sole. These effects are readily illustrated by experiment. Place the charged disc of an electrophorus on the cap of an electroscope. On placing the plate on the ebonite no effect is produced, but on touching the plate the leaves at once diverge widely, showing that a portion of the charge in the sole is now free and would pass to earth if communication offered.* If the plate be now removed the leaves will again collapse,† showing that the charge just liberated is now recalled into the sole.

It thus appears that, by the action of the electrophorus, a theoretically unlimited number of charges can be obtained from the single charge on the ebonite disc. This at once shows that electricity cannot, in itself, be a form of energy, for if it were, we could, by the action of this instrument, obtain an almost inexhaustible supply of energy from the finite quantity represented by the charge on the disc. If, however, the positively charged plate be held over a

* If at this stage the disc and plate be removed together without discharging the electroscope, the latter will be found to be charged with positive electricity.

† On trying this experiment it will be found that occasionally the leaves collapse and re-diverge. This is due to a partial escape of the freed positive charge from the edges of the leaves. On removing the plate an equivalent amount of negative is repelled to the leaves and causes the re-divergence. See Art. 12.

number of pith balls it attracts them vigorously, and may be thus made to do a considerable amount of work. The plate therefore possesses energy, and a question arises as to where the energy which it receives each time it is charged comes from. This question is readily answered. When the positively charged plate lies on the negatively charged ebonite, each attracts the other; hence in removing the plate work has to be done against this force of attraction, and the energy it possesses when separated from the ebonite is equivalent to the work done in effecting the separation, just as in lifting a weight from the ground the potential energy it possesses at any height is the equivalent of the work done in lifting it to that height. In the one case, work is done in moving electricity against electric force, and the potential energy of the charge may be called electric potential energy; in the other case, the work is done in moving matter against the attraction of the earth, and the potential energy of the weight is what is commonly called potential energy of position. If the electrical potential energy be expended in doing mechanical or chemical work, in producing heat, or is in any way transformed, its transformation is subject to the general law of the Conservation of Energy.

In the above, it should be noticed that the electric potential energy of the charge on the brass plate is the equivalent of the work done in separating it from its inducing charge; hence, in considering the potential energy of any charge, we may take it to be the equivalent of the work that would have to be done to separate it from its induced charge, wherever that exists, or the equivalent of the work that either would do in attracting the other up to it.

17. The Electric Spark. If the knuckle be presented to the charged plate of an electrophorus, it will be found that, on bringing it within, say, half an inch of the edge of the plate, a bright spark passes between the knuckle and the plate, and at the same time a sharp crack accompanying the spark is plainly heard, and a peculiar pricking sensation is experienced. On testing the plate after the passage of the spark, it will generally be found to be completely discharged. This spark discharge is due to the break-down

of the intervening air under the stress existing between the charge on the plate and the induced charge on the knuckle. As the distance between the plate and the knuckle decreases, the electric force at any point, say halfway between them, increases rapidly until finally the stress becomes too great for the medium to withstand, and finally the overstrained material gives way mechanically, and the strain energy in it is transformed into the heat, light, and sound of the spark.

CHAPTER III.

DISTRIBUTION OF ELECTRICITY ON CONDUCTORS

18. The Charge resides on the Outer Surface of a Charged Conductor. In Art. 3 it has already been indicated that electrification is a surface effect appearing at the surface of separation between the charged body and the surrounding insulating medium. In other words the charge on an electrified body is said to be distributed over the surface of that body. In the case of charged conductors the charge is found on the *outer* surface only; for example, if a charge be given to a hollow conductor no charge can be detected on its inner surface. This statement is readily established by a number of simple experiments. If a brass ball suspended by a silk thread be placed in contact with the inner surface of a charged metal can, it will be found on removal to possess no charge. This shows that no free charge resides on the inner surface of the vessel, for when in contact the surface of the ball formed a portion of this surface, and would therefore have removed a corresponding portion of the charge, had any such existed. The same fact may be more directly illustrated thus. Let an electroscope be placed inside a tin box, and let its cap be connected with the inner surface of the box. On charging the latter, the leaves of the electroscope, viewed through a small opening in the side of the box, show no divergence even when the box is very strongly charged.

It is not essential for the electroscope to be connected directly with the inner surface, for it will be seen on consideration that the inner surface is connected with the outer surface by the material of the box, and therefore the same effect will be produced whether the connection be made inside or outside, so long as the electroscope remains within the

hollow conductor. Another experiment, first made by Biot, illustrates this fact very neatly. An insulated brass ball, A (Fig. 23), is charged, and two hollow hemispherical conductors, B and C, of somewhat larger diameter, are then closed over it by the help of the insulating handles attached to them. So long as contact does not take place between the ball and the enclosing shell formed by B and C, the charge remains on the former, but immediately contact is made the charge passes to the outer surface. On removing B and C and testing them they will be found to be charged, while the ball may be shown to be completely discharged.

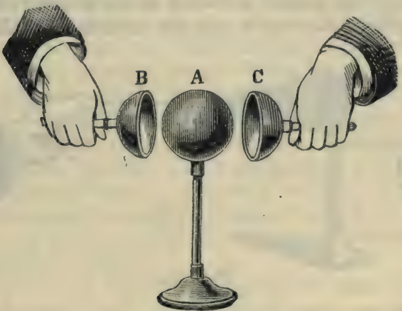


Fig. 23.

This is Biot's form of the experiment, but it may be more readily performed by lowering the charged body into a rather deep metal can—as in Faraday's ice-pail experiment. So long as the ball does not touch the vessel it retains its charge, but if contact is made, then it will be found that the vessel is charged and the ball is completely discharged, thus showing that a charge cannot remain on a conductor whose surface forms a portion of the inner surface of a hollow conductor. The result of this experiment has an important practical application. If one conductor is required to *completely* deliver up its charge to another, this latter should be made in the form of a hollow vessel and

insulated; then on placing the former *inside* it and in contact with its inner surface, the total charge at once passes into the outer conductor. This may be repeated as often as is desirable, and a considerable charge may thus be accumulated in a single conductor.

Many other experiments have been devised in support of the fact that no charge resides on the inner surface. Faraday's net experiment should be mentioned. A net of linen gauze is mounted on an insulating stand, as shown in Fig 24. The string, *ss*, is of silk, and is used to turn the net inside out. The net, extended as in the figure, is strongly charged. On testing with a proof-plane and an electroscope the charge is found to be accumulated on the outer surface, but none can be detected in the interior.

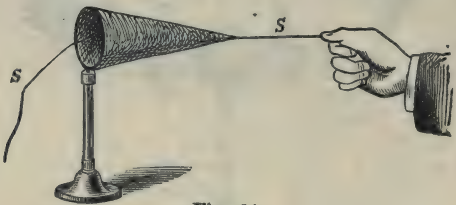


Fig. 24.

The net is now turned inside out, but the charge is still found on the outer surface, only having thus changed surfaces.

In applying the above law as to the distribution of the charge on the outer surface only, care must be taken that the proper interpretation is given to the term "outer." The ordinary meaning of the word is not always the one applicable. For the purpose of this law the outer surface of any conductor is the surface nearest the external induced charge. For example, if a metal pail be charged, the induced charge will, under ordinary circumstances, be on objects external to it, and, therefore, the charge will be on what we usually call its outer surface; but if a slightly smaller pail be placed inside it and connected to earth,

then the induced charge will be found chiefly on this pail, and the charge on the outer vessel will lie on what is usually called its inner surface, but which is now its electrical "outer" surface, that is, the surface nearest the induced charge.

From a consideration of this result, we shall be able in a general way to explain why the charge always resides on the "outer" surface of a conductor. Evidently the charge always distributes itself so as to be as near as possible to its induced charge, that is, in such a way as to possess minimum potential energy. Just as objects always tend to fall towards the earth and to rest at the lowest attainable level, so electricity passes to its position of lowest attainable potential energy on the outer surface of the conductor. For example, if mercury be poured into a vessel full of water, it passes through the water and takes up a position at the bottom of the vessel, between the water and the vessel; similarly, if a charge be given to the interior of a hollow conductor, it passes to the outer surface, and takes up its position between the conducting surface and the insulator surrounding it. Hence, if we choose, we may say that the charge of a conductor resides not on the conductor at all, but on the surface of the insulating medium adjacent to its "outer" surface.

The explanation may also be given in the language of lines or tubes of force. The tubes of force emanating from the charged body terminate on the nearest earth-connected surface, that is, the strain distributes itself in the medium between the "charged" surface and the nearest earth-connected surface. As the nature of the strain is such that the tubes tend to shorten it is evident that the distribution of the strain will be such that the surfaces bounding the positive and negative faces of the strained medium will be as close together as possible. If one or both of the bounding surfaces are irregular, the strain will naturally concentrate at the regions where the distance between the surface is least, that is, where the insulating medium is thinnest. The figures given below (25, 26, 27, 28) indicate in a general way the distribution of the strain in some typical cases. In each figure A represents the insulated charged body.

and B the earth-connected body or bodies. In Fig. 25 B is supposed to be at an infinite distance.

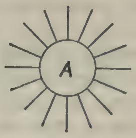


Fig. 25.

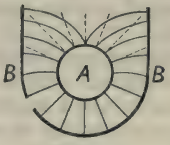


Fig. 26.

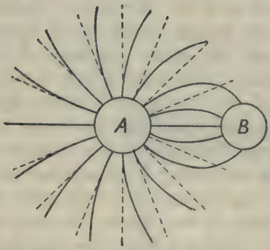


Fig. 27.

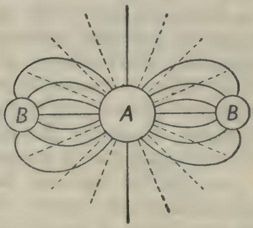


Fig. 28.

19. Distribution of the Charge on the Outer Surface of a Conductor. The general law given at the end of the preceding article may be applied to the consideration of this question. If all points on the outer surface of a conductor are positions of equal potential energy for the charge, then the distribution of the charge over that surface will be uniform. If, however, there are portions of the surface where the potential energy of the charge would be less than on other portions of the surface, then the charge will accumulate on those portions, until the distribution of the opposite induced charge is such that all points on the surface of the conductor are positions of equal potential energy for the charge. In this case the distribution of the charge on the surface will not be uniform, but the charge per unit area, or the *density* of the charge, will vary with the form of the surface, and with

the position and arrangement of earth-connected conductors in its neighbourhood.

The distribution of the "charge" on the charged conductor and also of the complementary "induced charge" may also be inferred from a consideration of the tubes of induction indicating the distribution of the strain in the medium intervening between the two charges. If, in a redistribution of strain in the medium, the area from which a tube of induction starts or at which it terminates decreases, this implies that the charge which, before the redistribution resided on a given area of the charged surface, resides, after the redistribution, on a smaller area; that is, narrowing of the tubes of induction at any part of either surface, indicates an accumulation of the charge at that part. Hence in force diagrams wherever there is concentration of strain in the medium the tubes of induction appear to shorten and narrow, and the charges, initial and induced, tend to accumulate at those positions of the charged surfaces which are separated by regions of greatest strain.

We shall only consider how the form of the conductor affects the distribution, assuming all other conductors to be removed to a distance. On a spherical conductor the distribution is uniform, as indicated by the dotted line in Fig. 29 (a). On a conical conductor, such as shown at (b),

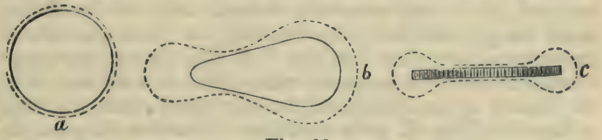


Fig. 29.

the distribution is indicated in a general way in the figure; the charge accumulates at the pointed end, where the curvature of the surface is greater than at other points. Similarly, in the case of the circular plate (c) the charge accumulates at the edges where the curvature is greatest, and, in general, experiment shows that the distribution

varies with the curvature of the surface, that is, the greater the curvature at any point the greater will be the quantity of electricity at that point, provided the surface be *convex* at the point and the latter not within a re-entrant hollow.

A study of the distribution of the strain in the medium surrounding these conductors will show that this question of distribution of the surface charge is only another aspect of the distribution of the strain in the medium. In Fig. 30 (*a*), the distribution of the tubes of induction surrounding the charged spherical conductor is seen to be symmetrical. If, however, the spherical conductor be supposed to take the form shown in (*b*) then the tubes of force shown in (*a*) take up the new forms and distribution shown in (*b*), and concurrent with this change of distribution of strain there is the redistribution of the charge indicated by the changes in the terminal areas of the tubes. In (*a*) the terminal areas are all equal, that is, the charge is uniformly distributed; in (*b*) the areas are unequal, that is, the charge is no longer uniformly distributed, but is densest where the terminal areas are smallest. In Fig. 30 (*c*), the redistribution of (*b*) is carried further, and it is evident that the density of the charge will be very great round the edge of the disc. Fig. 31 gives the tube of induction diagram for a conductor with a re-entrant hollow on its surface, and indicates why there is no charge in the hollow. When, instead of a hollow, there is a protuberant point or knob on the surface of the conductor, an accumulation of charge is found on the protuberance. This accumulation is an effect of strain redistribution similar in character to the absence of charge on the re-entrant hollow; the tube of induction diagram for this is given in Fig. 32.

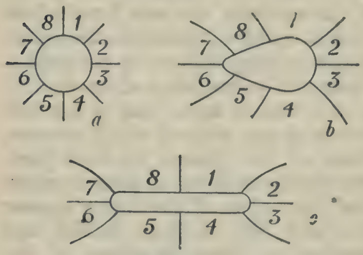


Fig. 30.

The simpler facts of charge distribution may be experimentally verified in the following manner. Let a large

Fig. 31.

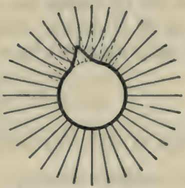


Fig. 32.

spherical conductor, such as is shown in Fig. 29 (a), be well insulated, and then strongly charged. By means of a proof-plane we can now test the distribution of the charge over the surface of the conductor. The proof-plane is laid on this surface at any point, and the charge received is communicated to an electroscope. The divergence of the leaves, which may be noted on a small scale mounted behind them, roughly indicates the magnitude of this charge. The charge which the proof-plane receives at any point on the surface of the conductor is, however, proportional to the quantity of electricity distributed over the surface at that point. Hence, by comparing the charges to be obtained at different points of the surface, the distribution over the whole surface may be investigated. In this case of a spherical surface, it will be found that the charge to be obtained from any point on the surface is the same, showing that the distribution is uniform.

If a conical conductor, such as is shown at b, be charged, and the distribution of the charge tested, it will be found that a much greater charge can be obtained from the pointed end than from the other end. Similarly, the distribution on any surface may be determined, and it will be found that the charge always tends to accumulate at points, corners, and edges, leaving the "flatter" portions of the surface less heavily charged, and the re-entrant portions almost or quite free from charge.

20. Surface Density. The surface density for the surface of a charged conductor is measured by the quantity of electricity distributed over unit area of surface. Thus, if a charge of Q units be uniformly distributed over a surface of area S units then the measure of the surface density (usually denoted by σ) is given by

$$\sigma = \frac{Q}{S}.$$

If the charge is not uniformly distributed, then $\frac{Q}{S}$ gives the *average* surface density. The surface density *at any point* on a surface is determined by the ratio $\frac{q}{s}$, where q is the small charge distributed over s, a very small area of surface which contains the given point.

The surface density from point to point on the surface of a conductor determines the distribution quantitatively. The dotted lines in Fig. 29 may thus be taken to indicate the variation of the surface density from point to point on the surface of the conductors there represented.

21. The Action of Points. From the preceding articles we have learnt that the charge on a conductor tends to accumulate at the points, corners, and sharp edges where the curvature of the surface is greatest. Thus, if a needle point project from the surface of a spherical conductor, the charge accumulates on this point, and the surface density there becomes very great. If the conductor is sufficiently charged, the density may become so great that particles of dust in contact with the point receive a charge. These particles are then repelled from the point, and their places taken by another set, which are in turn charged and repelled. Every particle repelled from the point thus carries away with it a minute charge, and, until the surface density falls too low, there is a constant rain of charged particles streaming away from the point. Hence, there is a continual discharge of electricity, sometimes called a *convection discharge*, from the point, and, in consequence the charge gradually decreases, and will continue to do so until the surface density on the

point is too small to charge the particles in contact with it. For the same reason a similar convection discharge takes place from the sharp edges of a charged conductor. This discharging action of points is readily illustrated by a number of experiments. If a pin point be soldered to the cap of an electroscope and the instrument be charged, it will be found that it rapidly loses its charge —the divergent leaves quickly close up and soon indicate complete discharge. Similarly, if a pointed wire be attached to the plate of an electrophorus, it will be found that on charging the plate no spark can be drawn from it; but if the point of the wire be at once presented to a steady flame, the "wind" due to the convection discharge will blow the flame aside. The discharging action of sharp edges can be illustrated by a very simple but somewhat striking experiment. If a strongly charged ebonite rod be held over the cap of an electroscope for a few seconds and then slowly removed, the divergent leaves will be seen to gradually collapse, and finally to open out as widely as before, but with a charge of opposite sign. The leaves at first diverge with negative electricity, inductively repelled by the charged rod. During the short time that the rod is held in position this repelled charge partially escapes from the very thin edges of the gold leaf. On slowly removing the rod the leaves gradually collapse as the bound positive charge on the cap is slowly liberated, and finally re-diverge with the excess of positive electricity corresponding to the negative which has escaped.

This action of points has some important applications. In the illustration just given the electroscope is charged by induction—the repelled induced charge escapes from the thin edges of the leaves, leaving the instrument charged with electricity of the sign opposite to that of the inducing charge. Similarly, if a pointed wire be attached to the cap of the instrument, and the inducing charge held for a short time over the point, the attracted induced charge will escape, leaving the instrument charged with electricity of the same sign as that of the inducing charge. Hence, if we wish to "collect" the charge from a charged body, it is only necessary to present a point or row of points

attached to a conductor to it—the charge induced on the points flows as a convection discharge on to the surface of the charged body, neutralising the charge there found, and leaving the conductor charged with electricity of the same sign. The final effect is thus the same as if the points directly collected the charge from the charged body. This effect may be illustrated by a few simple experiments. Let one end of a wire be attached to the cap of an electroscope, and the other end, carried in a split ebonite penholder, presented to any charged body—the leaves at once diverge, showing inductive action, and on removing the wire it will be found that the instrument is permanently charged. Similarly, if the point of a needle held in the hand be moved several times, close to the surface, from one end of a charged ebonite rod to the other, it will be found on testing that the rod is almost completely discharged. Again, if a large needle, connected to the cap of a very delicate electroscope, be carefully insulated in the open air,* it will be found that the leaves of the instrument will, in general, gradually diverge. The needle point "collects" the charge from the air around it, and the divergence of the leaves indicates the nature and extent of the electrification of the air at the point where the needle is placed. When this electrification is very great, the convection discharge often gives rise to a luminous glow diverging from the discharging point. This effect is often seen from the points of ship's masts, flagstaffs, etc., and has been called St. Elmo's fire. It may be seen on a small scale in a dark room from a needle point attached to a strongly charged conductor; for example, to the newly charged plate of an electrophorus, or better, the prime conductor of an electrical machine.

Owing to this action of points and sharp edges it is necessary to have the corners and edges of all conductors intended to retain charges carefully rounded and polished. The apparatus must also be kept scrupulously clean, for every particle of dust acts as a point, and therefore admits of escape of the charge.

* Some distance above the ground.

CHAPTER IV.

FRICTIONAL ELECTRICAL MACHINES.

22. The Frictional Electrical Machine. A frictional electrical machine is a mechanical arrangement for generating electricity by the friction of two suitable substances, and for collecting and storing the electricity so produced. Such a machine consists essentially of three parts, the *generator*, consisting of the body rubbed and the rubbers, the *collecting*

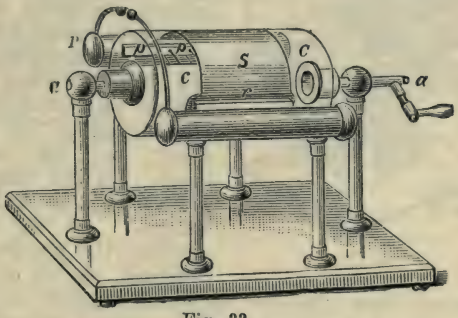


Fig. 33.

combs, and the *prime conductor* in which the charge collected by the combs accumulates. A very simple form of machine is shown in Fig. 33, which represents the essential parts of the ordinary cylinder electrical machine. The generator consists of a glass cylinder, C, capable of rotation about its axis, *a a*, and a rubber, *r*, made of a leather-covered pad coated with a mixture of an amalgam of zinc and mercury with grease. The rubber presses against the surface of

the cylinder, and on rotation the friction between the glass and the amalgam causes the surface of the former to become positively electrified, and that of the latter negatively. As the cylinder rotates the electrified portions leave the rubber behind, and other portions previously unelectrified come under its influence. Before again returning to the rubber the electrified surface of the glass passes close to the row of sharp-pointed wires which constitutes the collecting comb, *pp*. These points, acting in the way explained in Art. 21, "collect" the charge from the surface of the glass, which is thus rendered neutral and ready to be again electrified by the friction of the rubber. Connected with the comb is the prime conductor, P, an insulated conductor in which the charge collected from the cylinder accumulates. S is a silk flap attached to the rubber, and extending over the surface of the cylinder nearly to the collecting comb. Its use may be explained in a general way as follows. As the cylinder revolves the positive charge produced on its surface is separated from the corresponding negative charge on the rubber, but, owing to their mutual attraction, the former tends to slip back, over the surface of the glass, towards the rubber. The silk flap, however, acting as a rubber, has a small negative charge produced in it, and the attraction of this charge for the positive electricity on the glass is just sufficient to prevent any appreciable loss by reunion of the separated electricities. Fig. 34 illustrates a more usual form of electrical machine. It is known as the Plate Electrical Machine, and is in principle exactly similar to the one just described. A circular plate of glass

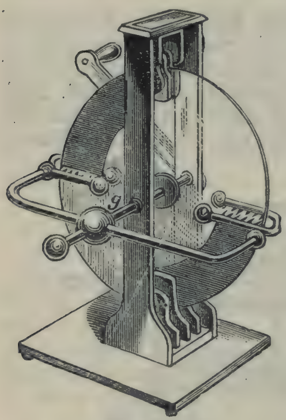


Fig. 34.

takes the place of the cylinder. The rubbers, two in number, are placed at the upper and lower extremities of the vertical diameter of the plate, each one being made up of two pads so arranged that the plate runs between them and consequently has both sides excited. The combs, having to collect from both sides of the plate are bent round it in the way shown in the figure, and are placed at the extremities of the horizontal diameter. The silk flaps and prime conductor perform the same function as in the cylinder machine. The prime conductor is here attached to the frame of the machine, and is insulated by the glass stem *g*, which should be well varnished and kept perfectly clean and dry. The action of the machine is readily understood. Following a portion of the surface of the plate from the lower rubber round through a complete revolution, we notice that as it leaves the rubber the glass is positively charged, and comes round to the right hand comb, where it is neutralised by the rain of negatively charged dust particles from the points; thus rendered neutral, it comes next under the influence of the top rubber, again becomes positively charged, and in passing the next comb is again neutralised, ready to be again excited by the friction of the bottom rubber. The continual discharge of negative electricity from the points of the combs leaves the prime conductor positively charged, and the magnitude of this charge increases as the machine is worked until the loss by defective insulation and convection discharge from its surface is equal to the gain from the combs. To reduce this source of loss as much as possible every part of the conductor is carefully rounded and polished, and the machine should be thoroughly clean and dry.

In Fig. 33 the rubber *r* is attached to the insulated conductor shown in the front of the figure, but as generally worked the rubbers of the machine are earth-connected, that is, they are not insulated, but directly connected with the framework of the machine. In this way no charge accumulates on the rubbers. If, however, they are insulated, then the negative charge accumulates, and finally becomes sufficiently great to prevent the positive electricity produced on the glass from reaching the combs,

and thus the machine ceases to work. If rubber and prime conductor are both insulated, but connected together by a conducting wire, then the negative of the rubber neutralises the positive charge induced in the prime conductor, and, however hard the machine may be worked, no sign of electrification can be detected in either rubber or prime conductor, thus proving that the quantities of electricity produced by friction are equal, but of opposite sign.

23. Experiments with the Electrical Machine. Let the machine be worked until the prime conductor is charged, then, on presenting the knuckle, or any conducting body to the prime conductor, a spark at once passes and the conductor is discharged. If the machine be worked in a dark room the action of the points of the collecting combs may be noticed. A bright luminous glow or brush will be seen to diverge from each point, indicating that there is a continuous convection discharge from the points on to the surface of the glass plate. Similarly, if a sharp-pointed rod or wire be attached to the prime conductor, it will be found impossible to get a spark of any length from the machine. but in the dark, a bright brush discharge may be seen to emanate from the pointed end of the wire. Also, if the point of a needle held in the hand be presented to the charged prime conductor of the machine, a silent discharge takes place from the point neutralising the charge on the conductor. If the discharging point in either of these experiments be directed towards a candle-flame, the "wind" of the discharge will blow the flame aside and may extinguish it. The electric whirl shown in Fig. 35 is a favourite illustration of the discharging action of points. It conclusively proves that the discharge is due to the repulsion of material particles from those

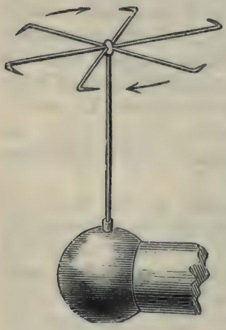


Fig. 35.

points, for in this arrangement the arms carrying the points are lightly pivoted on a central axis, and the reaction of the repelled particles drives them round in a direction opposite to that in which the pointed ends of the wire are bent.

The repulsion between similarly electrified bodies is strikingly shown by attaching a tassel of light thread, strips of paper, hair, etc., to the prime conductor. On working the machine the tassel opens out, each individual thread or hair repelling and being repelled by every other one.

Henley's quadrant indicator is a simple application of this effect. As shown in Fig. 36, it is often attached to the prime conductor of electrical machines, to indicate the magnitude of the charge here collected. The pith-ball *b*, carried by a light lever, is repelled from the similarly charged brass rod, B, to which it is attached, and the angular distance through which it is repelled may be read off on the circular scale, *ss*, and roughly indicates the state of electrification of the conductor.

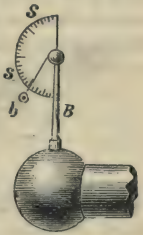


Fig. 36.

If a person stands on an insulating stool, that is, a stool with legs of glass or other insulating material, and grasps the prime conductor of the machine, he will practically form a part of the prime conductor, and will therefore gradually become charged as the machine is worked. Sparks can be drawn from any portion of his person, just as they may be drawn from the prime conductor, and by placing his finger near a gas jet, the gas may be lit by the spark which passes. A person thus charged will not retain a charge for many seconds after the machine stops working, for electricity escapes from the point of every hair and fibre of the clothing.

CHAPTER V.

ELECTRICAL POTENTIAL AND CAPACITY.

24. Electrical Potential. In Fig. 37 let A represent a positively charged body, and a and b any two points on the line A B. If a positive charge be placed at b, it will experience a force of repulsion due to the charge on A, and before it can be moved from b up to a this force of repulsion must

Fig. 37.

be overcome through the distance ba, and, therefore, work must be done. The work so done has its equivalent in the potential energy gained by the charge, in virtue of the change in its position relative to the charge on A. At a the potential energy of the charge is greater than at b, and if we consider *unit positive charge*, the difference for the two positions is called *the difference of the electrical potentials at the two points.* This difference of potential is due to the charge at A, and its value will therefore depend on the magnitude of this charge; hence, when we speak of the difference of potential at any two points, we imply the existence of a charge to which the difference of potential is due, The difference of the potentials at any two points *due to a given charge* may, therefore, be defined as the equivalent of the work done in moving *unit positive charge*, from one point to the other against the electrical force *exerted by the given charge.* A familiar illustration may help the student to realise the above explanations. If a pound mass is

lifted from a chair to a table, work is done, and the gain of potential energy by the mass is equivalent to the work done. In this case, the work is done against the force of attraction exerted by the earth, and the potential energy gained by the mass in virtue of the change of its position relative to the earth may be called the difference of the gravitation potentials due to the earth at the two points considered. So far, we have only considered difference of potential, but it is often convenient to speak of the electrical potential *at any point* due to a given charge. Now just as in Mechanics we can speak of the potential energy of a pound mass, at any level, as the equivalent of the work done in raising it from *the surface of the earth* to that level, so in Electrostatics we may speak of the electrical potential due to a given charge, at any point, as equivalent to the work done in bringing a unit positive charge from the earth to that point *against the electrical force* exerted by the given charge. This is evidently not the absolute potential at the point, but merely the difference between the potential due to the given charge at the point considered and at a point on the earth.

If the charge at A be taken as negative instead of positive, *then no work would have to be done* in bringing unit positive charge up to *a*, but the force of attraction which the charge exerts *would do work* in so bringing it up. Hence, the potential at *a* is said to be negative, for the unit charge, instead of having, as it were, to be pushed up to *a* against the force of *repulsion* exerted by the charge on A, is pulled up to *a* by the force of attraction due to that charge.

If the potential, at a point in the neighbourhood of two or more charged bodies, be considered, then the total potential at that point due to these charges is the sum of the potentials due to each of the charges taken separately, each with its proper sign, according to the sign of the charge.

Difference of potential may be defined in another way. Just as in Hydrostatics difference of level is the condition of flow of a liquid from one point to another, so in Electrostatics *difference of potential is the condition of flow of electricity*

from one point to another. It must, however, be noticed that a liquid flows from a high level to a low level, in virtue of the greater potential energy it possesses at the higher level, and this difference of potential energy per unit mass of liquid is the true analogue of difference of electrical potential, so that this method of defining difference of potential is practically identical with that given above. Further, although difference of level (implying difference of potential energy) is the condition of flow of a liquid from one point to another, this flow will not take place unless some path is open by which it can take place. Similarly, flow of electricity between two points at different potentials will not take place, unless these points are connected by conducting material through which the electricity can pass. From this it follows that no difference of potential can exist between any two points on a conductor, for if such a difference exist for an instant, a flow of electricity at once sets in and immediately establishes equality of potential,* Hence, all points on and in a conductor are at the same potential called *the potential of the conductor.* This potential may be defined as the equivalent of the work done in bringing a unit positive charge from the earth up to any point on the surface of the conductor.

The potential at any point in the interior of a hollow conductor is also the same as that of the conductor, for in bringing a unit positive charge up to any point in the interior, it must first be brought up to the outer surface, and then all the work to be done on it is done, for there is no electric force in the interior of the conductor, and therefore no more work will be done in moving it from the surface to any point in the interior. A neutral conductor is said to be at zero potential, for it offers no resistance to the approach of a unit positive charge.

Since difference of potential is the condition of transfer of electricity from one point to another, it follows that, if two conductors at different potentials are put in contact, electricity will flow from the one at the higher potential to

* This applies to electrostatics. In current electricity there is a constant supply maintaining the difference of potential and a *current* tending to equalise this difference.

the one at the lower potential until equality of potential is established. Similarly, if two conductors at considerably different potentials are placed *near* each other, electricity *tends* to pass from one to the other through the insulating air-gap separating them. If the difference of potential is sufficiently great, the electricity may break through this gap, producing a spark, and the potentials of the two conductors are at once equalised. The electric spark may be illustrated hydrostatically. Two communicating tubes, A and B (Fig. 38), are separated by a thin membrane, *m*. Mercury is poured into A up to a certain height, and then into B up to a higher level. The liquid now tends to pass from B to A, and the membrane, *m*, resisting the passage, is in a state of strain. If more mercury be poured into B this state of strain increases, and finally the membrane bursts, and the difference of level in the two tubes is at once equalised. If this experiment be performed, it will be noticed that after the breakdown of *m* the mercury oscillates rapidly up and down in the tubes, and finally comes to rest at the same level in each. It has recently been shown that an exactly similar phenomenon accompanies the electric spark: the duration of the spark is probably considerably less than the millionth part of a second, but in this short time the electricity oscillates to and fro across the spark path a large number of times before equality of potential is established.

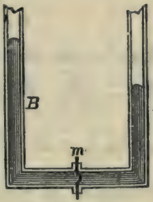


Fig. 38.

Considered mathematically, the potential at a point is measured by the work done in bringing a positive unit of electricity up to the point from an infinite distance, that is,

Fig. 39.

the point of zero potential is assumed to be at an infinite distance. The measure of the potential at any point on this assumption is obtained as follows. In Fig. 39 let a

positive charge of Q units be placed at O and let the points a, $b, c, \ldots n$ be supposed to be very close together along the line O B. The electric force at a, that is, the force which a unit positive charge at a would experience is $\frac{Q}{(Oa)^2}$ and similarly the electric force at b is $\frac{Q}{(Ob)^2}$. Therefore the average force for the very short distance $a\,b$ may be taken as $\frac{Q}{Oa\,.\,Ob}$ which is the geometric mean of the values at a and b. The work done when unit quantity of electricity is moved from a to b is therefore measured by $\frac{Q}{Oa\,.\,Ob}\,.\,a\,b$, that is, by

$$\frac{Q}{Oa\,.\,Ob}(O\,b - O\,a) \quad \text{or} \quad \frac{Q}{Oa} - \frac{Q}{Ob}.$$

That is, the difference of potentials at the points a and b is $\frac{Q}{Oa} - \frac{Q}{Ob}$. Similarly the difference of potentials for the points b, c is $\frac{Q}{Ob} - \frac{Q}{Oc}$. It follows from this that the difference of potential for the points a, and, n, separated by a finite distance $a\,n$, is

$$\left(\frac{Q}{Oa} - \frac{Q}{Ob}\right) + \left(\frac{Q}{Ob} - \frac{Q}{Oc}\right) + \left(\frac{Q}{Oc} - \frac{Q}{Od}\right) + \cdots \left(\frac{Q}{Om} - \frac{Q}{On}\right).$$

this evidently reduces to

$$\frac{Q}{Oa} - \frac{Q}{On}$$

and if n be assumed to be at an infinite distance from O, *the potential at a* is measured by

$$\frac{Q}{Oa} - \frac{Q}{\infty} = \frac{Q}{Oa}.$$

That is, the potential at a point at a distance r from a point charge Q is measured by $\frac{Q}{r}$ and the difference of potential between two points at distances r and r' respectively from O is $\left(\frac{Q}{r} - \frac{Q}{r'}\right)$. By taking the potential of the earth as

an arbitrary zero the measure of the potential at any point will differ from its mathematical measure by a fixed amount (the mathematical potential of the earth), but the measure of the difference of potential between any two points will not be altered.

It has been stated that the difference of potential between any two points is measured by the work done in conveying unit positive quantity of electricity from one point to the other. It is evident that this measure of difference of potential is independent of the path along which the unit positive charge is conveyed from one point to the other. For, in Fig. 39, suppose it to be possible that there is another path from n to a, for which the work done in moving a positive unit from n to a is greater or less than for the straight line path $n\,a$. Then, if the unit be moved from n to a by that path and back again to n by the straight path $a\,n$ the last state of the electric field is the same as the first, no energy has been gained or lost but the work done on moving the unit charge is not zero but positive or negative according as the work for the new path from n to a is less or greater than that for the straight path. This result is contrary to the theory of conservation of energy, and, therefore, there cannot be another path for which the work is different from that for the straight path $n\,a$. That is, by whatever path the charge is moved from n to a the work done is $\frac{Q}{Oa} - \frac{Q}{On}$ and the work done in bringing a positive unit charge from an infinite distance up to a is measured by $\frac{Q}{Oa}$, no matter by what path the charge reaches a.

25. Potential Energy of a Charge. In the preceding article we have considered the electrical potential at a point due to a given charge. This must not be confounded with the electrical potential energy of a charge placed at that point. The electrical potential at any point due to a given charge is the work that would have to be done in bringing *unit positive charge* from the earth up to that point *against the electrical force exerted by the given charge.* The potential energy of a charge placed at the point considered is the

equivalent of the total electrical work done in accumulating the charge at that point. Thus the potential of a charged conductor is the equivalent of the work necessary to bring a unit positive charge from the earth up to the conductor, against the electric force exerted by the charge of the conductor, but the potential energy of the charge on the conductor is the equivalent of *all* the electrical work done in charging the conductor.

Consider a conductor charged with a positive charge Q to a potential V. That is, the quantity of electricity with which it is charged is denoted by Q, and the work that would have to be done to bring a positive unit of electricity from the earth up to its surface when so charged is denoted by V. Hence, the quantity of work necessary to bring Q units of electricity from the earth up to the conductor, *supposing its potential to remain constant at* V during the process, is given by Q V. This, however, is evidently not the work done in charging the conductor *up to potential* V, with Q units of electricity, for during this process the potential is not constant at V, but rises from zero to V *in proportion as the charge is increased*, the latter value being acquired when the charging is complete.

The *average* value of the potential during the process of charging is therefore $\frac{V}{2}$, and the work done is the same as if the potential remained constant at this value during the process. Hence the work done in charging is given by $\frac{1}{2}$ Q V, and this therefore expresses the potential energy of a charge Q at potential V. If Q be expressed in the electrostatic units of quantity (Art. 6), and V in *ergs per unit of quantity*, then the energy of the charge is expressed in *ergs*.

26. Equipotential Surfaces. An equipotential surface is the locus of all points having the same potential. Imagine a charge to be concentrated at a point A (Fig. 40), then all points at the same distance from the charge have the same potential, that is, the equipotential surfaces in this case are a series of concentric spheres having their centre at the point A. The potential at each of the spherical surfaces is different, but for all points on any one surface it is the

same; that is, work has to be done to move a positive unit of electricity from one surface to another, but no work is done in moving it from any point on a given surface to another point on the *same* surface. From this it follows

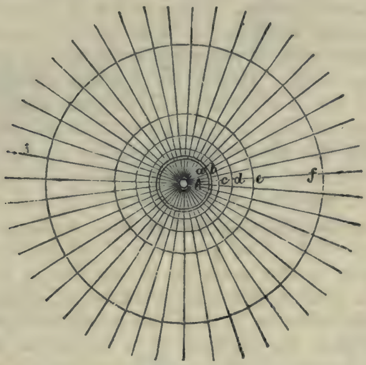


Fig. 40.

that the lines of force are at all points normal to the equipotential surfaces, for if no work is done in moving electricity from one point to an adjacent one on an equipotential surface, then the direction of motion must be everywhere perpendicular to the lines of force. In Fig. 40 the traces of the equipotential surfaces surrounding the point A are shown as concentric circles, and the lines of force in the plane of the paper are shown as straight lines radiating from the centre A. It is convenient in drawing equipotential surfaces to represent them so that unit quantity of work must be done in conveying unit quantity of electricity from any one surface to the next. When so drawn the distance between consecutive surfaces gradually increases as the distance from the charge increases. For the greater this distance the weaker the force exerted by the charge, and therefore the greater must be the distance through which it has to be overcome to do unit work.

If a charge be accumulated on a spherical conductor its action at all *external* points is the same as if the charge were accumulated at the centre of the conductor. Hence the equipotential surfaces surrounding a charged sphere are concentric spherical surfaces having the centre of the sphere as their common centre. The surface of the sphere is also an equipotential surface, for with statical electricity there can be no difference of potential between any two points of a conducting body. Hence, for a given charge, the actual potential of the conductor will depend upon its radius. For example, in Fig. 40, if the surface of the conductor coincide with the equipotential surface *a*, the potential of the conductor *and of all points inside it* will be that of surface *a*; if, however, it coincide with *b*, *c*, *d*, or any other surface, then its potential will be that of the surface with which it coincides. The potential of all points external to the conductor is the same as if the charge were concentrated at its centre, but the potential of all points in the interior is the same as that of the conductor itself.

The fact that the surface of a conductor must be an equipotential surface may be applied to explain electrostatic induction. In Fig. 10, C represents a positively charged conductor, and the potential at A due to the charge on C is therefore greater than the potential at B due to the same charge. As a consequence of this difference of potential between the points A and B on the conductor A B, positive electricity flows from A to B, until equality of potential is established; that is, the end B becomes positively charged and the end A is left negatively charged. The potentials at the points A and B are now equal, because the potential at A due to the charge on C is diminished by the potential due to the negative charge there induced, and the potential at B due to C is increased by the potential due to the positive charge induced at that point. Similarly, equality of potential is established all over the surface of the conductor, for the charge induced at any point is such that the algebraic sum of the potentials at that point due to the electrification on A and the charge on C is the same for all points on the surface.

The fact that the charge always resides on the outer

surface of a conductor may also be explained in terms of potential. In Fig. 41, let A represent a hollow spherical conducting shell of some thickness. If a positive charge be given to the inner surface, the potential of that surface will be greater than the potential due to this charge at the outer surface. Hence, since the inner and outer surfaces are connected by conducting material, the charge will at once pass to the outer surface, and no charge will be found on the inner surface.

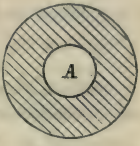


Fig. 41.

27. Electrical Capacity. In the preceding article it has been noticed that the potential produced in a spherical conductor by a given charge varies with the size of the conductor. For example, in Fig. 40, if a charge Q be given to a conductor of the same diameter as the equipotential surface *c*, a lower potential would result than if the same charge were given to a conductor of the same diameter as the surface *a*; that is, the potential of a spherical conductor due to its own charge decreases with the size of the conductor. Further, it can be shown by experiment that if the same charge be given to any two conductors differing in size or form, different potentials will be produced in these conductors. For example, if we charge two conductors of different size or form with equal charges, and then connect them by a thin wire, a charge will pass from one to the other indicating that they are at different potentials.

These facts may be expressed by saying that different conductors have different *electrical capacities*, and *the electrical capacity of a conductor* may be defined *as the ratio of the charge given to the conductor to the potential produced in the conductor by that charge.* That is, if a charge Q given to any conductor produce in it a potential V, then the capacity of the conductor is given by

$$C = \frac{Q}{V},$$

where C denotes the capacity of the conductor. Another way of expressing this definition is to say that *the capacity of a conductor is measured by the quantity of electricity*

necessary to raise the conductor to unit potential. Thus, if a charge of 10 units of electricity raise the potential of a conductor to 5 units, then the capacity of the conductor is given by the ratio $\frac{10}{5} = 2$, that is, the conductor requires 2 units of electricity to raise it to unit potential. Similarly, if a conductor at potential V receive a charge Q, raising the potential to V′, then $C = \frac{Q}{V'-V}$, that is, the capacity of a conductor is measured by the quantity of electricity necessary to raise its potential by unity. Thus, if a charge of 10 units of electricity raise the potential of a conductor from 2 to 7, then the capacity of the conductor is given by $\frac{10}{7-2} = \frac{10}{5} = 2$, that is, the conductor requires 2 units of electricity to raise its potential by unity.

These ideas will be more easily realised by considering a simple illustration. If the same quantity of water be poured into tubes of different bores, the level of the water in each tube will be different, and the capacity of any tube may be conveniently defined as the quantity of water necessary to raise the level of the water in the tube through unit distance. This definition of the capacity of the tube relative to quantity of water and level is analogous to the definition of electrical capacity relative to quantity of electricity and potential.

It can be shown (Arts. 24, 29) that the potential due to a charge Q on a spherical conductor, having all other conductors at an infinite distance from it, at any point outside the conductor at a distance r from the centre of the conductor is given by $\frac{Q}{r}$. Hence, if a charge Q be given to a spherical conductor of radius R, the potential of the conductor will be the potential at any point of its surface due to the charge Q, the latter being supposed to act from the centre of the conductor. That is, the potential of the conductor is given by $V = \frac{Q}{R}$, where Q denotes the charge and R the radius. From the relation given above, however,

we have $V = \frac{Q}{C}$, where Q denotes the charge and C the capacity of the conductor—hence, *in the case of a spherical conductor the capacity is measured by the radius of the conductor*. Thus, if V be expressed in ergs per unit of quantity, and Q in electrostatic units of quantity (Art. 6), then the radius of a spherical conductor in centimetres will express the capacity of that conductor.

From the ratio $C = \frac{Q}{V}$ we have $Q = V C$; that is, if a conductor of capacity C be charged to potential V, then the quantity of electricity with which it is charged is given by $Q = V C$. Thus, if a spherical conductor of radius 10 cms. be charged so that 10 ergs of work have to be done in bringing a positive unit from the earth up to its surface, that is, to potential 10, then the charge on the conductor is given by—

$$Q = 10 \times 10 = 100 \text{ electrostatic units of quantity.}$$

If two conductors charged to different potentials are brought into contact, they take up a common potential and share the combined charge in direct proportion to their capacities; for example, if two conductors of respective capacities 3 and 5 are made to share their charges, then one takes $\frac{3}{8}$ and the other $\frac{5}{8}$ of the combined charge. To determine the common potential which two or more conductors take on sharing their charges, we must remember two things—first, that the total quantity of electricity is the same before and after sharing; second, that the capacity of two or more conductors in contact with one another is the sum of the individual capacities of the conductors. Thus, if two conductors of capacities C_1 and C_2 be charged to potentials V_1 and V_2 respectively, then on being placed in contact they will take up a common potential V given by—

$$C_1V_1 + C_2V_2 = (C_1 + C_2)V;$$

or

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

The expression $C_1V_1 + C_2V_2$ gives the quantity of electricity before contact, and $(C_1 + C_2)V$ expresses the *same* quantity after contact and redistribution of the charge The conductors are here supposed to be put in contact in such a way that the distribution of the charge on one conductor is not influenced by the near presence of the other conductor. For example, the two conductors at a distance from each other and from other conductors might be joined by a long thin wire of negligible capacity.

The charges on the conductors will be in the ratio $C_1 : C_2$; that is, the charge on the conductor of capacity C_1 will be $\frac{C_1}{C_1+C_2}$ of the total charge on both conductors, and the charge on that of capacity C_2 will be $\frac{C_2}{C_1+C_2}$ of the total charge.

CHAPTER VI.

THE ELECTRIC FIELD

28. Normal Induction over a Surface in an Electric Field. Imagine any closed surface in an electric field, and at any point on it consider a very small area, a, containing that point. Let F be the electric force at this point, and let the direction of the force make an angle, α, with the *outward* drawn normal to the surface at the point. Then $F . \cos \alpha$, usually denoted by N, is the component of the force along the outward drawn normal, and $F . \cos \alpha . a$ or $N a$ is the flow of induction across the small area a. The total flow of induction or the total normal induction over the closed surface is obtained by supposing the whole surface to be divided up into a very large number of small areas such as a, and summing up the values of $N a$ for all these areas. That is, I, the total normal induction is given by

$$I = \Sigma N a.$$

It will be noticed that if α is less than 90° the component of F is outwards along the normal and $F \cos \alpha$ or N is positive, but if α is greater than 90° the component is inwards along the normal, and $F \cos \alpha$ or N is negative. This explanation of total normal induction enables us to consider a very important theorem in Electrostatics known as **Gauss's Theorem.** This theorem states that the total normal induction over any closed surface taken in an electric field is measured by 4π times the total quantity of electricity enclosed within the surface. This theorem is readily proved in the case of an electric field due to a single point charge. Let the point charge be at O (Fig. 42), *within*

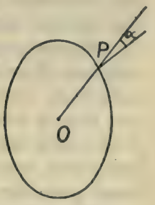


Fig. 42.

the closed surface, and let the small area, a, be taken on the closed surface at the point P. Then the normal induction over the small area is $F.\cos\alpha.a$ or $F.a\cos\alpha$. But $F = \frac{Q}{(OP)^2}$ and $a\cos\alpha = (OP)^2\,\omega$, where ω is the solid angle subtended at the point O by the area a. Hence the normal induction over a, given by $F.a\cos\alpha$, is equal to

$$\frac{Q}{(OP)^2}\,.\,OP^2.\,\omega = Q\,\omega.$$

It follows by dividing the whole surface into small elementary areas that the total normal induction I is given by $I = \Sigma\,(Q\,\omega) = Q\,\Sigma\,(\omega)$. When O is within the closed surface $\Sigma\,(\omega)$ is evidently equal to $4\,\pi$, and, therefore, the total normal induction over the closed surface is measured by $4\,\pi\,Q$, that is, by $4\,\pi$ times the quantity of electricity within the surface.

If the point O be *without* the closed surface (Fig. 43) then a straight line, O P′P, drawn from O will cut the surface at *two* points P′ and P, and if a cone with a very small solid angle ω at O be drawn with O P′P as axis, the normal induction over the small area defined on the surface by intersection with the cone at P is measured $Q\,\omega$, as above, and the induction over the corresponding area at P′ is measured by $-Q\,\omega$, the minus sign being used because the direction of the normal component at P′ is opposite to that of the *outward* drawn normal at that point. Hence the normal induction over the two surfaces at P and P′ is zero, and this is true for all the cones that may be drawn from O through the surface. It follows that the total normal induction over the surface is zero, and as, in this case, the total quantity of electricity within the surface is zero, this result is in accordance with the theorem.

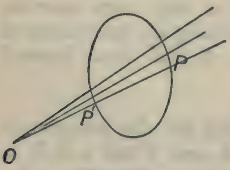


Fig. 43.

If the charge Q is not a point charge, that is, a charge supposed to be concentrated at a point, but a charge

distributed over the surface of a conductor or over a number of conductors, it can be shown that the theorem still holds. For, if the charge Q is, say, on a conductor inside the surface, then the charge may be divided into a number of elementary charges q_1, q_2, q_3, etc., at contiguous points on the surface of the conductor, and the total normal induction over the surface for these elementary charges is $4\pi q_1$, $4\pi q_2$, $4\pi q_3$, etc., and therefore the total normal induction for the whole charge, Q, is $4\pi q_1 + 4\pi q_2 + 4\pi q_3 +$ etc. or $4\pi (q_1 + q_2 + q_3 +$ etc.$)$, or $4\pi\, \Sigma(q)$, that is, $4\pi Q$, which is the required result. Similarly for a charge, Q, on a conductor outside the surface the induction is zero. It follows from this that whatever be the distribution of charges to which the field is due the total normal induction over a closed surface drawn in the field is measured by 4π times the quantity of electricity within the surface.

29. Application of Gauss's theorem. One important application of the theorem is for the determination, in some special cases, of the electric force at a point in the neighbourhood of a charged conductor. Thus, to find the electric force at a point, P, outside the surface of a uniformly charged sphere, let the charge on the sphere be Q, and take, as the closed surface in the field, a sphere through P concentric with the charged sphere. By the theorem the total normal induction over the closed spherical surface through P is $4\pi Q$; also as the charge is uniformly distributed over the charged surface, and symmetrically placed in relation to the closed spherical surface through P, the direction of the electric force at any point on this surface is along the outward normal to the surface at that point. Hence if F be the electric force at any point, $F . 4\pi (O P)^2$ is the total normal induction over the surface. Therefore we have

$$4\pi Q = 4\pi F . (O P)^2.$$

or

$$F = \frac{Q}{O P^2}.$$

That is, the force at P, due to the charge Q uniformly distributed over a sphere with centre at O, is the same as that due by the law of inverse squares to a point charge,

Q, at O, the centre of the sphere. Similarly, if the point P be taken inside the charged sphere we have $4\pi F.(OP)^2 = 0$, for the total normal induction over the spherical surface through P is zero, because there is no charge inside it. This gives $F = 0$, that is the electric force inside a uniformly charged sphere is zero. This is a particular case of a general result which applies to all closed charged conductors, *which contain no charges.* The general case may be established as follows: the surface of any charged conductor is necessarily an equipotential surface, and therefore the direction of the electric force at any point on the surface is normal to the surface at that point. If the interior of the conductor is everywhere at the potential of the surface it is evident that the electric force inside the conductor is zero, for, as no work would be done in moving a unit charge from one point to another inside the conductor, the electric force must everywhere inside the conductor be zero. If, however, the force be supposed not to be zero, it ought to be possible to draw equipotential surfaces inside the surface of the conductor. Imagine such an equipotential to be drawn very close to the surface of the conductor. Its potential must be higher or lower than that of the surface, and work must be done in moving electricity inwards or outwards from one surface to the other. That is, the normal electric force between the surfaces and inside the charged surface is not zero and its direction is either outwards or inwards, and therefore the total normal induction over the surface is a finite positive or negative and not a zero quantity. But the total normal induction over the surface is equal to 4π times the quantity of electricity inside the surface, and if this quantity is zero the induction must also be zero and cannot have a finite positive or negative value. That is, there can be no difference of potential inside a closed charged surface which contains no charge, and therefore the electric force inside this charged surface is zero.

The case of a uniformly electrified **plane surface** of infinite extent is an important one. It is evident from symmetry that the force at any point in the field must be normal to the surface, and that the magnitude of the force

must be the same for all points at the same distance from the surface. To apply Gauss's theorem imagine a small closed cylindrical surface in the field, bounded by plane ends, parallel to the electrified surface, and having its cylindrical surface normal to the surface. If this closed surface lies wholly on one side of the electrified surface, as in Fig. 44, the total normal induction over the surface is given by $N a - N' a$, where N is the electric force at any point in the end, A, of the closed surface, N′ the electric force at any point in the end B, and a the area of the ends. At A the direction of the force is along the outward drawn force and $N a$ is therefore positive, but at B the direction of the force is along the normal inwards, and therefore $N' a$ is negative. The flow of induction across the cylindrical surface is zero, for the direction of the force at every point in the surface lies in the surface, being everywhere normal to the electrified plane surface. Hence the total normal induction over the whole closed surface is given, as stated above, by $N a - N' a$. But, by Gauss's theorem, the total normal induction is in this case zero, for the closed surface contains no electricity. Hence we get $N a - N' a = 0$, and therefore $N = N'$, that is, the magnitude of the electric force is the same at all points in the field, and, as has already been assumed from symmetry, its direction is everywhere normal to the electrified surface.

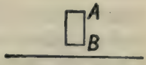


Fig. 44.

If the closed cylindrical surface lies, as in Fig. 45, with the ends A and B on opposite sides of the electrified surface, then the total normal induction over the surface is evidently given by $N a + N a$ or $2 N a$, where N denotes the electric force at any point in the field and a the area of the ends of the cylindrical surface. In this case, however, the closed surface encloses the quantity of electricity distributed on the portion of the electrified plane intercepted within the cylindrical wall of the surface, and, by Gauss's theorem, the total normal induction over the surface is equal to 4π times this quantity. If

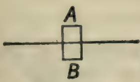


Fig. 45.

σ denote the surface density of the charge on the plane, then the quantity enclosed by the cylindrical surface is $a\,\sigma$, and therefore we have

$$2\,N\,a = 4\,\pi\,a\,\sigma$$

and

$$N = 2\,\pi\,\sigma$$

that is, the force at any point outside a uniformly electrified plane surface of infinite extent is equal to $2\,\pi$ times the surface density of the charge, and is everywhere normal to the electrified surface.

The case of a uniformly electrified infinitely long cylindrical surface is also of interest. To find the magnitude of the force at a point, P, outside the surface imagine a closed cylindrical surface, coaxial with the electrified surface, described through P, and having plane ends at right angles to the axis. The force at any point in the field is, by symmetry, the same for all points at the same distance from the axis, and is everywhere normal to the axis. Hence, the total normal induction over this closed surface is $2\,\pi\,r\,l\,F$, where r denotes the distance of P from the axis, l the length of the closed surface parallel to the axis, and F the magnitude of the electric force at any point at a distance r from the axis. If Q denote the charge per unit length on the electrified cylindrical surface, then $Q\,l$ denotes the quantity of electricity enclosed within the closed surface, and, by Gauss's theorem, the total normal induction over the surface is given by $4\,\pi\,Q\,l$. Hence we have

$$2\,\pi\,r\,l F = 4\,\pi\,Q\,l$$

or

$$F = \frac{2\,Q}{r}.$$

If P be a point *infinitely close to the surface of the electrified cylinder*, and σ denote the surface density of the charge, then $Q = 2\,\pi\,r\,\sigma$ and $F = \dfrac{4\,\pi\,r\,\sigma}{r} = 4\,\pi\,\sigma$. That is, the electric force at a point very close to the charged surface is $4\,\pi\,\sigma$. Similarly, in the case of the electrified spherical surface the force at a point infinitely close to the surface, deduced from the result $F = \dfrac{Q}{O\,P^2}$,

becomes $F = \frac{4\pi r^2 \sigma}{r^2} = 4\pi\sigma$, if r denote the radius of the surface and σ the surface density of the charge. It should be noticed that in the case of the uniformly electrified plane surface of infinite extent, the result $F = 2\pi\sigma$ is not restricted to points infinitely close to the surface. The results obtained so far assume the mediums of the field to be **air**.

30. Coulomb's Law. The results given above are special cases of a general result which Coulomb enunciated as an important law in Electrostatics. It states that the magnitude of the electric force at any point infinitely close to the surface of a charged conductor, *surrounded by air*, is equal to $4\pi\sigma$, where σ is the density of the charge at the point, and its direction is, at every point, normal to the surface of the conductor at that point. The surface of the conductor is an equipotential surface, and therefore the direction of the force at any point must be normal to the surface at that point. To determine the magnitude of the force at a point, P (Fig. 46), infinitely close to the charged surface, imagine a very small closed surface placed as A B with its ends A and B infinitely close to and parallel to the surface at the point considered, one end inside and the other outside the charged surface and bounded laterally by a tubular surface determined by lines drawn normal to the charged surface through all points on the boundary of each of the ends. Then if F denote the electric force at P, and a the area of the ends of the closed surface and also, since the three surfaces are infinitely close together, of the portion of the electrified surface intercepted within the closed surface, the total normal induction over the surface is evidently $F a$, and this by Gauss's theorem is equal to $4\pi a \sigma$, and therefore

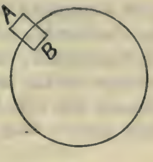


Fig. 46.

$$F a = 4\pi a \sigma$$

or

$$F = 4\pi\sigma.$$

This is a general result, true for a closed charged

conductor of any form but requires to be modified when the insulating medium surrounding the conductor is not air.

31. Relation between Tubes of Force and the intensity of the Electric Field. The electric force at any point in the field is proportional to the number of tubes of force that cross unit area of a surface drawn at right angles to the lines of force at the point, and is measured, when air is the medium of the field, by 4π times that number. Let the force at any point, P, in the field be F units. Consider a small area at P, at right angles to the direction of the force, and produce the tubes of force that pass through this area back to the positively charged surface from which they start. Let a' be the small area on the charged surface from which the tubes of force start, and let F′ be the force at a point very near the surface of this area. Then, by Gauss's theorem, we have

$$\mathrm{F}\,a - \mathrm{F}'\,a' = 0, \quad \text{or} \quad \mathrm{F}\,a = \mathrm{F}'\,a'.$$

But assuming the surrounding medium to be air, we have by Coulomb's law $\mathrm{F}' = 4\pi\sigma$, where σ is the surface density on the area a'. Hence we get $\mathrm{F}'a' = 4\pi\sigma a'$. But $\sigma a'$ is the charge on the area a', and is therefore equal to N, if N denote the number of unit tubes of force which start from this area. We therefore get

$$\mathrm{F}\,a = 4\pi\mathrm{N},$$

or, if n denote the number of unit tubes *per unit area* crossing the area a at P, then $\mathrm{N} = n\,a$, and $\mathrm{F}\,a = 4\pi n\,a$ or $\mathrm{F} = 4\pi n$; that is, the electric force at any point in the field, where air is the surrounding medium, is equal to 4π times the number of tubes of force per unit area crossing a very small area taken at right angles to the direction of the force at that point.

32. Mechanical Force per unit of Surface Area of a Charged Conductor. The electric force at any point in air, very near but *outside* the surface of the charged conductor, is $4\pi\sigma$, where σ is the surface density at that point. If the point be very near but *inside* the charged surface the force is zero. Now for a point outside the charged surface the force may be considered as made up of two parts, F_1

and F_2, F_1 being due to the charge on a very small area round the point, and F_2 to the charge on the rest of the surface. Similarly, if the point is just inside the charged surface the value of F_2 is practically unchanged, but that of F_1 is reversed in direction. Hence we get

$$F_1 + F_2 = 4\pi\sigma$$

and

$$-F_1 + F_2 = 0,$$

and, therefore,

$$F_1 = F_2 = 2\pi\sigma.$$

If a denote the area of the very small surface carrying the charge to which F_1 is due, then the charge on this surface is $a\sigma$, and the total force experienced by this, due to the charge on the rest of the surface, is $2\pi\sigma \cdot a\sigma = 2\pi a\sigma^2$. That is, the mechanical force per unit of area is $2\pi\sigma^2$. Since the surface is an equipotential one the direction of the force is outwards along the normal to the surface. Hence the surface of a charged conductor is subject to an outward tension, and the tension per unit area at any point where the surface density is σ is $2\pi\sigma^2$, when the surrounding medium is air. The greatest value of σ possible in air is about 8 in C.G.S. units, so that the maximum value of $2\pi\sigma^2$ is about 400 dynes per sq. cm., or, taking an atmosphere pressure to be 10^6 dynes per sq. cm., about the $\frac{1}{2500}$th part of an atmosphere pressure.

33. Energy in the Electric Field. It has been shown that the potential energy of the charge on a charged conductor is measured by $\frac{1}{2}QV$, where the charge is Q units, and the potential of the conductor V units. If there are several charged conductors in the field, the potential energy of each charge is measured in the same way, and the total energy in the field is given by $\Sigma \frac{1}{2}QV$, the summation being for all the charges in the field. This result is true whatever be the dielectric medium of the field. If the system of charged conductors are isolated from all external sources of energy, then for any displacement in the system the charge on each conductor remains unchanged, but the potentials change, and the work necessary to effect the displacement is provided by the loss of potential energy in the system. As the potential energy of a system always

tends to a minimum, it is therefore evident that the system tends to undergo any displacement which is associated with a decrease in the potential energy of the system. If, however, the system of charged conductors is connected with sources of electrical energy, so that during any displacement the potentials can be kept constant, then the system tends to undergo displacement, causing an increase in the potential energy of the system, and the external sources of energy are required to supply not only the energy necessary to effect the displacement, but also to provide for the increase in the potential energy of the system. If, for example, the conductors in the field be connected each to one pole of a battery, the other pole of which is earthed, the conductors will, for any displacement, maintain constant potentials, and the energy during displacement will be derived from the batteries. It can be proved that for a given *small* displacement the loss of potential energy, when the displacement takes place at constant charges in a system isolated from external sources of energy, is equal to the gain of energy in the system if the displacement take place at constant potentials in a system connected to external sources of electrical energy. Since the mechanical work done against opposing forces in effecting the displacement is, by the principle of conservation of energy, equal to the loss of potential electrical energy, when the charges are kept constant during the displacement, it follows that it is also equal to the gain of potential energy when the potentials are kept constant, and therefore, for a displacement at constant potentials, the external source of energy has to provide an amount of energy equal to twice the work done during the displacement.

To prove that, for a *very small* displacement, the loss of energy at constant charges is equal to the gain of energy at constant potentials, let $Q_1, Q_2, Q_3, \ldots$ and $V_1, V_2, V_3, \ldots$ be the initial charges and potentials of the conductors of the system. If, for a small displacement with constant charges, the potentials change to $V'_1, V'_2, V'_3, \ldots$ then the loss of potential energy is measured by

$$\tfrac{1}{2} Q_1(V_1 - V'_1) + \tfrac{1}{2} Q_2(V_2 - V'_2) + \text{etc.}$$

Similarly, if for the same small displacement with constant

potentials the charges change to Q'_1, Q'_2, Q'_3 . . . then the gain of potential energy is measured by

$$\tfrac{1}{2}\,V_1(Q'_1 - Q_1) + \tfrac{1}{2}\,V_2(Q'_2 - Q_2) + \text{etc.}$$

The difference between the expression for the loss of energy in the one case and the gain of energy in the other case is equal to

$$\tfrac{1}{2}\{(Q_1 - Q'_1)(V_1 - V'_1) + \ . \ . \ . \ \} + \tfrac{1}{2}\{(Q_1V_1 - Q'_1V'_1) + \ . \ . \ . \ \}$$

or

$$\tfrac{1}{2}\{\Sigma(Q - Q')(V - V')\} + \tfrac{1}{2}\{\Sigma(Q\,V - Q'V')\}.$$

Now, if the displacement is very small, the quantities $(Q - Q')$ and $(V - V')$ denoting the changes in the charges and potentials resulting from the displacement are very small quantities, and therefore their product is negligible; that is, the first term of the above expression is negligibly small. Again the quantities Q, V' and Q', V are for the same conductor in the same configuration of the system, and therefore $\frac{Q}{V'} = \frac{Q'}{V}$ or $Q\,V = Q'V'$, for the capacity of any conductor is the same when the configuration of the system is the same. Hence $Q\,V - Q'V' = 0$, and therefore the second term of the above expression is zero—whether the displacement be large or small.

Hence, the difference between the expressions for the change of energy is negligibly small if the displacement is small, and therefore with this condition the loss of energy with constant charges is equal to the gain of energy with constant potentials, and each quantity is equal to the mechanical work done during displacement. The theory of the quadrant electrometer given in Art. 36 illustrates the application of this principle.

So far the potential energy in the field has been spoken of as the potential energy of the charges on the charged conductors, but if we regard the medium as the active agent in the electrical phenomena of the field, we must regard the medium as the seat of the potential energy, which, as already indicated, may be assumed to exist in it as energy of strain. It is, therefore, of interest to consider the distribution of the energy in the medium, and to find an expression for the energy per unit volume of the medium

at any point in the field. In dealing with the distribution of the energy throughout the medium it is convenient to associate it with Faraday's tubes of force. Taking a complete unit tube in the field, it will have a unit positive charge at one end and a unit negative charge at the other end. Let V and V′ denote the potentials of the two corresponding ends of the tube. The energy of these unit charges is measured by $\frac{1}{2}V - \frac{1}{2}V'$ or $\frac{1}{2}(V - V')$, and this energy may be supposed to be in the portion of the medium bounded by the tube of force. That is, the energy located in each tube of force is measured by one half the difference of potential between the ends of the tube. This difference of potential may be written as $\frac{1}{2}\Sigma F . d$, where F is the electric force at any point on the axis of the tube, and d a very short length of the tube taken at that point, the summation to extend from one end of the tube to the other. Hence we may write the energy located in the tube equal to $\frac{1}{2}\Sigma F. d$, which is equivalent to stating that the energy associated with a tube of force, *per unit length of the tube*, is at any point in the length of the tube numerically equal to one half the electric force at that point. To determine the amount of energy per unit volume of the medium at any point in the field, consider a very short length, d, of a unit tube of force, taken at the point. Let F denote the electric force at that point. Then the energy located in this short length of the tube is $\frac{1}{2}F\,d$. But if n denote the number of tubes per unit area at this point, the area of one tube is measured by $1/n$, and since by Art. 31 $F = 4\pi n$ in air, we get

$$n = \frac{F}{4\pi} \text{ or } \frac{1}{n} = \frac{4\pi}{F}.$$

The volume of the short length of the tube may now be calculated, for its length is d and its area of cross section $4\pi/F$, and therefore its volume is $4\pi d/F$ The energy located in this volume was found to be $\frac{1}{2}F d$ Therefore the energy per unit volume must be given by

$$\frac{Fd}{2} \Big/ \frac{4\pi d}{F} \text{ or } \frac{F^2}{8\pi},$$

which is the usual expression for the energy per unit volume of the air medium at a point where the strength of the electric field is measured by F.

If we imagine the medium to be mapped out into equal tubes of force crossed by equipotential surfaces corresponding to successive equal differences of potential (Fig. 47), it will evidently be divided up into small blocks or cells, each containing the same quantity of energy. The energy in each cell is measured by one half the difference of potential between its ends multiplied by the number of unit tubes which it includes, and these quantities are the same for all the cells. If a small uncharged conductor is placed in the field, then a loss of energy results to the medium, due to the disappearance of the state of strain within the space occupied by the conductor. In accordance with the principle that the potential energy of a system always tends to a minimum, it is evident that if the conductor is free to move it will tend to move towards the strongest part of the field, where the loss of energy due to its presence in the medium is a maximum. The force urging the conductor in any direction is measured by the rate of decrease of the potential energy of the medium with displacement in the given direction. It should be noted that the loss of energy to the medium is here due, not only to the disappearance of the strain in the space occupied by the conductor, but also, in some measure, to the readjustment of the strain necessary to set the lines of force in the medium external to the conductor at right angles to the surface of the conductor. For example, in the case of a field due to a simple point charge, Q, the force at any point at distance r from the charge is Q/r^2, and therefore $F^2/8\pi$, the energy per unit volume at this point, is equal to $\frac{Q^2}{8\pi r^4}$. The rate of decrease of this for displacement towards the charge is $\frac{Q^2}{2\pi r^5}$,*

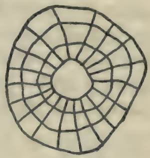


Fig. 47.

* Differentiate with respect to

and this gives a lower limit for the value of the force per unit volume urging a small conductor, placed at this point, towards the charge. This result is only a lower limit because the *loss* of energy attendant on the redistribution of the strain external to the conductor has been neglected, that is, it has been assumed that the value of F is the same after the introduction of the small conductor into the field as before. From the value obtained above for the force per unit volume on the small conductor it can only be stated that the force on say, a very small spherical conductor of radius ρ placed at a distance r from a point charge Q is *greater than* $\frac{Q^2}{2\pi r^5} \times \frac{4\pi\rho^3}{3}$ or $\frac{2 Q^2 \rho^3}{3 r^5}$. The real value is $\frac{2 Q^2 \rho^3}{r^5}$, just three times this quantity.

34. Tension and Pressure in Faraday Tubes of Force. It has already been proved (Art. 32) that the pull or tension per unit area at the surface of a charged conductor is measured by $2\pi\sigma^2$ in air, or in any medium by $\frac{1}{2}F\sigma$, at a point where σ is the surface density of the charge. Now from this unit area of the surface, σ tubes of force are supposed to emanate, and if we imagine each tube to exert a pull equal to $\frac{1}{2}F$, the required tension on the surface would be obtained. Hence, we may regard the tension at the surface of the conductor as due to a pull arising from the fact that the Faraday tubes originating at the surface are in a state of tension, the magnitude of the tension at any point being numerically equal to one half the electric force at that point. That is, the tension at any point in a tube of force has the same measure as the energy per unit length of the tube at the same point. Since the area of cross section of the tube is $1/n$, where n is the number of tubes of force per unit area, the tension per unit area at any point in a tube of force is $\frac{1}{2}Fn$. But $F = 4\pi n$ (Art. 31), that is, $n = \frac{F}{4\pi}$, and therefore the tension per unit area, at a point on the tube where the electric force is F, is measured by $\frac{F^2}{8\pi}$, which is also the

measure of the energy per unit volume of the medium at the same point. It can be shown that if this tension is supposed to exist along the tubes of force in the field it is necessary, in order to maintain equilibrium in the medium, that a pressure of magnitude $\frac{F^2}{8\pi}$ per unit area shall also exist in a direction at right angles to the tubes of force, that is in the plane of the equipotential surface at any point.

This system of tension along the tubes of force and pressures at right angles to the tubes indicates the nature of the stress in the medium of an electric field and may be regarded as the cause of the forces which act on a charged conductor placed in the field.

35. Coefficients of Potential, Capacity, and Induction. In the case of a system of conductors in the same field the potential of any conductor is due not only to its own charge but to all the other charges in the field. This may be expressed for a number of conductors with charges and potentials denoted by Q_1, Q_2, Q_3 . . . and V_1, V_2, V_3 . . by writing.

$$V_1 = {}_1p_1Q_1 + {}_1p_2Q_2 + {}_1p_3Q_3 + \ldots$$
$$V_2 = {}_2p_1Q_1 + {}_2p_2Q_2 + {}_2p_3Q_3 + \ldots$$

Here the quantities, ${}_1p_1$, ${}_1p_2$, ${}_1p_3$ . . . indicate the extent to which V_1 depends upon Q_1, Q_2, Q_3 . . ., and these quantities are called *coefficients of potential.*

Similarly, the charge on any conductor of the system is related not only to the potential of that conductor but also to the potentials of all the other conductors. That is we may write

$$Q_1 = {}_1q_1V_1 + {}_1q_2V_2 + {}_1q_3V_3 + \ldots$$
$$Q_2 = {}_2q_1V_1 + {}_2q_2V_2 + {}_2q_3V_3 + \ldots$$

Here the quantities ${}_1q_1$, ${}_1q_2$, ${}_1q_3$ . . . indicate the extent to which Q_1 depends upon V_1, V_2, V_3, . . . and these quantities are called *coefficients of capacity* or *coefficients of induction,* according as the suffixes are the same or different, that is, ${}_1q_1$, ${}_2q_2$, ${}_3q_3$ are coefficients of capacity, ${}_1q_2$, ${}_1q_3$, etc., coefficients of induction.

If the set of equations involving the coefficients of

potential be solved for Q_1, Q_2, Q_3 . . ., it is evident that the second set, involving the coefficients of capacity and induction, can be obtained, and that therefore these coefficients can be expressed in terms of the coefficients of potential. It can also be proved generally that reciprocal coefficients such as ${}_1p_2$ and ${}_2p_1$, ${}_1p_3$, and ${}_3p_1$, ${}_1q_2$, and ${}_2q_1$, ${}_1q_3$ and ${}_3q_1$ are equal.

As an example of the use of these coefficients, consider the case of two insulated concentric spherical conductors of radii a and b charged with quantities Q_1 and Q_2 to potentials V_1 and V_2 for the inner and outer conductors respectively. Now the charge Q_1 on the inner conductor produces a potential Q_1/a for that conductor, and Q_1/b for the outer conductor, that is,

$${}_1p_1 = \frac{1}{a} \text{ and } {}_2p_1 = \frac{1}{b}$$

Similarly the charge Q_2 on the outer conductor produces a potential Q_2/b for that conductor, and Q_2/b also for the inner conductor, that is

$${}_2p_2 = \frac{1}{b} \text{ and } {}_1p_2 = \frac{1}{b}.$$

We may therefore write for

$$V_1 = {}_1p_1Q_1 + {}_1p_2Q_2$$

$$V_2 = {}_2p_1Q_1 + {}_2p_2Q_2$$

the equations

$$V_1 = \frac{1}{a}Q_1 + \frac{1}{b}Q_2,$$

$$V_2 = \frac{1}{b}Q_1 + \frac{1}{b}Q_2.$$

Solving these equations for Q_1 and Q_2 we get

$$Q_1 = \frac{ab}{b-a}V_1 - \frac{ab}{b-a}V_2,$$

$$Q_2 = -\frac{ab}{b-a}V_1 + \frac{b^2}{b-a}V_2.$$

That is

$${}_1q_1 = \frac{ab}{b-a};\ {}_1q_2 = \frac{-ab}{b-a} = {}_2q_1,\ {}_2q_2 = \frac{b^2}{b-a}.$$

An interesting example of the use of coefficients of capacity and induction is found in the explanation of the action of **electric screens.** Consider the three conductors, A, B, C (Fig. 48), and imagine A to be enclosed in C, and B outside it. Let Q_1, Q_2, Q_3, and V_1, V_2, V_3 be the charges and potentials of these conductors, then we may write—

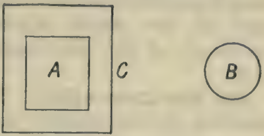


Fig. 48.

$$Q_1 = {}_1q_1V_1 + {}_1q_2V_2 + {}_1q_3V_3$$
$$Q_2 = {}_2q_1V_1 + {}_2q_2V_2 + {}_2q_3V_3$$
$$Q_3 = {}_3q_1V_1 + {}_3q_2V_2 + {}_3q_3V_3.$$

If now C be connected to earth its potential will be zero, and if A be also supposed to have no charge its potential in the interior of C must also be zero. That is, if V_3 and Q_1 are zero V_1 must also be zero, and the first equation be-becomes $0 = {}_1q_2V_2$, which shows that ${}_1q_2 = 0$ for all values of V_2. Hence, if A and B be charged, and C earth-connected, the above equations reduce to

$$Q_1 = {}_1q_1V_1$$
$$Q_2 = {}_2q_2V_2$$
$$Q_3 = {}_3q_1V_1 + {}_3q_2V_2,$$

for V_3 is zero in value, and it has just been proved that ${}_1q_2 = 0$ and ${}_2q_1 = {}_1q_2$.

From these simplified equations it is evident that when C is earth-connected the charge on A (Q_1) depends upon its own potential V_1 only, and similarly the charge on B (Q_2) depends upon its potential V_2 only. That is, the action between A and B is completely stopped by the earth-connected conductor, C, which may thus be said to screen A from the inductive action of the charge on B.

The same result is obvious from a consideration of the distribution of the tubes of force in the field of A, B, and C. The tubes starting from A must all terminate on C; those from B will terminate on C and on other adjacent earth-connected objects, but no tubes cross from A to B or from B to A.

The method of coefficients is somewhat roundabout for simple problems in electrostatics, just as the use of equations may be for simple algebraic problems, but in general problems of the electrostatic field they are found serviceable.

36. The Kerr effect. The strain produced in a dielectric medium by electrostatic stress has been shown by Dr. Kerr to be such as to produce double refraction in the medium. A piece of glass, subjected to electric stress by placing it between the terminals of an influence machine, was found to doubly refract a beam of light passed through it. The axis of double refraction is parallel to the direction of the electric field, and the difference of phase produced between the ordinary and extraordinary waves is approximately proportional to the square of the electric force.

CHAPTER VII.

CONDENSERS.

37. Electrical Condensers. In Fig. 49 let A represent a spherical conductor of radius R_1, to which a positive charge, Q, is given. The potential of the conductor due to this charge is $\frac{Q}{R_1}$. Let this conductor be surrounded by a hollow conducting shell, B, of radius R_2. This shell is connected to earth, and the space between the conductors is supposed to be filled with air. The positive charge Q acting inductively across the layer of air between A and B induces a negative charge, $-Q$, on the inner surface of B, and repels a positive charge equal to Q to earth.

Fig. 49.

The potential of A is now due to its own charge *and to the charge on* B. The potential due to its own charge is $\frac{Q}{R_1}$ and that due to the charge on B in $\frac{-Q}{R_2}$; hence, the potential of A is $\frac{Q}{R_1} - \frac{Q}{R_2}$, that is, its original potential $\frac{Q}{R_1}$ is now diminished by a quantity equal to $\frac{Q}{R_2}$. This result shows that the capacity of A has been *increased* by surrounding it with the earth-connected conductor B, for the same quantity of electricity, Q, raises it to a lower potential

than at first, and, therefore, a greater quantity would now be required to raise it by unit potential. The capacity of A by itself is R_1, but after surrounding it with B, it is greater than this, and its value may be determined from the fact that the charge Q raises it to potential $\frac{Q}{R_1} - \frac{Q}{R_2}$, that is, if C denote the increased capacity we have

$$\frac{Q}{C} = \frac{Q}{R_1} - \frac{Q}{R_2},$$

for $\frac{Q}{C}$ also expresses the potential of the conductor. This gives

$$\frac{1}{C} = \frac{1}{R_1} - \frac{1}{R_2},$$

or

$$C = \frac{R_1 R_2}{R_2 - R_1}.$$

From the fact that the capacity of A is increased by the presence of the earth-connected conductor B, this arrangement of conductors is called an electrical *condenser*. A greater quantity of electricity must now be given to A to raise it to a given potential, that is, the arrangement of the conductors A and B as a *condenser* enables A to receive a greater charge for a given rise of potential than it can receive as a simple conductor. For example, if A be connected to the prime conductor of an electrical machine it will be charged to the potential of the prime conductor, but the magnitude of the charge will be much greater when it forms a condenser with B than when it is charged as an isolated conductor.

It is not at all necessary that a condenser should have the form indicated in Fig. 49. Any two conductors separated by a layer of insulating substance will act as a condenser. One conducter must be insulated, and the other must be earth-connected—the former receives the charge, the latter, by the reaction of its induced charge, diminishes the rise of potential due to the charge, and admits of greater accumulation of electricity than if no

earth-connected conductor were present. A familiar form of condenser is the Æpinus air condenser shown in Fig. 50.

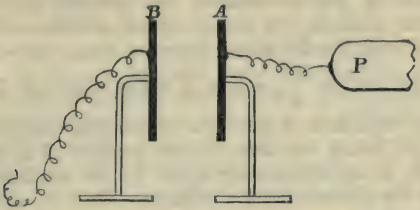


Fig. 50.

This consists of two metal discs, A and B, mounted on insulating stands. They are set parallel to each other, and the layer of air between them forms the separating insulating medium. Imagine A connected to the prime conductor, P, of an electrical machine, and B connected to earth. Then A receives a positive charge, and its potential gradually rises. At the same time a negative charge is induced on B, and *the negative potential at A* due to this charge also increases.

Hence, the rise of potential in A, due to the charge coming from P, is retarded by the *negative* potential, due to the induced charge on B, and thus a large quantity of electricity will flow into A before it rises to the potential of P. This action of the condenser may also be seen in this way. Imagine B to be removed, and A connected with the prime conductor; then electricity will flow into A, and it will quickly attain the potential of P. If B be now placed in position a negative charge is induced in it, and the potential at A due to this induced charge will reduce the potential of A to a value below that of P, and, as a consequence, a further flow of electricity will take place from P to A, until equality of potential is again established.

From the relation, $C = \frac{R_1 R_2}{R_2 - R_1}$, obtained above, it is

evident that the capacity of the spherical air-condenser there considered increases as the difference (R_2-R_1) diminishes; that is, the thinner the layer of air between the plates of the condenser, the greater the capacity. Also, if R_1 and R_2 are nearly equal, and therefore large compared with the difference between them, (R_2-R_1), then we may write $C = \frac{R^2}{d}$, where R is the mean of R_1 and R_2 and d the thickness of the layer of air between the plates. But $4\pi R^2$, being the area of the surface of a sphere of radius R, is approximately equal to the surface area of either of the plates of the condenser; that is, $4\pi R^2 = S$, or $R^2 = \frac{S}{4\pi}$, where S denotes the area of the surface of one of the plates of the condenser. Hence, substituting this value for R^2 in $C = \frac{R^2}{d}$, we have $C = \frac{S}{4\pi d}$, that is, the capacity of the condenser *per unit area* of surface is $\frac{1}{4\pi d}$. This result is specially applicable where the plates of the condenser are parallel plane surfaces at a distance d apart, for such a condenser may be considered as a part of a spherical condenser with surfaces of infinitely long radii, and consequently the assumption that R is infinitely greater than d is strictly true. Hence the capacity of a parallel plate air condenser may be taken as $\frac{1}{4\pi d}$ per unit of area, and this agrees with the result obtained by the method of Art. 41.

It can be shown that this result is roughly true for a condenser of any form, so long as d is small compared with the other dimensions. Hence we see that the capacity of such a condenser varies directly as the surface area of its plates, and inversely as the thickness of the layer of air between the plates; that is, the larger the plates and the closer they are together the greater will be the capacity of the condenser they form.

Further, it is found by experiment that the capacity of a condenser varies with the nature of the insulating

medium or *dielectric* separating its plates. For example, if the capacity of a condenser with air as dielectric be C, then the capacity of the *same* condenser with, say, shellac as dielectric will have a different value, C_1, and with paraffin as dielectric another different value, C_2, and so on. This variation of capacity with the nature of the dielectric employed depends upon the *specific inductive capacity* of the dielectric; and *the ratio of the capacity of a condenser with some given substance as dielectric to the capacity of the same condenser with air as dielectric is a measure of the specific inductive capacity of the given substance.* Hence, if the capacity of an air condenser be taken as $\frac{S}{4\pi d}$, then the capacity of that same condenser with a substance of specific inductive capacity K as dielectric is given by $\frac{KS}{4\pi d}$; that is, the capacity of a condenser of given form and dimensions increases with the specific inductive capacity of the dielectric between its plates.

There are several simple forms of apparatus constructed to illustrate the action of a condenser. If a small square of tinfoil be pasted on the middle of one surface of a pane of glass it will act as an insulated conductor, but its capacity is very small, that is, it can take only a very small charge. This may be shown by transferring the charge from the prime conductor of a machine to the tinfoil by means of a proof-plane. At the first contact a small spark passes, indicating the transfer of the charge from the proof-plane to the foil; at the second contact the spark is much feebler, and after a few times no spark can be seen or heard, indicating that the foil has been raised to the potential of the proof-plane, and that no further transfer can be effected. If, however, another similar piece of foil be pasted on the other side of the pane, directly opposite the first piece, and connected to earth, the two pieces of tin-foil act as a condenser, and a much larger charge may now be given to the first. If the charging be effected as before by means of the proof-plane, it will be found that a very great number of contacts have

to be made before the cessation of the spark indicates that the charging is complete. The two pieces of foil are now charged, the one positively and the other negatively; but if either be touched with the finger, the other being insulated, no discharge takes place, because each charge is, as it were, bound by the attraction of the other. Hence a condenser cannot be discharged by touching or "earthing" either of its plates, but if the two plates be connected discharge at once takes place, that is, the two opposite charges unite and neutralise each other. If the discharge takes place through the human body, for example, if the plates of a condenser be connected by placing one hand on one plate and the other on the other plate, the discharge takes place through the arms and shoulders, and a convulsive muscular jerk, commonly called the electric "shock," is experienced. This is more or less severe according to the energy of the discharge.

38. The Leyden Jar. A very familiar form of condenser is called the Leyden Jar. It consists, as shown in Fig. 51, of a glass jar having an inner and outer coating of tinfoil. These coatings cover the bottom and sides up to about three-quarters of the height of the jar, and act as the plates of the condenser. To exclude dust, and for convenience in use, the mouth of the jar is closed by a lid made of dry well-varnished wood, and contact is made with the inner coating by means of a brass rod, which passes through the centre of the lid. This rod terminates above in a rounded knob and carries below a loose piece of brass chain, the lower end of which rests on the tinfoil. To charge the jar the inner coating is put in connection with the prime conductor of an electric machine, that is, the knob of the jar is connected to the prime conductor or is put in direct contact with it, and the outer coating is connected to earth, for example, by being held in the hand. The inner coating thus receives, say, a positive charge, and an equal negative charge is induced on the inner surface of the outer coating, the corresponding positive

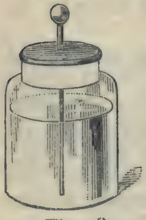


Fig. 51.

charge being repelled to earth. These charges in the jar exert mutual attraction, and must therefore reside on the surfaces of the coatings nearest the glass, or on the inner and outer surfaces of the glass itself where they are as near as possible to each other. That this is actually the case may be shown experimentally by means of a jar with movable coatings (Fig. 52). The jar is charged in the usual way and placed upon an insulating stand, for example, on a sheet of dry varnished glass. The inner coating is now lifted out by means of an insulating loop of silk thread or ribbon and placed on the sheet of glass. The glass jar is now lifted out of the outer coating and also laid on the glass. If either of the metallic coatings be now removed and tested by the electroscope, it will be found to possess no charge whatever; but if the jar be put together again as before, it will be found to be as strongly charged as at first. This shows that the charge may be located on the surfaces of the glass which forms the dielectric of the condenser. If a Leyden jar be charged and placed upon an insulating stand, either coating may be connected to earth, for example, by touching it with the hand, without discharging it, because the charges are mutually bound, and neither can escape because of the attraction exerted by the other. If, however, the two coatings be connected, discharge at once takes place; for example, if the little finger be placed on the outer coating, and the thumb be gradually brought near the knob of the inner coating, the strain in the air-gap separating the two will ultimately become too great, and a spark will pass, effecting the discharge of the jar and producing a convulsive shock in the muscles of the hand.

Fig. 52.

Similarly, if a jar be charged and laid on the table, and a person standing on the floor present his knuckle to the knob, a spark passes and discharge takes place through table, floor, and the person of the experimenter. A safer and

more pleasant method of discharging a jar is by means of the discharging tongs of Fig. 53, or by a bent piece of stout copper wire held in the hand.

Fig. 53.

The capacity of a Leyden jar,* as of any other form of condenser, varies with its size: the larger it is the greater is its capacity, and for a given size the capacity is inversely proportional to the thickness of the glass jar, and directly proportional to the specific inductive capacity of the kind of glass used. It is not convenient to construct or work with very large jars, so when a jar of very large capacity is required, the expedient of uniting several small jars together is adopted. The outer coatings are all connected together, generally by placing them in a box lined with tinfoil, and the inner coatings are all joined by short brass rods passing from knob to knob. This arrangement of jars is called a *battery* of jars, and if n jars are so arranged, they are in reality equivalent to one jar n times as large, and consequently of n times the capacity of one of the components of the battery. The capacity of a jar is also increased by using thin glass, but the glass must not be too thin, or it may break down under the strain produced in it when the jar is charged.

If a jar be strongly charged, and then discharged, it will be found that if allowed to stand for some time a second small spark may be obtained by discharging it again. This charge which thus collects in the jar on standing is called the *residual charge.* It is due to the gradual recovery of the glass from the strain produced by the stress to which it is subjected while the jar is charged. Hence, the more highly the jar is charged and the longer it remains charged before being discharged the greater will be the residual charge. The magnitude of this charge also varies with the nature of the dielectric: thus an air condenser

* The Leyden Jar being a condenser in which the thickness of the glass is small compared with its other dimensions, its capacity is approximately given by $C = \frac{KS}{4\pi d}$, where S denotes the area of either coating, K the sp. ind. cap., and d the thickness of the glass.

exhibits no residual charge, the air appearing to recover instantaneously from the strain to which it is subjected.

39. The Condensing Electroscope. The condensing electroscope is an application of the principle of the condenser which is sometimes very convenient. The instrument shown in Fig. 54 is an ordinary gold-leaf electroscope, with a somewhat larger cap than usual. The cap C acts as the insulated plate of a condenser, and another similar brass disc, D, acts as the other plate. This disc is usually attached to an insulating handle, *h*, and is carefully varnished, so that when laid on the cap of the electroscope the two plates form a condenser with a very thin layer of dielectric made up of the varnish and a film of air. If it be required to test the electrification of a body of very large capacity, but at a very low potential, the ordinary electroscope would be of no use. For, on placing the body in contact with the cap of the instrument, the common potential of the system would be slightly less than the initial potential of the body, and the capacity of the leaves is so small that the quantity of electricity necessary to charge them to this low potential is not sufficient to cause any appreciable divergence. If, however, a condensing electroscope be used with its plates C and D in position as a condenser, and the body be placed in contact with C, while D is connected to earth, the plate C will receive a very large charge. The common potential of the system will, however, be much lower than the initial potential of the body, and consequently the charge in the leaves is not sufficient to cause any divergence. When the condenser is charged, let the body be removed and the plate D slowly raised. As D is raised, the potential of C rises, owing to the

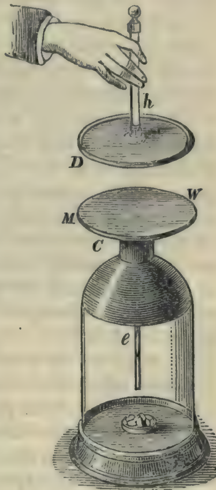


Fig. 54.

recession of the opposite charge on D, and electricity flows down to the leaves and ultimately produces divergence. When D is entirely removed, the capacity of C is so much reduced that the charge it received *as the plate of a condenser* is sufficient to raise it and the gold leaves to a very high potential, and thus the latter receive a sufficient charge to produce divergence.*

The *potential of a condenser* is the potential of the insulated plate when the other is earth-connected, or, more generally, it is *the difference of potential* between the two plates. The *capacity* of a condenser is therefore equal to the quantity of electricity necessary to produce unit difference of potential between its plates. If V be the potential of a condenser, and C its capacity, then the charge in it is given by $Q = V\,C$, and the potential energy of that charge or the energy of discharge is, as in the case of a simple conductor, given by $\frac{1}{2}\,Q\,V$.

It will be noticed that the capacity of a condenser depends upon three things. 1. The size of the plates—the greater the size the greater the capacity. 2. The thickness of the dielectric—the greater the thickness the less the capacity. 3. The specific inductive capacity of the dielectric—the greater the specific inductive capacity the greater the capacity.

40. Capacity of Spherical Condenser with Inner Conductor connected to Earth. In thiscase, if the inner sphere be earth-connected and the outer one insulated, the question is slightly complicated. The inner sphere is at potential zero and the outer one at potential V_1, say. The potential of the surroundings of the outer sphere may also be assumed to be zero. Hence the lines of force emanating from the outer sphere will run from its inner surface to the inner sphere and also from its outer surface to the surrounding objects, that is, there will be a positive charge on both the inner and outer surfaces of the sphere. Since all the lines emanating from the inner surface must terminate

* The charge on the leaves depends upon *their* capacity and the potential to which they are raised ($Q = C\,V$). Their capacity is very small; hence, for an appreciable charge, the potential must be high. The capacity of *the leaves* is not affected appreciably by the position of the condensing plate.

on the inner sphere it follows that the negative charge on the latter is equal to the positive charge on the former and similarly the positive charge on the outer surface is equal to the negative charge distributed over the surroundings on which the tubes of force emanating from the outer surface terminate. Let R_1, R_2, and R_3 denote the radii of the charged spherical surfaces, in order from the centre. Then, since the arrangement practically consists of a spherical condenser with surfaces of radii R_1, R_2, and a spherical conductor of radius R_3, the combined capacity of the arrangement is $\frac{R_1 R_2}{R_2 - R_1} + R_3$. The potential of each of the two inner surfaces is *equally* affected by the charge on the outer surface and therefore the *difference* of these two potentials is independent of the outer charge.

Also the potential at the outer surface is not affected by the charges on the inner surfaces, since the algebraic sum of these charges is zero. Hence, it follows that the capacity of the condenser formed by the two inner surfaces is unaffected by the third outer charge, and also that the capacity of the outer spherical surface is unaffected by the inner charges, and therefore the capacity of the system is the sum of the capacities of the two constituent parts. The capacity of the outer spherical surface is measured by R_3 only when the surrounding earth-connected objects are at an infinite distance from it. If one or more earth-connected conductors are in the near neighbourhood of the surface the magnitude of the charge on this surface will be increased by an amount depending upon the conditions of the case. As an extreme case, for example, if the system be surrounded by a fourth concentric spherical surface, of radius R_4, connected to earth, the system evidently becomes a double spherical condenser, the radii of the surfaces being R_1 and R_2 for one condenser, and R_3 and R_4 for the other, and the capacity of the system is

$$\frac{R_1 R_2}{R_2 - R_1} + \frac{R_3 R_4}{R_4 - R_3}.$$

41. Capacity of a Cylindrical Air-condenser. A cylindrical condenser is made up of two coaxial cylindrical

plates. If the inner plate is charged and the outer plate connected to earth, the induced charge on the outer plate will be equal in magnitude and opposite in sign to that on the inner plate; for it is evident that all the tubes of force emanating from the inner plate must terminate on the inner surface of the outer one. Let the charge *per unit of length* on the inner plate be Q units. Then, as shown in Art. 29, the electric force at a point at a distance r from the axis of the cylinders is $\frac{2\,Q}{r}$. Let the radii of the inner and outer charged cylindrical surfaces be a and b units respectively. Then the work done in conveying unit quantity of electricity from the outer to the inner surface is given by

$$-\int_b^a \frac{2\,Q}{r}\,d\,r \quad \text{or} \quad 2\,Q \log_e \frac{b}{a}.$$

But this is the difference of potential between the two plates. Hence the capacity of the condenser *per unit length* is

$$Q/2\,Q \log_e \frac{b}{a} \quad \text{or} \quad 1/2 \log_e \frac{b}{a}.$$

If the difference between b and a is very small compared with a, so that $b = a + d$, where d is very small, then the result reduces to $1/2 \log \frac{a+d}{a}$ or $1/2 \log \left(1 + \frac{d}{a}\right)$ or $1/2 \frac{d}{a}$ (approximately), that is, to $\frac{a}{2d}$. This approximate result is easily obtained more directly. If a and $a+d$ are the radii of the cylinders, and d is very small compared with a, then the electric force between the cylinders is approximately $\frac{2\,Q}{a}$ and the work done in conveying unit quantity of electricity from one plate to the other is approximately $\frac{2\,Q\,d}{a}$, and as this is approximately the difference of potential between the plates, the capacity per unit length is approximately equal to $Q\Big/\frac{2\,Q\,d}{a}$ or $\frac{a}{2\,d}$. This result may be written as

$\frac{2\pi a}{4\pi d}$, and $2\pi a$ is evidently the area of unit length of the charged cylindrical surface, so that in this case also the capacity of the condenser is given by $\frac{S}{4\pi d}$, and the capacity *per unit area* is $\frac{1}{4\pi d}$, where d is the distance between the plates, and is assumed to be very small compared with the radius of the plates.

42. Grouping of Condensers. When individual condensers are connected together as the jars in a battery of Leyden jars, that is, when the plates of the compound condenser are formed by joining together the corresponding plates of the individual condensers, they are said to be connected in parallel, and the capacity of the compound condenser is the sum of the capacities of the individual condensers. For if C denote the capacity of the compound condenser and V the difference of potential between its plates, then the total charge in it is given by $Q = CV$. But if C_1, C_2, C_3, etc., denote the capacities of the individual condensers and Q_1, Q_2, Q_3, etc., the individual charges, we have

$$Q = Q_1 + Q_2 + Q_3 + \ldots$$

and therefore

$$VC = VC_1 + VC_2 + VC_3 + \ldots,$$

that is

$$VC = V(C_1 + C_2 + C_3 + \ldots),$$

and therefore

$$C = C_1 + C_2 + C_3 + \text{etc.},$$

or

$$C = \Sigma C.$$

Leyden jars, insulated and connected in the way shown in the figure (Fig. 55), are said to be joined in *series* or in *cascade*. The free plates at the ends of the series form the terminals of the compound condenser. When condensers are joined in series the capacity of the compound condenser is less than that of the individual condensers, and can be shown to be equal to the reciprocal of the

Fig. 55.

sum of the reciprocals of the capacities of the component condensers. Taking the case of the jars, the charge on the outside of the first is evidently equal to that on the inside of the second, for the jars are insulated, and the algebraic sum of the two charges is zero. Similarly for the other jars. Also the charge on the inside of any jar is equal to the charge on the outside. It therefore follows that the charge in each jar is the same. Let this charge be Q units, then, with the same notation as above we get

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3} \ldots,$$

where V_1 V_2 V_3 . . . are the differences of potentials between the coatings of the individual jars.

But the difference of potential for the combination of jars is the difference between the potentials of the inside coating of the first jar and the outside coating of the last. That is, if this difference be denoted by V,

$$V = V_1 + V_2 + V_3 + \ldots$$

But $V = Q/C$, where C denotes the capacity of the combination, for Q is the total charge in the condenser. Hence we get

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \ldots$$

or

$$\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right),$$

that is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

or

$$\frac{1}{C} = \Sigma \frac{1}{C}$$

which establishes the rule that the reciprocal of C is the sum of the reciprocals of C_1, C_2, C_3, etc.

CALCULATIONS.

THE potential at any point at a distance r from a charge q is $\frac{q}{r}$ when the intervening medium is air, and $\frac{q}{K r}$ when the intervening medium has a specific inductive capacity K. (Art. 24.)

If a conductor be charged with a quantity of electricity, Q, to a potential V, then the potential energy of the charge is given by $\frac{1}{2} Q V = \frac{1}{2} V^2 C = \frac{1}{2} \frac{Q^2}{C}$. (Art. 25.)

If Q be expressed in electrostatic units of quantity (Art. 6), and V in ergs per unit of quantity, then the energy of the charge is expressed in ergs.

The electrical capacity of a conductor is defined as the ratio of the charge given to the conductor to the potential to which the conductor is raised by that charge. This is,

$$C = \frac{Q}{V},$$

where Q denotes the quantity of electricity constituting the charge, V the potential to which the conductor is raised, and C the capacity of the conductor. (Art. 27.)

In the case of a spherical conductor of radius R, the potential to which it is raised by a charge Q is given by—

$$V = \frac{Q}{R}. \qquad \text{(Art. 26.)}$$

But from the relation just given we have—

$$V = \frac{Q}{C}.$$

Therefore, $$C = R,$$

that is, the capacity of a sphere is measured by its radius. (Art. 27.)

From the ratio $C = \frac{Q}{V}$ we have $Q = V C$.

That is, if a conductor of capacity C be charged to a potential V, then the quantity of electricity with which it is charged is given by $Q = V C$. (Art. 27.)

Similarly, if a quantity of electricity Q raise the potential of a conductor from V to V′, then the capacity of that conductor is given by

$$C = \frac{Q}{V'-V}. \qquad \text{(Art. 27.)}$$

If two conductors of capacities C_1 and C_2 be charged to potentials V_1 and V_2, then on being placed in contact they will take up a common potential V, given by

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}. \qquad \text{(Art. 27.)}$$

That is, for any number of conductors

$$V = \frac{\Sigma(C\,V)}{\Sigma(C)}$$

After contact the charge on the conductor of capacity C_1 is $\frac{C_1}{C_1+C_2}$ of the total charge on *both* conductors, and the charge on that of capacity C_2 is $\frac{C_2}{C_1+C_2}$ of the total charge. The *ratio* of the charges on the two conductors is therefore $C_1 : C_2$, and each conductor has the same potential V.

The capacity of an *air condenser* (that is, a condenser in which air is the insulating medium between the plates) formed of two concentric spherical surfaces of radii R_1 and R_2 is given by

$$C = \frac{R_1 R_2}{R_2 - R_1}. \qquad \text{(Art. 37.)}$$

If V denote the potential of a *condenser*, and C its capacity, then the charge in it is given by $Q = V\,C$, and the energy of this charge is $\frac{1}{2} Q V = \frac{1}{2} V^2 C = \frac{1}{2} \frac{Q^2}{C}$.

The capacity of an air condenser of any form is approximately given by $\frac{S}{4 \pi d}$, where S denotes the area of one of the plates, and d the thickness of the air layer between the plates, if d is very small compared with the other dimensions of the condenser.

If the dielectric is of specific inductive capacity K, then under the same conditions the capacity is approximately given by $\frac{K\,S}{4 \pi d}$. This result implies that the capacity of the condenser per unit area of surface is

$$\frac{K}{4 \pi d}. \qquad \text{(Art. 37.)}$$

This result is strictly true for a parallel plate condenser for regions on the plates not too close to the edges.

The capacity of a cylindrical condenser is given by

$$\frac{K}{2\log_e \frac{b}{a}}$$

per unit length of the cylinder, where K is the specific inductive capacity of the dielectric, a the radius of the inner surface, and b the radius of the outer surface. (Art. 41.)

In the grouping of condensers the capacity of the compound condenser is given by

$$C = \Sigma C \text{ for parallel grouping,}$$

and by

$$\frac{1}{C} = \Sigma \frac{1}{C} \text{ for series grouping.} \qquad \text{(Art. 42.)}$$

The results of Arts. 29, 30, 31 are of fundamental importance. It must be clearly understood however that only the special case in which **air** is the field medium is dealt with in these articles.

In Art. 32, the mechanical force per unit of area on a charged surface is shown to be measured by $2\pi\sigma^2$.

The loss of energy for a given small displacement with the charges in the field maintained constant is equal to the gain of energy for the same displacement when the potentials are maintained constant, each quantity being equal to the mechanical work done during displacement, which is therefore the same in each case. (Art. 33.)

The energy per unit volume of electric field in air is given by

$$\frac{F^2}{8\pi} \qquad \text{(Art. 33.)}$$

Examples II.

1. The potential at a point 10 cm. from the centre of a charged sphere of 5 cm. radius is 1 C.G.S. unit. Find the energy of the charge on the sphere.

Here, if Q denote the charge on the sphere, we have

$$\frac{Q}{10} = 1, \text{ or } Q = 10 \text{ C.G.S. units of quantity.}$$

Hence, the potential of the sphere is given by

$$\frac{10}{5} = 2 \text{ C.G.S. units,}$$

and for the energy of the charge we therefore have

$$E = \frac{1}{2} QV = \frac{1}{2} . 10 . 2 = 10 \text{ ergs.}$$

2. A charge of 50 C.G.S. units raises the potential of a spherical conductor from 10 to 15 units. Find the radius on the conductor.

The capacity of the conductor is from the definition given by

$$C = \frac{50}{15-10} = \frac{50}{5} = 10,$$

and since the capacity of a spherical conductor is measured by its radius, the radius of the given conductor is 10 cm.

3. A spherical conductor of 5 cm. radius, and charged to a potential of 100 units, is placed inside a larger uncharged spherical conductor of 10 cm. radius, and made to touch its inner surface. Find the potential to which the larger conductor is raised.

Here, *all* the charge from the smaller sphere passes to the larger. Hence we have

$$5 \times 100 = 10 \times V,$$

where V denotes the required potential of the larger sphere.

That is,

$$10\,V = 500,$$

or

$$V = 50 \text{ C.G.S. units.}$$

4. The smaller conductor of question (3), after having been discharged by contact with the inner surface of the larger conductor, is taken out and connected with the latter by a long thin wire. Find the common potential which the conductors attain, and the charge on each.

Here, if V denote the common potential of the conductors, we have

$$V(5+10) = (50 \times 10) + 0.$$

That is,

$$15\,V = 500,$$

or

$$V = \frac{500}{15} = 33{\cdot}3 \text{ units of potential.}$$

The charge on the larger conductor

$$= \frac{10}{15} \times 500 = 333{\cdot}3 \text{ units of quantity,}$$

and that on the smaller

$$= \frac{5}{15} \times 500 = 166{\cdot}7 \text{ units of quantity.}$$

5. A spherical air condenser, the radii of whose spherical surfaces are 8 cm. and 10 cm. respectively, is charged so that the difference of potential between its plates is 80 units. Find its charge.

The capacity of the condenser is given by

$$C = \frac{10 \times 8}{10 - 8} = 40,$$

and since $Q = VC$ we have

$$Q = 80 \times 40 = 3200 \text{ units of quantity.}$$

6. A Leyden jar charged to a potential of 200 units is made to share its charge with a spherical conductor of 20 cm. radius. The potential of this conductor is then found to be 100 C.G.S. units. Find the capacity of the Leyden jar and the energy of its discharge before and after sharing its charge with the spherical conductor. Also find the *energy* lost when the charge is shared.

If C denote the capacity of the jar, we have, since no electricity is lost during the sharing of the charge,

$$200\,C = 100\,C + (100 \times 20),$$

or

$$100\,C = 2000,$$

that is,

$$C = 20 \text{ units.}$$

The charge of the jar before sharing with the spherical conductor is given by—

$$Q = 20 \times 200 = 4000 \text{ units of quantity,}$$

and the energy of its discharge would therefore be—

$$\frac{1}{2} \times 4000 \times 200 = 400000 \text{ ergs.}$$

After sharing with the spherical conductor, the jar retains a charge equal to

$$\frac{20}{40} \times 4000 = 2000 \text{ units of quantity,}$$

and the energy of its discharge would now be

$$\frac{1}{2} \times 2000 \times 100 = 100000 \text{ ergs.}$$

The energy lost when the charge is shared will be given by the difference between the energy of the charge in the jar before sharing with the spherical conductor, and the sum of the energy of the jar after sharing and the energy of the charge on the spherical conductor. This latter charge is equal to $\frac{20}{40} \times 4000 = 2000$ units of

quantity. Hence its energy is $\frac{1}{2} \times 2000 \times 100 = 100000$ ergs, and we therefore have—

$$\begin{aligned} \text{Energy lost} &= 400000 - (100000 + 100000) \\ &= 400000 - 200000 \\ &= 200000 \text{ ergs.} \end{aligned}$$

7. A conductor of capacity C is charged to a potential V. Show that the energy of its charge is given by $\frac{1}{2} V^2 C$.

8. Four equal charges, each of 10 units, are placed one at each of the four corners of a square of 5 cm. side. Find the potential at the centre of the square and at the middle point of either of the sides.

9. A sphere of 10 cm. diameter charged with 50 C.G.S. units of electricity is placed in contact with an insulated tin can, and the potential of the conductors after contact is found to be 5 C.G.S. units. Find the capacity of the tin can.

10. Two equally charged spherical conductors of radii a and b are connected by a long thin wire. Find the ratios of their common potential after contact to the potential of each before contact.

11. Find an expression for the energy lost when a charged Leyden jar is made to share its charge with another exactly equal and similar jar. In what sense is the energy lost? what becomes of it?

12. The thickness of the air layer between the two coatings of a spherical air-conductor is 2 cm. The condenser has the same capacity as a sphere of 120 cm. diameter. Find the radii of its surfaces.

13. Two small equal insulated metal spheres are charged with quantities of electricity in ratio 3 to 7 respectively, and, when placed at a distance from each other equal to several times their diameter, they are found to repel each other with a certain force: they are then made to touch each other and afterwards separated to 3 times their previous distance. Compare the force now exerted between them with that exerted before they were brought in contact.

14. Give the theory of the Leyden jar and show how to charge it.

A quantity of electricity (5 units) is conducted into the interior of a Leyden jar of surface (2 units); and a quantity of electricity (6 units) is conducted into the interior of a similar jar of surface (3 units). Compare the heat developed by discharging each.

15. What is the law of attraction and repulsion between small bodies charged with electricity?

If a number of insulated bodies, some charged positively and some negatively, be suspended within an insulated tin canister, what will be the condition of the outside of the canister, and under what circumstances will it possess no charge?

16. Explain the electrical terms *potential*, *capacity*, and *specific inductive capacity*.

Assuming that the quantity of electricity produced by a plate machine is proportional to the number of turns of the disc, explain how the capacities of two condensers may be compared.

17. If a metal ball, hung by a dry silk thread, be made to touch the inside of an electrified metal jar and then carried to an electroscope, the electroscope is not affected; but, if the electroscope is connected with the inside of the jar by a wire, it receives a charge. Account for the difference between the two results.

18. A, B, and C are three Leyden jars, equal in all respects. A is charged, made to share its charge with B, and afterwards to share the remainder with C—both B and C being previously without charge. The three jars are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge.

19. It is usually said that two portions of electricity attract or repel each other with a force that is inversely proportional to the square of the distance between them. Does it follow that the attraction or repulsion between two electrified bodies is always in the inverse ratio of the square of their distance? If not, show why not: and explain generally the conditions under which the law of inverse squares does or does not apply to the force between two electrified bodies.

20. How would you compare the electric capacities of two Leyden jars? A Leyden jar is charged and caused to share its charge with another jar whose linear dimensions are double its own. How is the charge distributed, and how is the potential related to the original potential?

21. An insulated conductor, elongated in shape, is placed near a charged sphere. How would you investigate the distribution of the electricity on its surface?

Describe some method of charging two bodies so that they may contain equal quantities of electricity of the same kind.

22. Two small equal insulated spheres, placed 10 centimetres apart, are charged respectively with 5 and 20 units of electricity. The spheres are made to touch and then replaced; what are the forces between them before and after contact respectively?

23. Two spheres, of diameters 9 and 3, are connected by a long thin wire, and 144 units of electricity are shared between them. Compare their charges; and describe how to study the distribution of electricity on their surfaces when they are brought nearer together. From which sphere would a brush discharge first occur? Give a reason.

24. Given a large fixed insulated sphere, how would you proceed to test its charge (i) for sign; (ii) for approximate amount? In what units would you express the quantity of electricity on it?

CHAPTER VIII.

ELECTROMETERS.

THE quantitative comparison of potential is made by means of instruments called *electrometers.* Most electrometers are instruments of very elaborate construction, so that here we can give only brief descriptions of the forms most generally used.

43. The Quadrant Electrometer. The *Quadrant Electrometer* consists essentially of four quadrantal boxes (such as might be obtained by cutting a shallow cylindrical brass box into four quadrants) and a light needle of aluminium foil, suspended by a fine silver wire, so that it hangs inside these boxes. Fig. 56 shows three of the quadrants, *a, b,* and *c,* and the needle, *n,* the fourth quadrant, *d,* being removed to show the needle.

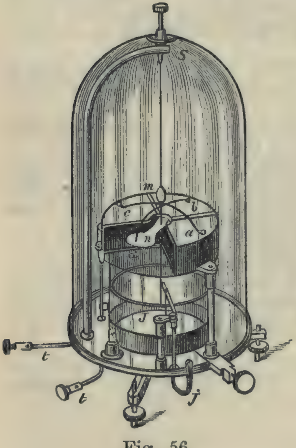


Fig. 56.

The four quadrants are mounted horizontally on insulating glass stems, and the suspension wire of the needle is attached, as shown in the figure, to a suspension head, S. The principle of action of the instrument is simple. The needle, *n,* is charged to a comparatively high constant potential, by connecting it to the inner coating of a charged

Leyden jar : thus charged, it is attracted by the neutral brass quadrants ; but if it lie, as in Fig. 57, symmetrically along one of the lines of separation of the quadrants, the forces of attraction are in equilibrium, and the needle is not displaced. If, however, the four quadrants are divided into two pairs by connecting *opposite* quadrants (a to c and b to d), the needle may be deflected by charging these pairs to unequal potentials.

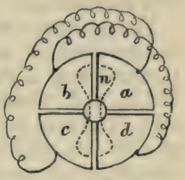


Fig. 57.

For example, if the needle be charged positively, and the quadrants a and c positively, and b and d also positively but to a lower potential than a and c, then, since the capacity of each pair of quadrants is the same, the charge on b and d is less than on a and c, and therefore the force of repulsion between b and d and the needle is less than between a and c and the needle. As a consequence, the needle is deflected towards b and d by a couple equal to the difference between the couples due to the charges on b, d, and a, c. This difference depends ultimately on the potential of the needle and the difference of potential between b, d, and a, c, and therefore the deflection depends upon the potential of the needle and the difference of potential between the pairs of quadrants.

If the potential of the needle be maintained at a *constant high* value, and if the deflection be small, it can be shown that *the difference of potential between the two pairs of quadrants is proportional to the deflection.* If one pair of quadrants be connected to a charged conductor, the conducting system made up of the conductor and the two connected quadrants will be at a potential very slightly less than that of the conductor before connecting it with the quadrants. Now, if the other pair of quadrants be connected to earth, or *earthed*, the difference of potential between the quadrants will practically be that of the charged conductor, and the deflection of the needle will be proportional to that potential. Similarly, with any other charged conductor, a deflection proportional to its

potential may be obtained, and the ratio of the two deflections is thus a measure of the relative potentials of the two conductors.

For convenience the Leyden jar, with which the needle is connected, is arranged in the base of the instrument. It consists of a glass vessel, J (Fig. 56), coated outside with tinfoil, and, when in use, half filled with strong sulphuric acid. The acid being a conductor serves as the inner coating of the jar, and also helps to secure good insulation by absorbing all moisture. The needle is connected to this inner coating by means of the piece of platinum wire to which it is attached, and which is long enough to dip into the acid. The opposite quadrants are connected by short pieces of fine copper wire, and the brass rods, *t*, *t*, which are carried outside the glass case covering the instrument, are connected to two *adjacent* quadrants, and thus serve to make contact with the two pairs of quadrants; they are called the *terminals* of the instrument. Another similar brass rod, *j*, communicates, by means of a short length of platinum wire suspended from it, with the acid in the jar, and its terminal thus represents the knob of the jar. As in the torsion balance, the couple deflecting the needle is, in the position of equilibrum, balanced by the opposing couple due to the torsion on the wire, which, to secure the necessary sensitiveness, must be very fine. As the deflection is in general very small, the lamp and scale method illustrated in Fig. 58 is chosen. The light from the lamp passes through a small circular aperture just below the scale, and, passing slightly upwards, falls on a small concave mirror, *m*, attached to the wire carrying the needle. It is thence reflected back on to the scale, where an image of the aperture is formed, and if a fine wire be stretched vertically across the slit the image of the wire enables the position of the reflected "spot" of light to be accurately read off on the scale. When the needle is deflected even through an extremely small angle, the *spot* on the scale moves through an appreciable distance, and for small deflections the angle of deflection may be taken as proportional to the deflection of the spot as measured by the divisions on the scale. Fig. 58 shows the scale and lamp adjusted to the mirror, *m*,

but the student can appreciate all the details of the arrangement only by seeing and working with the apparatus

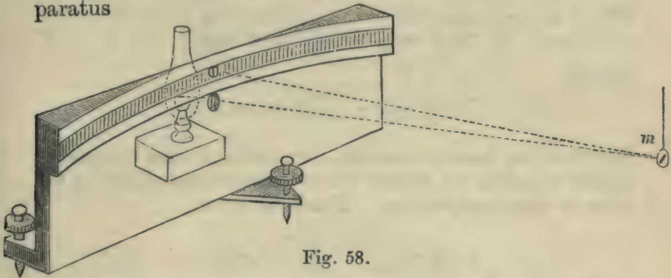


Fig. 58.

The elementary theory of the action of the Quadrant Electrometer may be given as follows. Let V, v_1 and v_2 denote the potentials of the needle and the two pairs of quadrants. Let θ be the deflection, and T the corresponding couple exerted on the needle by its torsion fibre or bifilar suspension. It will be seen that the quadrant-needle system constitutes a double condenser, each pair of quadrants forming a condenser with the part of the needle that lies within them. Imagine the deflection increased from θ to $\theta+a$, a being a very small angle. Then we may assume, for an ideally symmetrical arrangement, that the capacity of one condenser is increased and that of the other decreased by a definite equal amount proportional to a. Let c denote the change of capacity of each condenser for unit angular displacement of the needle. Then, for the displacement a in a direction causing an increase of capacity in the condenser formed by the needle and the quadrants at potential v_2, the change in the energy of the condenser system is given by—

$$\tfrac{1}{2} c a (V - v_2)^2 - \tfrac{1}{2} c a (V - v_1)^2$$

or

$$\tfrac{1}{2} c a \{(V - v_2)^2 - (V - v_1)^2\}$$

or

$$c a \left\{ V - \frac{v_1 + v_2}{2} \right\} \{v_1 - v_2\}.$$

* Method of Virtual Work. See *Tutorial Statics*, p 281.

This change of energy * is equal to the work done against the couple T during the extra displacement α. Hence we get

$$T\alpha = c\alpha\left\{V - \frac{v_1+v_2}{2}\right\}(v_1 - v_2),$$

that is,

$$T = c\left(V - \frac{v_1+v_2}{2}\right)(v_1 - v_2).$$

But T is the torsion moment for a twist θ, and is equal or nearly equal to $\kappa\,\theta$, where κ is a constant depending on the wire or other suspension. Hence we have

$$\kappa\,\theta = c\left(V - \frac{v_1+v_2}{2}\right)(v_1 - v_2)$$

or

$$\theta = \frac{c}{\kappa}\left(V - \frac{v_1+v_2}{2}\right)(v_1 - v_2).$$

If V be very large compared with v_2 and v_1 then $\left(V - \frac{v_1+v_2}{2}\right)$ is approximately equal to V, and we have as a rough approximation

$$\theta = \frac{cV}{\kappa}(v_1 - v_2).$$

That is, the deflection, θ, is directly proportional to $(v_1 - v_2)$, the difference between the potentials of the two pairs of quadrants.

It should be noted that if the potential of one pair of quadrants be the same as that of the needle, for example, if $v_1 = V$, we get

$$\theta = \frac{c}{\kappa}\left(\frac{V - v_2}{2}\right)(V - v_2).$$

* This change of energy, it will be noted, is an *increase* of energy. In a case of this kind, where the displacement takes place at constant potential, there is an increase of energy equal to the work done in the displacement. The battery or generator, which must be supposed connected with the system in order to adjust the several charges so as to maintain all the potentials constant, therefore supplies energy equal to twice the work done in the displacement. If the system is left to itself and the displacement takes place at constant charges the work done is the same as before, but is now, by the principle of conservation of energy, equal to the loss of energy by the system. See Art. 33.

or

$$\theta = \frac{c}{2\kappa}(V - v_2)^2.$$

In the relation given above θ is of the same sign as $(v_1 - v_2)$ but in the result just obtained θ does not change sign with $(V - v_2)$, being proportional to $(V - v_2)^2$.

If one pair of quadrants be earthed so that we may write $v_2 = 0$, then the formula given above reduces to

$$\vartheta = \frac{cV}{\kappa} v_1$$

and

$$\vartheta = \frac{c}{2\kappa} V^2 *$$

Usually c is small, and κ comparatively large even for a thin wire, so that if θ is to be of easily measurable

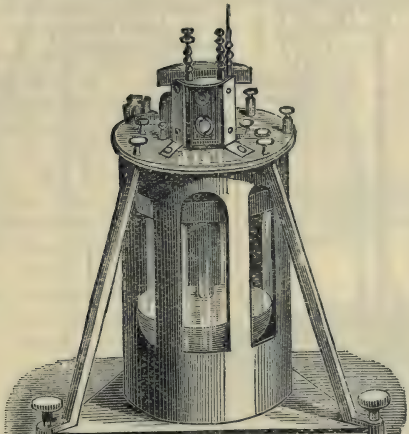


Fig. 59.

* This formula cannot be obtained from the one above it directly, for the latter is obtained on the assumption that v_1 is small compared with V.

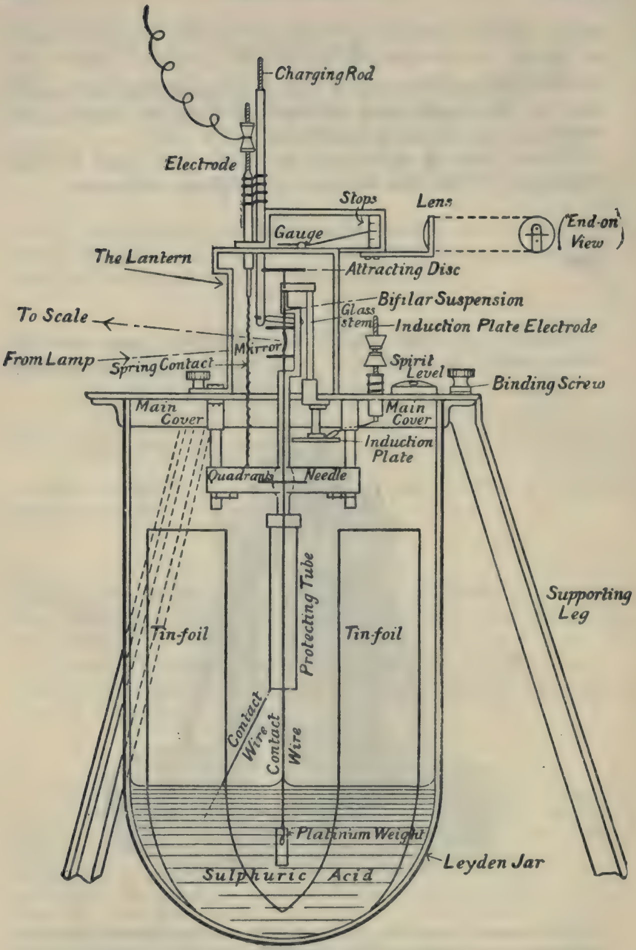

Charging Rod
Electrode
Stops
Lens
Gauge
'End-on' View
The Lantern
Attracting Disc
Bifilar Suspension
Glass stem
Induction Plate Electrode
To Scale
Mirror
From Lamp
Spring Contact
Spirit Level
Binding Screw
Main Cover
Main Cover
Induction Plate
Quadrants
Needle
Protecting Tube
Supporting Leg
Tin-foil
Tin-foil
Contact Wire
Contact Wire
Platinum Weight
Leyden Jar
Sulphuric Acid

Fig. 60.

magnitude when v_1 is small, V must evidently be large. If, however, the potential to be measured is fairly large the arrangement giving the second formula which involves V^2 usually gives a satisfactory value for θ. This arrangement can also be used when V is alternating in value.

The Kelvin form of the Quadrant Electrometer is shown in Figs. 59 and 60, and the Dolezalek form in Fig. 375.

44. The Attracted-disc Electrometer. The *absolute* measurement of difference of potential, that is, its direct determination in ergs per unit of quantity, can be effected by means of Thomson's *Attracted-disc Electrometer.* The principle of this instrument consists in balancing the attraction between two parallel discs at different potentials, and at a known distance apart, by a weight or other force which can be expressed in absolute units of force. The first instrument of this kind was used by Sir William Snow-Harris; the essentials of this instrument are shown in Fig. 61. The parallel discs are shown at a and b, and the attraction between them, when at different potentials, is measured by weights placed in scale-pan P, which, when empty, exactly balances the disc b, when there is no electrical attraction between it and the disc a. The attraction between a and b is thus measured in absolute units of force, and, as this attraction can be expressed in terms of the difference of potential between a and b, it is evident that this difference can be expressed in absolute C.G.S. units of potential.

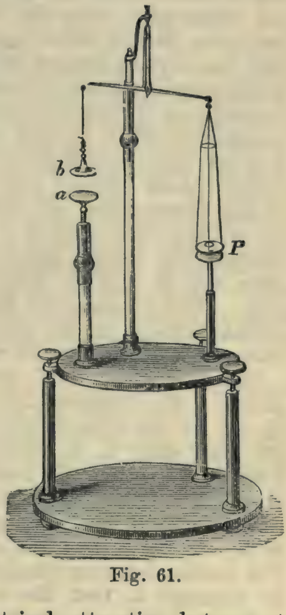


Fig. 61.

A serious defect in this form of the instrument is the want of uniformity in the surface density of the charges on the parallel discs.

In the case of a plate of finite area the surface density is not uniform. As already explained, the distribution of the charge is such that the density increases as the edge of the plate is approached. Over the central portion of the plate, however, the surface density is practically uniform, and if this central portion be separated mechanically, but not electrically, from the surrounding edge-ring it will provide a plate of finite small size charged in such a way that the surface density of the charge is uniform. This is the *guard-ring* arrangement due to Lord Kelvin. The construction of the arrangement will be understood by imagining a small circular plate to be cut centrally out of a larger plate and slightly reduced in size so that it fits freely without contact into the ring left in cutting it out of the larger plate. If this plate and ring be connected by a fine wire and the arrangement insulated and charged the surface density of the charge on the small plate will be uniform when it is in the same plane as the ring. The presence of the ring prevents increase of surface density at the edge of the plate, and for this reason it is called a guard-ring.

The modern form of the instrument is due to Lord Kelvin. The balance part of Snow-Harris' form is replaced by the arrangement indicated in Fig. 62. The attracted disc, C, is surrounded by a guard-ring, B B, forming the bottom of a metal box, A A, which protects the disc from external influences. The disc is suspended from the top of the box by a spring, and its normal position is slightly above the plane of the guard-ring bottom, B B. The distance between the disc and the lower plate, P P, can be adjusted so that the attraction on the disc is just sufficient to bring it exactly into the plane of the guard-ring B B, and as the force in dynes necessary to do this is readily found by direct experiment the force of attraction on the disc for a known distance between the plates is determined.

Before describing how the instrument is used, it will be

necessary to show briefly how the force of the attraction between two plates at different potentials can be expressed in terms of their difference of potential. In the arrangement of Fig. 62, let the plate C be charged to potential V, and let the surface density of the charge be denoted by σ. If P P be now connected to earth it remains at zero potential, but a charge of opposite sign to that on C is induced on it, and the surface density of this charge will be denoted by $-\sigma$. The difference of potential between the plates is therefore V, that is, V units of work would be done in conveying unit quantity of electricity from P P to C. Hence if f denote the *electric force* in the space between the plates, and d the distance between the plates, we have $fd = V$, for the field of force between the plates is uniform. But the electric force between the plates is given by $4\pi\sigma$, for a positive unit of electricity would be repelled downwards by C with a force $2\pi\sigma$, and it would be attracted downwards by P P with a force $2\pi\sigma$. Hence the total force which it would experience would be $4\pi\sigma$ downwards, and giving this value to f we get $4\pi\sigma d = V$, or $\sigma = V/4\pi d$.

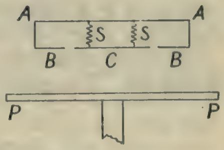


Fig. 62.

Now each unit of electricity on C is attracted downwards by P P, with a force $2\pi\sigma$, and if S denote the area of C the charge on it is $S\sigma$, and the total force with which P P attracts it down is $S\sigma \times 2\pi\sigma = 2\pi S\sigma^2$. If this force be denoted by F we have

$$F = 2\pi S\sigma^2,$$

But,

$$\sigma = V/4\pi d.^*$$

Therefore,

$$F = \frac{2\pi S V^2}{16\pi^2 d^2} = \frac{S V^2}{8\pi d^2},$$

* The result given above that $\sigma = V/4\pi d$ indicates that the charge per unit area for unit value of V is $1/4\pi d$, that is, the capacity of the arrangement per unit area of plate is $1/4\pi d$.

and

$$V = \sqrt{\frac{8\,\pi\,d^2\,F}{S}} = d\sqrt{\frac{8\,\pi F}{S}}.$$

That is, if F is constant, the difference of potential between the plates is proportional to their distance apart.

One form of Kelvin's attracted disc electrometer is shown in Fig. 63. The guard-ring box and disc cover, arranged as

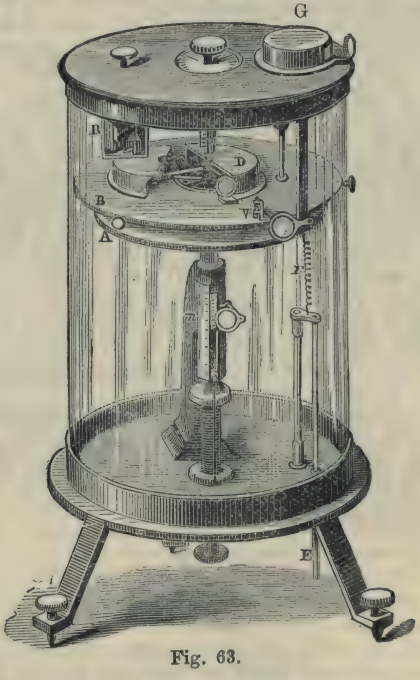


Fig. 63.

in Fig. 62, is shown at D. The disc, C, is inside D in the plane of the guard plate B and the sensitiveness of its adjustment can be controlled by means of the screw shown

in the centre of the cover of the instrument. In order to maintain the disc and guard-ring at the constant potential V the system is connected to the inner coating of a Leyden jar formed by the glass case of the instrument. The potential of this coating is maintained by the action of a small replenisher (p. 161), shown at R, and a gauge shown at G indicates the constancy of the potential. This gauge is itself a small attracted disc arrangement. A small disc or plate, *c*, Fig. 64, of aluminium, is carried at one end of the lever L, and fits into the plate, P P, the guard-ring.

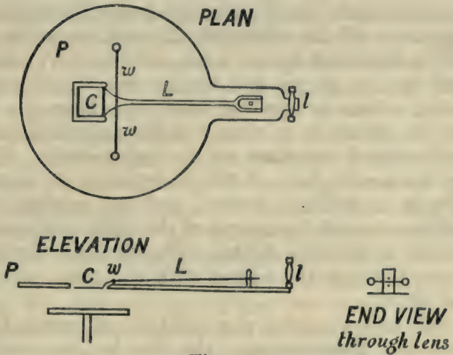


Fig. 64.

The lever L is attached rigidly to the wire, *w w*, in such a way that the torsion of the wire makes the normal position of the plate, *c*, to be just a little above the plane of its guard-ring. At the other end the lever L forks, and a fine hair fixed across the two prongs of the fork moves with the lever up and down before a white scale with two small black dots on it. These dots are so placed that when the hair lies exactly between them the disc *c* is in the plane of its guard-ring. A lower plate, below and parallel to *c* and its guard-ring, is arranged at a *fixed* distance from it. If this lower plate is earthed or connected to the outside of the case of the instrument, and the disc and guard-ring of

the guage to the inner coating of the Leyden jar, then, since the distance between the lower plate and the disc is fixed, the force of attraction necessary to bring the disc into the plane of the guard-ring is also fixed, and corresponds to a definite constant potential for the inner coating of the jar. The attainment of this potential is indicated by the gauge hair index. Hence, by the use of the replenisher, and by keeping watch on the indication of the gauge, the potential of the jar and everything connected to it may be maintained at a definite constant value for any length of time. A similar gauge is shown in Fig. 60, attached to the Kelvin Quadrant Electrometer.

The position of the attracted disc relative to its guard-plate, B, is indicated by an index, shown at V, similar to that used in the gauge, and to increase the accuracy of observation a lens is fitted to each index in such a way as to allow the observer to view a clear magnified image of the fine hair and the dots on the index scale. The rod carrying the lower plate of the instrument, A, is movable vertically in a sliding socket, and its motion can be accurately measured by means of the micrometer arrangement shown at *m* in the figure. The terminal for making connection, through the wire *r*, with the lower plate, A, is shown at E.

We can now describe the method of using Kelvin's Attracted-disc Electrometer; the disc is always adjusted by sighting the indicator to the plane of the guard-ring, but the adjustment is made by moving the lower plate up or down until the force of attraction between the plates is equal to the known force necessary to bring the gauge indicator into the sighted position, that is, the distance between the plates is varied so that the force of attraction between them always has a constant known value. Hence, one way of using the instrument would be as follows: let the conductor, whose potential is to be measured, be connected to disc and guard-plate,* and let the lower plate be earthed. Then let the position of the latter be adjusted

* The disc and the guard-plate are connected by a wire joining the guard-ring, with the metal supports on which the lever L is pivoted.

until the disc occupies the sighted position and the distance d between the plates measured. The required potential of the conductor will then be given by the relation obtained above

$$V = d\sqrt{\frac{8\pi F}{S}},$$

where d, F and S (the area of the disc) are known. This direct method, however, does not give good results, because it is difficult to measure d accurately, so the following indirect method has to be adopted.

The disc and the guard-ring are maintained at a constant potential, and the lower plate is first connected to earth and adjusted in position until the disc is in position. The reading of the micrometer screw attached to the pillar supporting the lower plate is then taken, and the connection between the plate and the earth broken. The plate is next connected to the conductor whose potential has to be measured, and again adjusted for the equilibrium of the disc. The reading of the micrometer-screw is again taken, and the *difference* of the two readings noted; this difference gives the distance through which the lower plate has been moved in effecting the second adjustment. Now, if V denote the constant potential at which the disc and guard-ring are maintained, and v the potential to be measured, then we have, since the potential of the lower plate for the first ajustment is zero,

$$V = d_1\sqrt{\frac{8\pi F}{S}}, \qquad (1)$$

where d_1 denotes the distance between the plates when the first adjustment is made. Similarly we have

$$V - v = d_2\sqrt{\frac{8\pi F}{S}}, \qquad (2)$$

where d_2 denotes the distance between the plates when the second adjustment is made. Hence, subtracting (2) from (1) we get

$$v = (d_1 - d_2)\sqrt{\frac{8\pi F}{S}},$$

where v denotes the required potential, and $(d_1 - d_2)$ is

accurately given by the micrometer screw, as the *difference* of two readings. The factor $\sqrt{\frac{8\pi F}{S}}$ may be determined once for all as a constant of the instrument.

This factor is usually determined experimentally as a whole but it may be calculated from experimentally determined values of S and F. In determining the value of S, the area of the plate C, the width of the gap between the plate and its guard-ring should be considered. The gap is very narrow and we may therefore assume that the surface density on the lower plate in the region opposite the gap is uniform and unaffected by the gap. It may also be assumed from symmetry that the distribution of the lines of force between the upper and lower plates in the region of the gap is as indicated in Fig. 65, where B and C denote the guard-ring and plate with the gap between. From this it appears that the total charge on C is equal to the charge on a portion of the lower plate equal in area to the plate C *plus* half the area of the gap between the guard-ring and the plate. That is, the total charge on the plate is $(S + a)\,\sigma$, where $2\,a$ denotes the area of the gap. The extra charge $a\,\sigma$ is evidently distributed round the edge of the plate and is a source of error in working the instrument. Usually, how-ever, the gap is inappreciable in width and its effect is very small.

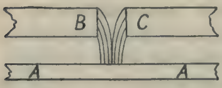


Fig. 65.

45. Comparison of Capacities. The electrometer in some form or other is *the* measuring instrument in electrostatics. Its use in comparing potentials has already been referred to, but it may also be used in the measurement of capacity and electrical quantity. As an example of the method of measuring capacity, let it be required to determine the capacity of a given conductor, say a tin can. A spherical conductor of known radius* is carefully

* The diameter of the sphere should be about equal to the largest dimension of the given conductor.

insulated and connected to one of the terminals of the electrometer, the other terminal being put to earth. This conductor is then charged until the needle of the electrometer experiences a considerable deflection, and the corresponding deflection of the spot on the scale is read off, and noted down. The given conductor is now insulated and, put in contact with the charged spherical conductor. A division of the charge then takes place, and the potential of the compound conductor takes a value lower than that first indicated by the electrometer. The reduced deflection of the spot is then read off and noted for comparison with the initial deflection. Let d_1 and d_2 denote these deflections in the order obtained, and let V_1 and V_2 denote the potentials to which they correspond, that is, the initial potential of the charged spherical conductor, and the potential of the system made up of the two conductors in contact. Then, if Q denote the charge given to the spherical conductor, C_1 its capacity, and C_2 the required capacity of the given conductor, we have

$$Q = V_1 C_1 \tag{1}$$

also

$$Q = V_2(C_1+C_2), \tag{2}$$

for (C_1+C_2) is the capacity of the compound conductor made up of the two conductors in contact. Hence, from (1) and (2) we have

$$V_1 C_1 = V_2(C_1+C_2),$$

that is,

$$\frac{V_1}{V_2} = \frac{C_1+C_2}{C_1}.$$

But

$$\frac{V_1}{V_2} = \frac{d_1}{d_2},$$

and therefore

$$\frac{C_1+C_2}{C_1} = \frac{d_1}{d_2},$$

that is,

$$\frac{C_1}{C_2} = \frac{d_2}{d_1-d_2}.$$

But C_1 is known, for it is given by the radius of the

spherical conductor; also d_1 and d_2 have been noted. Therefore C_2 can be calculated from this relation.

Now that potential and capacity can be measured, it is evident that quantity can also be measured, for, to determine the magnitude of the charge on any conductor, it is only necessary to determine its potential, V, and capacity, C, and then to apply the relation $Q = V\,C$.

In the description given above of the method of measuring capacity, it has been assumed that the capacity of the quadrants of the electrometer is negligible, compared with the capacities of the conductors employed. This is not always the case, but it is always easy to determine the capacity of the quadrants, and to include it in working out the relation between C_1 and C_2. A simple method of effecting this determination is as follows. Let two spherical conductors of capacities C_1 and C_2, measured by their radii, be charged to the same potential, and their potentials compared when joined successively to the quadrants of the electrometer. Let c denote the capacity of the quadrants of the instrument, V the common initial potential of the two conductors, and V_1 and V_2 the potentials assumed by these conductors when connected with the quadrants; then, if d_1 and d_2 denote the deflections corresponding to these potentials, we have

$$V_1\,(C_1+c) = VC_1$$

and

$$V_2(C_2+c) = VC_2,$$

that is,

$$\frac{V_1(C_1+c)}{V_2(C_2+c)} = \frac{C_1}{C_2}.$$

But

$$\frac{V_1}{V_2} = \frac{d_1}{d_2},$$

and therefore

$$\frac{d_1}{d_2} \cdot \frac{C_1+c}{C_2+c} = \frac{C_1}{C_2},$$

or

$$c = \frac{C_1C_2(d_2-d_1)}{d_1C_2-d_2C_1}.$$

In this relation C_1 and C_2 are known, d_1 and d_2 are given by the experiment, and c is therefore completely determined.

The capacities of two condensers may also be compared experimentally by the same method. Let the condensers be, for example, two Leyden jars of capacities C_1 and C_2, and let the one of capacity C_1 be charged to potential V. The quantity of electricity with which this jar is charged is given by $Q = C_1 V$ (1); and if it be made to share its charge with the other jar by putting the two knobs in contact no electricity will be lost, but the common potential of the two jars after contact will be less than V. Let this common potential be denoted by v. Then, as in Art. 27, we have

$$Q = (C_1 + C_2)\, v, \tag{2}$$

for during contact the two jars form one condenser of capacity $(C_1 + C_2)$.

Hence, from (1) and (2) we have

$$C_1 V = (C_1 + C_2)\, v,$$

or

$$C_1 (V - v) = C_2\, v,$$

that is,

$$\frac{C_1}{C_2} = \frac{v}{V - v}.$$

This relation expresses the required ratio of the capacities of the two condensers in terms of the potentials V and v, and these potentials can be compared, as explained above, by means of the Quadrant Electrometer. An experimental comparison of the capacities of any two condensers can thus be made.

To compare the potentials V and v by means of the Quadrant Electrometer, it is only necessary, after adjusting the instrument, to earth one terminal, that is, to connect it to iron, gas or water pipes by means of a wire, and to connect the knob of the jar to the other terminal. The outer coating is earthed, and the deflection of the needle

is then proportional to the potential of the inner coating, that is, to the potential of the jar as a condenser.

If therefore the deflection of the spot, given by the jar of capacity C_1, before sharing the charge with the other jar, is denoted by D, and the deflection given by *either* jar after the sharing of the charge by d, then, as before,

$$\frac{V}{v} = \frac{D}{d},$$

and

$$\frac{C_1}{C_2} = \frac{v}{V - v} = \frac{d}{D - d}.$$

Compound condensers of very large capacity may be made by joining together a large number of simple condensers in the way indicated in the battery of Leyden jars. For example, if a large number of sheets of tinfoil and paraffined paper be laid alternately on one another, a condenser may be formed by connecting together the 1st, 3rd, 5th, etc., sheets of foil as one plate of the condenser, and the 2nd, 4th, 6th, etc., sheets as the other plate. The sheets of paraffined paper act as the dielectric; but in the better forms of condensers, thin sheets of mica are used instead.

CHAPTER IX.

SPECIFIC INDUCTIVE CAPACITY.

46. Generalisation of the Fundamental Electrostatic Theorems. From the definition of specific inductive capacity it is possible to deduce the effect of the magnitude of this quantity on the magnitude of the other measurable quantities of the electric field.

Consider two parallel plate condensers of exactly equal dimensions, one an air condenser, and the other with a dielectric of specific inductive capacity K. By definition, the capacity of the latter will be K times that of the air-condenser. Let both condensers receive equal charges, then since the capacity of one condenser is K times that of the other, the difference of potential between its plates will be only $1/K^{th}$ of the difference of potential between the plates of the other—the air-condenser. That is, if V denote the difference of potential for the air-condenser, then V/K denotes the difference of potential for the condenser in which the dielectric is of specific inductive capacity, K. For the uniform field between the plates the force at any point is measured by the difference of potential per unit length of the distance between the plates. That is, in the air condenser the electric force between the plates is V/d and in the other condenser where the dielectric is of specific inductive capacity K, the force between the plates is $V/K\,d$ where d is the distance between the plates. The charges in the two condensers are equal, and the number of unit tubes of induction crossing the dielectric from one plate to the other is exactly the same for the two, and consequently the number of unit tubes of induction crossing unit area between the plates is exactly the same for each condenser. It, therefore, follows that for a given number of unit tubes

of induction per unit area in a medium of specific inductive capacity K, the electric force is 1/K times the force in air for the same number of unit tubes of induction per unit area. The general form of the formula, $F = 4\pi n$, established in Art. 31 should therefore be $F = \frac{4\pi}{K} n$, where F is the electric force, and n the number of tubes of induction per unit area at a point in a medium of specific inductive capacity K.

In the general treatment of the electric field it is usual to introduce a quantity known as the **polarisation** of the dielectric or **electric displacement** in the dielectric. Polarisation at any point in the dielectric is defined, with reference to any direction, as the algebraic sum of the number of unit tubes of induction per unit area crossing a plane taken at right angles to the given direction at the given point, the tubes in the given direction being taken as positive, and those in the opposite direction as negative. If P denote the polarisation in the medium in a given direction at the given point and if F and n denote, for the same direction, the electric force and the algebraic sum of the number of tubes of induction per unit area at the same point, then the relation $F = \frac{4\pi}{K} n$ may evidently be written $F = \frac{4\pi}{K} P$, for P and n are here identical.

If we consider any closed surface in the field, the polarisation at any point on the surface along the outward drawn normal to the surface is measured by the algebraic sum of the number of tubes of induction that cross the surface per unit area, the direction of the outward drawn normal being taken as positive. If a denote a very small area taken at the point, and P be the measure of the polarisation, then the value of ΣPa, taken over the whole closed surface, is defined as the total normal polarisation for the surface. In the case of a closed surface it is evident that tubes of force which start and end outside the surface must, if they cut the surface at all, cut it an even number of times, alternately positively and negatively, and therefore contribute nothing to the total normal polarisation over the surface. In the case of tubes that originate inside the surface

and pass out they must cut the surface an odd number of times, and must therefore contribute positively to the total normal polarisation. Similarly tubes that start outside the surface and terminate inside it contribute negatively to the polarisation. Hence, if N tubes start inside and N′ end inside the closed surface, $N - N'$, measures the total normal polarisation over the surface. But each tube of force is associated with a positive unit of electricity at its origin and a negative unit at its termination and therefore $N - N'$ measures also the quantity of electricity inside the surface. That is, the total normal polarisation over the surface is measured by the quantity of electricity enclosed by the surface.

If at any point on the surface F denote the electric force normal to the surface and P the polarisation, then, as shown above, we have $F = \frac{4\pi}{K} P$, and taking a very small area a at the point we get, for the whole surface, $\Sigma F a = \frac{4\pi}{K} \Sigma P a$, that is, the total normal induction over the surface is equal to $4\pi/K$ times the total normal polarisation, that is, to $4\pi/K$ times the total quantity of electricity enclosed by the surface. This relation is the general form of Gauss' theorem, given in Art. 28, for air, for in air $K = 1$ and $4\pi/K$ becomes 4π. In the form given above the theorem is applicable in any medium.

The expression of Coulomb's law will in the same way be changed from the form $F = 4\pi\sigma$, given for air in Art. 30, to $F = \frac{4\pi}{K}\sigma$ for a medium of specific inductive capacity K. For, if σ denote the surface density at a point on the surface of the conductor, then the number of tubes of force per unit area crossing the adjacent surface of the dielectric at the same point is also σ, that is, σ measures the polarisation of the dielectric at the point in the direction of the normal to the surface, and therefore if the normal electric force at that point be F units, then, as above, $F = \frac{4\pi}{K} P$, that is, $F = \frac{4\pi}{K}\sigma$.

The expression for the energy per unit volume of the medium must also involve K, the specific inductive capacity of the medium. The result given in Art. 33 is true only for air, but to generalise the result it is only necessary to substitute for the relation $F = 4\pi n$, involved in the proof, the general relation $F = \frac{4\pi}{K} n$. This substitution at once gives the energy per unit volume of the medium as equal to $K F^2/8\pi$ for a medium of specific inductive capacity K. This shows that in a medium of specific inductive capacity K the energy per unit volume at any point is, *for the same value of the electric force at that point*, equal to K times the energy per unit volume in air. If P denote the polarisation of the medium at any point then, since $F = \frac{4\pi}{K} P$, the energy per unit volume is also given by $2\pi P^2/K$, which shows that, *for the same polarisation*, the energy per unit volume, in a medium of specific inductive capacity K, is only $1/K$ times its value in air. In the same way the tension per unit area in the tubes of force and the pressure per unit area across the tubes, shown to be measured by $F^2/8\pi$ in air, are evidently given by $K F^2/8\pi$, in a medium of specific inductive capacity K.

The appearance of K in the formula

$$f = \frac{1}{K} \frac{qq}{d^2}$$

of Art. 6, will now be understood. Consider a spherical surface of radius d round the point charge q. The polarisation of the medium normal to the spherical surface is uniform over the whole surface, and if P denote the normal polarisation at any point then $4\pi d^2 P$ gives the total normal polarisation over the surface. But it has been shown above that the total normal polarisation over a closed surface is equal to the total quantity of electricity enclosed by the surface, and here, therefore, we have $4\pi d^2 P = q$,

or $P = \frac{q}{4\pi d^2}$. But if F denote the electric force along the normal at any point on the surface, then, since

$$F = \frac{4\pi}{K} P$$

we have

$$F = \frac{4\pi q}{4\pi K d^2} = \frac{1}{K} \cdot \frac{q}{d^2}.$$

That is, the electric force at any point at a distance d from a point charge q in a medium of specific inductive capacity K is given by $F = \frac{1}{K} \cdot \frac{q}{d^2}$, and therefore the force exerted on a charge q' placed at this point is measured by

$$F = \frac{1}{K} \cdot \frac{q q'}{d^2},$$

as stated in Art. 6.

It follows from this that if F be the electric force at any point in a field in air, then the force at the same point, when air is replaced by a dielectric of specific inductive capacity K (the charges in the field being supposed constant), is F/K, and under the same conditions, if the potential at the point is V in air it will be V/K in a dielectric of specific inductive capacity K. If, on the other hand, conductors in the field are maintained at constant potential instead of at constant charge, then the values F and q, the electric force and charge in air, change to K F and Kq in a dielectric of specific inductive capacity K.

Thus in a parallel plate condenser, if the dielectric between the plates has a specific inductive capacity K, the electric force between the plates is $4\pi\sigma/K$, where σ is the surface density of the charge on the plates, and the mechanical force on either of the plates per unit area is $2\pi\sigma^2/K$. That is, if σ remains constant the force of attraction between the plates is, with the dielectric of specific inductive capacity K, the Kth part of what it is in air. Also, since $4\pi\sigma/K = V/d$, we have $\sigma = K V/4\pi d$, where d is the distance between the plates, and V the difference of potential in the condenser.

This gives the force of attraction between the plates $2\pi\sigma^2/K$, equal to $KV^2/8\pi d^2$ which shows that if V, the difference of potential, is kept constant, the force of attraction is K times its value in air.

47. Refraction of Tubes of Force. When a field of force contains more than one dielectric medium it is of interest to determine the conditions which must obtain at the boundary of two media of different specific inductive capacities. Let A B (Fig. 66) represent the trace of a small portion of the boundary between two media of specific inductive capacities K_1 and K_2.

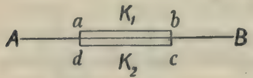


Fig. 66.

The electric force in each medium at a point in the boundary surface may be resolved into two components, one parallel to or tangential to the surface at the points and the other normal to the surface. The tangential components must be equal; otherwise it would be possible to obtain an infinite supply of energy by moving a charge of electricity round the cycle indicated by *a b c d*, the cycle being so taken that the work done on the charge along *a b* is greater than that done by the charge along *c d*, if we assume the tangential force in the upper medium greater than that in the lower medium. The cycle is infinitely small and the algebraic sum of the work for the paths *d a*, *b c*, of the cycle may be taken as zero. One condition which must obtain at the surface of separation is, therefore, that at any point in the surface the tangential component of the force in one medium must equal the tangential component of the force in the other medium. That is, if T_1 and T_2 denote the two tangential components then

$$T_1 = T_2.$$

Again, if there is to be no free charge at the surface of separation of the two media, it is evident from the definition of polarisation that at any point the *normal* polarisation in one medium must be equal to the normal polarisation in the other medium. That is if N_1 and N_2 are the normal components of the electric force in the

upper and lower media, then the corresponding normal polarisations, P_1 and P_2, are

$$P_1 = \frac{K_1}{4\pi} N_1 \quad \text{and} \quad P_2 = \frac{K_2}{4\pi} N_2$$

and the second condition which must obtain at the bounding surface is given by the relation $P_1 = P_2$ or

$$\frac{K_1}{4\pi} N_1 = \frac{K_2}{4\pi} N_2.$$

When a line of force passes from one medium to the other the law of its refraction can be determined from the conditions specified above. Let F_1 and F_2 denote the magnitudes of the electric force in the two media, and let the directions of these forces make angles, ϕ_1 and ϕ_2 with the bounding surface. Then the tangential components and normal components are $F_1 \sin \phi_1$, $F_2 \sin \phi_2$, and $F_1 \cos \phi_1$, $F_2 \cos \phi_2$ and the necessary relations between these quantities are

$$F_1 \sin \phi_2 = F_2 \sin \phi_2$$

$$\frac{K_1}{4\pi} F_1 \cos \phi_1 = \frac{K_2}{4\pi} F_2 \cos \phi_2.$$

This gives

$$\frac{\tan \phi_1}{K_1} = \frac{\tan \phi_2}{K_2}$$

or

$$\frac{\tan \phi_1}{\tan \phi_2} = \frac{K_1}{K_2},$$

a relation which determines the refraction of a line of force in passing from a medium of specific inductive capacity K_1 to one of specific inductive capacity K_2. From this relation it is evident that if K_1 is greater than K_2 then ϕ_1 is greater than ϕ_2, that is, when a line passes from one medium to another of smaller specific inductive capacity the line is bent towards the normal, as in Fig. 67.

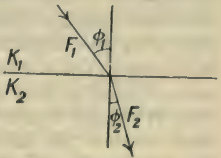


Fig. 67.

This question of refraction of the lines of force may also

be associated with the distribution of energy in the media of the field. It has been shown that, for a given intensity of polarisation, the energy per unit volume of a medium is smaller the greater the specific inductive capacity of the medium. Hence it follows that, since the energy in the field always tends to a minimum, the tubes of force will pass as far as possible through the media of greatest specific inductive capacity, and the law of refraction from one medium to another is that each tube of force is refracted so as to take the path of minimum potential energy possible for it.

Fig. 68 illustrates this for the case of a ball of sulphur or other dielectric of high specific inductive capacity placed in the initial uniform field in air between two charged parallel plates. The tubes of force are refracted so as to crowd into the dielectric of high specific inductive capacity, and each one seeks a path of less energy than it initially possessed in the uniform field in air. There is a limit to the number of tubes which are drawn through the ball, because for tubes some distance out the necessary increase in air path, in order to pass through, would involve a greater increase in energy than would be compensated for by the decreased energy of the path through the ball. It will be seen that the crowding of the tubes of force through the ball causes, for a portion of their path, an increase in the area of cross section of some of the adjacent external tubes. This evidently means that at points of increased area of cross section the intensity of polarisation of the medium has decreased, and, therefore, the energy per unit volume has decreased, that is, although the volume of the tube has increased for a portion of its path, the energy for that portion can be less than before.

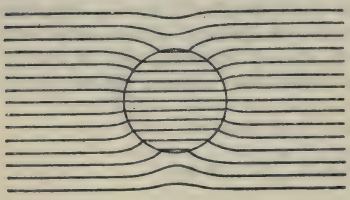


Fig. 68.

When a piece of conducting material is placed in an electric field it has been assumed that there is no energy in the space occupied by the conductor. That is, the conductor acts as if its surface enclosed a dielectric of infinite specific inductive capacity. Thus, if a conducting sphere be supposed to take the place of the ball of sulphur in the field shown in Fig. 68, the distribution of the tubes of force will still be of the same nature, but, in accordance with the tangent law of refraction and with the assumption that the conducting surface acts as if it enclosed a dielectric of infinitely large specific inductive capacity, the lines of force as shown in Fig. 69 cut the surface of the conductor everywhere in a direction normal to that surface.

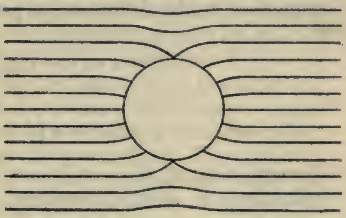


Fig. 69.

The explanation given in Art. 33 of the force acting on a neutral conductor placed in an electric field is evidently capable of extension. If a piece of dielectric of greater specific inductive capacity than that of the surrounding medium be placed in a non-uniform electric field it is evident that the piece will, like the conductor, be urged towards the region where the intensity of the field is a maximum. Similarly, if the specific inductive capacity of the piece of dielectric be less than that of the surrounding medium it will be urged towards the region of minimum electric force. The lower limit of the force acting in each of these cases can evidently be determined by the method indicated in Art. 33 for the case of a small conductor in the field.

48. Parallel Plate Air Condenser with intervening slab of other Dielectric. Assuming that there is no disturbance due to edge distribution on the plates, the field of force between the plates is uniform, and the lines of force are parallel straight lines crossing normally from one plate to the other. If a slab of dielectric with plane

faces parallel to the plates be interposed between the plates there will be no change in the distribution of the tubes of force and the polarisation in the air, and the dielectric will be the same. Now the force in air between the plates is $4\pi\sigma$, where σ is the surface density of the charge on the plates, and, the polarisation being the same, the force in the dielectric will, if σ be unchanged, be $4\pi\sigma/\mathrm{K}$, where K is the specific inductive capacity of the dielectric. Hence, if d denote the distance between the parallel plates and t the thickness of the slab of dielectric, then the work done in conveying unit quantity of electricity from one plate to the other, that is, the difference of potential between the plates, is

$$4\pi\sigma(d-t)+\frac{4\pi\sigma}{\mathrm{K}}t$$

or

$$\mathrm{V}=4\pi\sigma\left(d-t+\frac{t}{\mathrm{K}}\right).$$

The capacity of the condenser per unit area of plate is evidently σ/V, and is therefore given by

$$\frac{1}{4\pi(d-t+t/\mathrm{K})} \text{ or } \frac{1}{4\pi d'}$$

where

$$d'=(d-t)+t/\mathrm{K}.$$

That is, the capacity of the condenser with the intervening slab of dielectric is the same as if the dielectric of thickness t were replaced by a layer of air of thickness t/K, where K is the specific inductive capacity of the dielectric.

The force exerted on a unit area of either plate is, when the medium surrounding the plates is air, given by $2\pi\sigma^2$ (Art. 32), and is, therefore, unchanged by the introduction of the slab of dielectric if σ is unchanged, that is, if the charges on the plates are unchanged. If the charges remain constant with a surface density σ the potential difference for the plate must change from $4\pi\sigma d$ to $4\pi\sigma d'$ as explained above. If the potential difference be maintained constant then the charges must change, and we get

$$4\pi\sigma d=4\pi\sigma' d',$$

so that

$$\sigma' = \frac{d}{d'}\sigma = \frac{d\sigma}{d - t + t/\mathrm{K}}$$

and the force exerted on either plate per unit of area is given by

$$2\pi\sigma'^2 \text{ or } 2\pi\sigma^2\left(\frac{d}{d - t + t/\mathrm{K}}\right)^2$$

Hence if the quantity

$$\frac{d}{d - t + t/\mathrm{K}}$$

is greater than 1 the force is increased, that is, the force is increased or decreased by the introduction of the slab of dielectric according as K is greater or less than 1, that is, according as the specific inductive capacity of the dielectric is greater or less than that of air. Obviously, if the dielectric slab be replaced by a conducting plate the effect is merely to reduce the air thickness from d to $(d - t)$, and this is consistent with the assumption that the value of K for conducting material may be taken as infinite.

The potential energy of the condenser is given by $\frac{1}{2}$ Q V, and is evidently changed by a change in Q or V, and therefore it is changed by the introduction of the slab of dielectric, both for constant charges and constant difference of potential. The magnitude of this change of energy may be determined from the values of Q and V indicated above. A consideration of the distribution of the energy in the medium between the plates also indicates that the introduction of the slab of dielectric will change the amount of energy. Taking the case of constant charges the energy per unit volume in air is $\mathrm{F}^2/8\pi$ or $16\pi^2\sigma^2/8\pi$ or $2\pi\sigma^2$, while in the dielectric it is $2\pi\sigma^2/\mathrm{K}$. Before the introduction of the dielectric the energy in the air between the plates per unit area of plate is $2\pi\sigma^2 . d$; after the introduction of the slab the corresponding value is $2\pi\sigma^2(d-t)$ and $2\pi\sigma^2 t/\mathrm{K}$ or $2\pi\sigma^2(d-t+t/\mathrm{K})$. That is, there is a decrease of energy if K is greater than 1. The same result may be got by taking the difference of the two values of $\frac{1}{2}$ Q V. For unit area of plate $\mathrm{Q} = \sigma$ and V is equal to $4\pi\sigma d$

before the slab is introduced and to $4\pi\sigma(d-t)+4\pi\sigma t/K$ or $4\pi\sigma(d-t+t/K)$ after its introduction. Hence the two required values of $\frac{1}{2}QV$ are $2\pi\sigma^2 d$ and $2\pi\sigma^2(d-t+t/K)$ as obtained above.

49. Measurement of Specific Inductive Capacity. In Art. 37 it is explained that the specific inductive capacity of a given substance is measured by the ratio of the capacity of a condenser with that substance as dielectric to the capacity of the same condenser (or an exactly similar one) with air as dielectric. Hence, the problem of measuring the specific inductive capacity of any substance resolves itself into one of comparing the capacities of condensers.

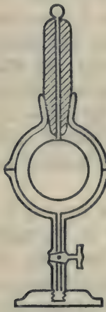


Fig. 70.

The first measurements of specific inductive capacity were made by Faraday. He constructed two similar spherical condensers, each of which consisted, as shown in Fig. 70, of an inner spherical brass ball and an outer spherical shell, made in two halves, so that the space between might be readily filled with any given dielectric. To the inner sphere was attached a thin brass rod carrying a small brass knob, and this rod, insulated by a plug of shellac, passed through the outer shell and formed the pole by which the condenser could be charged—that is, it performed the same function as the knob of a Leyden jar. In one of these condensers air was the dielectric, and into the other different substances, whose specific inductive capacities were to be determined, were successively introduced. In each case the capacities of the two condensers were compared by the method given in Art. 45, and the specific inductive capacity of the substance used as dielectric determined.

Since Faraday's time many new determinations of specific inductive capacities have been made. Modern experimenters have endeavoured to effect improvements mainly in two directions. Firstly, in the forms of condensers used and the methods of comparison. Guard-plate and cylindrical condensers of adjustable capacity are used in preference

to spherical condensers, and null methods of comparison of capacity are employed instead of the simple, direct method of Art. 45. Secondly, efforts are made to eliminate the effect of what is known as *electrical absorption*. When a condenser is charged to a given potential it is found that the charge which it takes tends to increase with the time of charging. It was at first thought that the charge was absorbed by the dielectric—hence the term *absorption*—but it has been shown that the effect results from a change in the state of the dielectric, which requires time to take place, and which is analogous to the "set" which a wire takes when subjected to torsion.* The existence of this effect evidently complicates the question of comparing capacities; for the capacity of a condenser will vary with the time of charging, and the amount of this variation will also depend on the nature of the dielectric. To avoid this complication methods of comparison have been adopted in which the condensers are charged only for an extremely small fraction of a second, so that the effects of absorption are reduced to a minimum.

From what has been said it will be seen that if, in any condenser system or system of conductors, air as a medium be replaced completely by a medium of specific inductive capacity K, and the potentials maintained constant, the force exerted between the various parts of the system will be changed in the ratio 1 to K. Thus, if in a quadrant electrometer air be replaced by, say, paraffin oil, while the potentials of the needle and quadrants remain unchanged, the moment of the couple twisting the needle will be K times in the oil what it was in air. This result may evidently be applied in determining the value of K for an insulating liquid.

The change in the capacity of a parallel plate condenser by the introduction of a slab of dielectric between the plates may evidently be conveniently used to determine the specific inductive capacity of the dielectric. Thus, if the charge in the condenser remain constant, the difference of potential between the plates will change when the slab

* The recovery of the dielectric from its electrical "set" explains the existence of the *residual charge*.

of dielectric substance is placed between the plates. If, however, the distance between the plates be adjustable this distance may be varied until the difference of potential, as indicated by an electrometer, is the same as before. When this is the case the capacity is the same as before, and in the notation of Art. 48 we have

$$\frac{S}{4\pi d} = \frac{S}{4\pi(d'-t+t/K)}$$

or

$$d = d'-t+t/K,$$

that is

$$K = \frac{t}{(d-d')+t}.$$

If the distance between the plates is adjusted by means of an micrometer screw the distance $(d-d')$ can be very accurately determined, and, as t can also be accurately measured, this method would give very good results were it not for the difficulty of preventing leakage and keeping the charge constant.

CHAPTER X.

INDUCTION ELECTRICAL MACHINES.

50. Introductory. In induction electrical machines the supply of positive and negative electricity is the result of continuous inductive action between the charged moving parts of the machine. The electrophorus described in Art. 16 is a simple form of induction machine supplying only positive electricity; its action, however, is not continuous, and the manipulations of the operator take the place of the automatic action of a machine. A careful consideration of the following experiment will enable the reader to grasp the principles involved in the action of induction machines. Let two metal vessels, A and B, be insulated and charged, A positively and B negatively, and let a brass ball, C, suspended by a silk thread, be lowered into one of them, say A. If C be now, while inside A, connected to earth for an instant, it becomes negatively charged by the inductive action of the charge on A, and may be raised out of A, carrying its negative charge with it. While still negatively charged, let it be lowered into B and allowed to touch the inner surface of that conductor; *it at once gives up its charge* to B (Art. 18), *and the negative charge on* B *is thereby increased.* The ball C is now uncharged; but if connected to earth for a moment while it is still inside B, it will acquire a *positive* charge, as the result of the inductive action of the negative charge on B, and this charge may be transferred to A by removing C from B, and putting it in contact with the inner surface of A. The operations just described may evidently be repeated indefinitely, *and at each operation the initial charges on A and B are increased.* This accumulative action will go on until the conductors A and B are charged

to such a potential that their loss of charge, owing to imperfect insulation, is as great as their gain from *the carrier* C. If the motion of C were controlled by machinery the arrangement just described might be considered as a form of induction machine, in which the conductors A and B represent the positive and negative poles respectively, and if these conductors were furnished with knobs nearly in contact a spark would pass between them when the difference of potential between the knobs became sufficiently great. After the passage of this spark, however, the action of the arrangement would, in general, cease, for the conductors A and B would be almost completely discharged. We shall now proceed to consider a simple modification of the above apparatus, from which the possibility of continuous action will be more easily realised; but, before going further, the reader should be careful to fully understand the principles of action involved in the above arrangement. These are, first, the inductive action of the charged conductors, A and B, on the carrier C, *when the latter is connected to earth*; second, the gradual increase of the initial charges on A and B by the successive transfer of the induced charges from C.

In the arrangement of Fig. 71, let the conductors A and B be represented by two short arcs of brass tube split along the inner edge, and the carrier C by the small brass ball C, attached to the end of an insulating rod pivoted at O, the centre of the circle, of which the axes of the tubes A and B are arcs. The ball C, when rotated, passes freely through the tubes A and B, the insulating arm passing through the split on the inner side of these tubes. To effect the necessary contacts with the earth and the conductors A and B, two sets of springs, *a*, *b* and *e*, *e*, are provided; *a* and *b* make contact with A and B respectively, and *e* and *e* both with the earth. Let A receive a small positive charge, and imagine C to rotate in the direction indicated by the arrow. In passing through A the carrier C first touches the spring *a*, but has not yet received any charge to give up; it next touches *e*, and is for an instant connected to earth; the positive charge on A now acts inductively on it, and, as a result, C leaves A with a

negative charge induced on it. This charge is carried round towards B, and when C touches the spring b, which is on the inner surface of B, it gives up nearly all its charge to this conductor. The charge on B is now increased, and when C touches the spring e in B it is acted on inductively by this increased charge, and the induced charge on C is carried round to the spring a, where it is delivered up to A. This action goes on until A and B are charged as

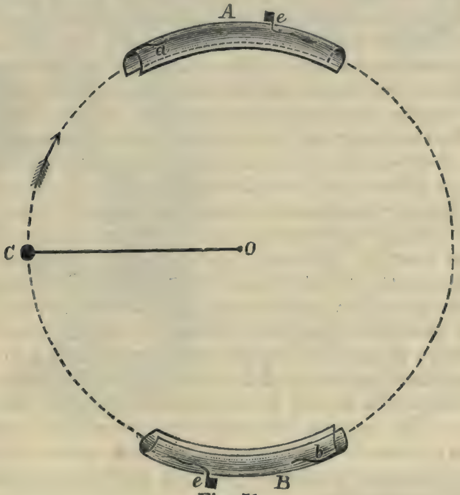


Fig. 71.

highly as the circumstances of their insulation will permit. If these conductors are provided with knobs which can be brought near each other by means of sliding rods to which they are attached, successive sparks may, under favourable conditions, be made to pass between these knobs, for, after the passage of one spark, although A and B are nearly discharged, a sufficient residual charge in general remains in each to maintain the action of the machine, and the conductors A and B are again quickly charged up to the

necessary difference of potential to give the spark discharge between the knobs. In machines constructed on this principle, however, it is not usual for the parts here represented by A and B to form the poles between which spark discharge takes place. In this relation let us consider a further modification of the arrangement of Fig. 71. Let the carrier C, and the insulating arm which carries it, be replaced by an insulating rod pivoted through its centre at O, and bearing at *each* end a carrier C. Further, let the springs *e*, *e*, be insulated and connected with one another, instead of being connected to earth. The rod pivoted at O is supposed to revolve in the same way as that bearing the single carrier C, and the electrical action of the arrangement is exactly similar to that described above; when one end of the revolving rod is in contact with the spring *e* in A, the other end is in contact with the spring *e* in B, and it is evident that the inductive actions of the charges on A and B work together to induce a negative charge in the end of the rod near A, and a positive charge on the other end near B. If now a gap be made in the connection betweeen *e* and *e*, it will be seen that, if the inductive action is strong enough, a spark may pass across this gap, and in most induction machines this is where the spark discharge takes place. When the machine is starting to work, the connection between *e* and *e* should be continuous; but as the charges on A and B accumulate, and the inductive action on the carriers becomes stronger and stronger, a break may be made in the connection, and spark discharge take place across this break, without interfering with the action of the machine.

Sir William Thomson, in 1867, constructed a small apparatus known as *Thomson's Replenisher*, which works in exactly the same way as the arrangement just described. The construction of the instrument is sufficiently explained by the Figs. 72 and 73, which are lettered to correspond with the diagrammatic figures given above. The darkly shaded parts are of ebonite, and the rest of metal. The springs *e*, *e*, are connected by the strip of brass M, which runs round the ebonite base, on which the spindle for rotating the carrier C C rests. There is no gap in this

connection between the springs *e*, *e*, for the instrument is not required to produce sparks. It is used in Thomson's electrometers to maintain the Leyden jar at a constant potential—one of the *inductors*, A, B, is connected to the inner coating of the jar, and the other to earth; and when the potential of the jar shows signs of falling a few turns of the carrier quickly restores it to its original value, which is indicated by a suitably constructed gauge.

The principles involved in the action of the usual forms of induction machines are completely illustrated by the

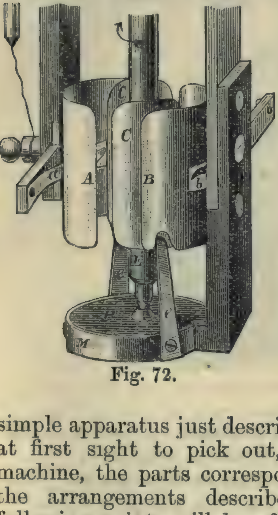


Fig. 72.

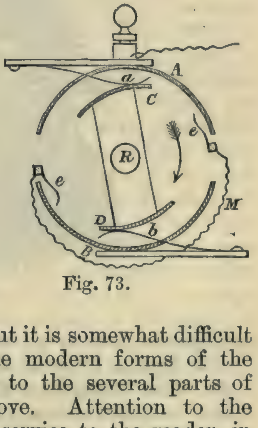


Fig. 73.

simple apparatus just described, but it is somewhat difficult at first sight to pick out, in the modern forms of the machine, the parts corresponding to the several parts of the arrangements described above. Attention to the following points will be of some service to the reader, in this respect. It is evident that the apparatus of Fig. 71 will work more efficiently if we have a number of carriers, instead of only one or two. Now, the most convenient way of arranging a number of insulated carriers, for revolution round a central axis, is to fix a number of metal studs round a circle, near the edge of a circular glass plate which can be rotated on an axis passing through its centre.

Hence a varnished glass plate with a ring of metal studs fixed to the glass in a circle near its edge is the characteristic feature of an induction or *influence* machine, and constitutes the carrier system of the machine. In some cases, however (for example, in the first forms of the Holtz machine), a plain glass plate without any metal studs on it forms the carrier system; and, on the other hand, in the most recent form of the machine (the Wimshurst machine) the studs on the plates act the parts of the inductors A and B as well as that of carriers. The springs *a*, *b*, are represented by points, or fine wire brushes, while the

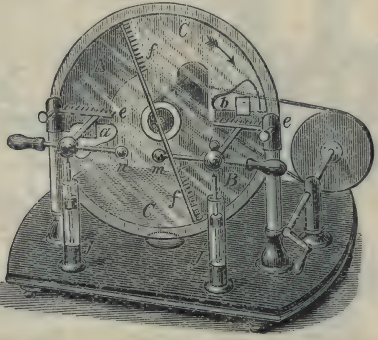


Fig. 74.

springs *e*, *e*, are in all cases represented by combs of fine points, similar to those used in frictional machines.

51. The Holtz and Voss Influence Machines. The Holtz machine, shown in Fig. 74, was the first form of machine that came into general use, as a substitute for frictional machines. Its construction will be readily understood from the figure, which is lettered in accordance with the notation used above. The inductors A, B (usually called *armatures*), are pieces of paper fastened to the back of the fixed plate of glass, shown at the back of the figure. The pointed tongues *a*, *b*, of these armatures, which protrude through openings in the plate, represent the springs

a, *b*, of Fig. 41, and are the means of electrical communication between the armatures and the carriers. The smaller glass plate C, C, in front is the "carrier" plate of the machine, and its rotation is effected by means of the strap and pulley arrangement shown. In most forms of the Holtz machine, this carrier plate is a plain plate of varnished glass, having no metal studs fixed on it; the surface of the plate acts as a continuous carrier, much in the same way as if a ring of tinfoil with radial slits were pasted round it, near the outer edge. In front of this plate are arranged the combs, *e*, *e*, which communicate with one another across the spark gap *m n*. To work the machine a small charge is given to one of the armatures, the knobs *m*, *n*, are put in contact, and the carrier plate C, C, is rotated; in a short time it will be found that it is much more difficult to effect the rotation, and if the room is darkened, brushes of light will be seen diverging from the teeth of the combs. If the knobs *m*, *n*, be now separated, sparks will at once break across the gap. When the difference of potential between A and B is very great, then the action of the single pair of combs *e*, *e*, on the rapidly rotating plate may not be sufficient to admit of complete inductive action at each point of the plate, or on each carrier on the plate, as it passes the comb. To remedy this, a second pair of combs connected by a continuous insulated conductor D, is used, and the armatures extended so that this pair of combs may be placed, as at *f*, *f*, in such a position that it may complete the action of the pair *e*, *e*. A machine provided with this second pair of combs may evidently be started without *m* and *n* being in contact.

The small Leyden jars J, J, are connected to the combs in order to increase their capacity, and thus a larger charge can be accumulated on them, before spark discharge takes place across *m n*, and when this spark does take place it is much louder and brighter than without the jars.

The Voss machine is a slight modification of the Holtz machine. The carrier-plate is usually furnished with a ring of metal studs as carriers, but the characteristic of the machine is that it is self-starting; small brushes of fine wire are fixed on the combs, so as to touch the carrier

studs as they pass, and the friction between them causes the latter to become very slightly charged, and thus the initial charge is obtained. The other details of the machine are similar to those of the Holtz.

52. The Wimshurst Machine is the most recent form of influence machine. A simple form is shown in Fig. 75.

In construction it is very simple and it is practically a

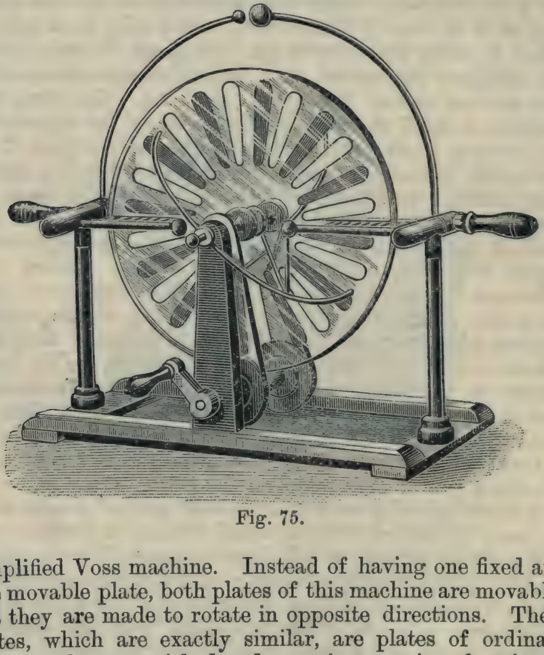


Fig. 75.

simplified Voss machine. Instead of having one fixed and one movable plate, both plates of this machine are movable; but they are made to rotate in opposite directions. These plates, which are exactly similar, are plates of ordinary window glass, varnished and carrying a series of strips of metal fixed in a ring round the plate, with their lengths arranged radially. These strips serve as carriers and as armatures, the carriers of one plate being in succession armatures for the carriers on the other plate.

The arrangement of brushes in this machine is, however, somewhat different from that adopted in the older forms. The combs are arranged as in the Voss machine, with small brushes attached to them to give the initial charge to the carriers, but instead of the tongues of the armatures there are the two *uninsulated* conductors shown in the figure, one for each plate, and each carrying a small brush of fine brass wire at each end. These conductors are arranged at right angles to each other, and at an angle of about 45° with the combs. By this arrangement each carrier is connected to earth twice in each revolution of the plate, and at each contact receives a charge by the inductive action of the charge on the carrier directly opposite it on the other plate. In this way the metal strips on the plates perform the functions of both carriers and armatures, and from the method of action it is evident that the plates of the machine should be made to revolve in opposite directions; for the charges on the carriers of one plate being due to the inductive action of those on the carriers of the other plate, the sign of the charge of one set of carriers must be opposite to that of the other set, and therefore one plate should be revolving towards one comb, say that at which positive electricity is collected, and the other plate towards the other comb. Similarly it will be seen that, on each plate, the sign of the charges of all the carriers *above* the combs must be opposite to those on the carriers below the combs. This last result, however, obtains in all forms of influence machine; in the simple machine of Fig. 71 the charge on the carrier reverses each half-revolution.

The machine shown above has only two plates, but compound machines are now made with four, six, eight and more plates. These machines can be made to give sparks several inches in length and are very convenient for general use; for, unlike the frictional machines, they can be made to work under almost any atmospheric conditions.

53. Spark and Brush Discharge. When two conductors at different potentials are made to approach one another a passage of electricity takes place between them across the intervening dielectric accompanied by heat, light, and a crackling sound. The phenomenon is in general

momentary and the potentials of the conductors become more or less equalised, but by connecting the latter to an electrical generator, as in the case of the knobs of a Wimshurst machine, it is possible to restore the original potential difference as fast as it runs down and the discharge then becomes practically continuous.

The appearance presented may be either that of a single or branched line of light (straight, curved, or zigzag) passing from one conductor to the other, when it is called a "spark," or that of a brush-like glow with branches diverging from a stem springing from one of the conductors when it is called a "brush."

The distance which separates the two conductors when the discharge takes place is called their *striking distance*, and for a given pair of conductors similarly placed it is found to be very nearly proportional to their potential difference, but for differently shaped pairs (or even for the same pair with differently shaped portions of their surfaces facing one another), it depends also upon the shape. Thus for a pair of spheres the striking distance is about one inch per 100,000 volts, while for a point facing a flat plate it is much greater, viz., about one inch per 20,000 volts, and for a pair of points greater still. There appears to be no practical difference so far as striking distance is concerned between the "spark" and the "brush," that is, taking a given pair of conductors in a given relative position and at a given difference of potential, and causing them gradually to approach, we do not get a brush at one distance and a spark at another but we get *either a spark or a brush* at a definite distance. Which of the two forms the discharge assumes depends partly on the capacity of the conductors, but mainly on the shape of their opposing surfaces; rounded surfaces tend to produce sparks, and pointed ones brushes.

It should be carefully noted that the term "striking distance" refers to the *starting* of the discharge and not to its *maintenance*. For example, suppose we have two spheres which by means of a machine are kept at a potential difference of 10,000 volts, then in order to start a spark between them they must be brought to within

$\frac{1}{10}$ inch (the striking distance), but after the spark has once started they may be gradually separated to a much greater distance and it will still continue. This circumstance finds application in electric arc lamps where a mechanism is employed to bring the carbons into close proximity in order to start the light and afterwards to separate them.

The numbers given above suppose the discharge to take place in air at the ordinary pressure. For solid (and liquid) dielectrics the striking distance is *ceteris paribus* much less owing to their greater mechanical rigidity. For the same pressure the striking distance is under similar conditions much the same in all gases, but when the pressure is diminished the striking distance increases up to a certain point, and afterwards diminishes until when the exhaustion is very complete no discharge occurs however great the difference of potential. For high but not extreme exhaustion the appearance is neither that of the spark or brush, but of delicate striae, and is known as the electric "glow."

All these phenomena appear to indicate that the discharge (in whatever form it occurs) is not a simple passage of "electricity" from one of the conductors to the other, but that this passage is effected by means of the material molecules of the dielectric. According to the kinetic theory of matter, the molecules of every substance are in a state of rapid vibration and mutual collision, the length of the journey performed by each molecule (called its "free path") being extremely small, and the greater the mechanical rigidity the shorter the path. Hence if we regard the discharge as due to the handing on of electricity from one molecule to another during the vibratory movements an explanation is suggested of the connection between rigidity and striking distance. Moreover, in the case of gases, the kinetic theory renders it certain that as the pressure diminishes the free path increases, which explains the corresponding increase of striking distance.

A remarkable fact with regard to the discharge is its *oscillatory* character; although the spark is practically instantaneous, in reality it occupies a finite though very

small time, and when it is examined by a revolving mirror it is found to consist of a series of discharges alternately in opposite directions. If a piece of paper or cardboard be held between the knobs of a Wimshurst so as to allow the spark to pierce it, it makes a hole, which, instead of being indented on one side and protuberant on the other like a pin-prick, is *protuberant on both sides,* the molecules having apparently oscillated to and fro, and on the whole moved outwards in both directions.

CHAPTER XI.

ELECTRIC DISCHARGE.

54. Discharge through Conductors. An electric discharge through a conductor is accompanied by all the effects produced by an electric current in the conductor. The heat developed by a discharge may be made to produce sudden fusion or volatilisation of the metal of the conductor. Thus a strong discharge sent through a thread made up of strands of silk, silver, gold, and other metals, volatilizes the metal threads with explosive violence. Similarly, if a sheet of gold leaf and a piece of white silk, separated by a thin card with a perforated design on it, be clamped tightly between two pieces of glass, and a discharge sent through the gold leaf in a direction parallel to its surface, it is at once volatilised, and the vapour passing through the perforations of the design, traces it out on the white silk in a dark violet colour.

The magnetic effects of a discharge are found to be exactly the same as those of a continuous current. A magnetic field is developed round the conductor, and a magnetic needle is deflected, or a piece of iron or steel may be magnetised by the influence of this field. Thus if a discharge is passed through a long solenoid coil, in the interior of which a steel needle has been placed, it will be found that the needle is permanently magnetised. The usual electrolytic effects due to the passage of a current through an electrolyte are also found to accompany electric discharge through the electrolyte, and the laws of electrolysis given later apply to the electrolytic effects of a discharge. When the conductor is of bad conducting material, and offers a very high resistance to the passage of the discharge, violent mechanical effects are often produced.

Thus a sheet of glass may be pierced by the discharge between two points placed one on each side of it. In the case of a substance, like green wood, containing water or other easily vaporised liquid, the sudden vaporisation of the liquid by the heating effect of the discharge may produce a violent shattering effect. In the case of bad conductors the discharge is often found to pass over the surface of the substance. This is usually due to the fact that the surface layer is, on account of moisture or some other deposit, a better conductor than the substance of the conductor. A Leyden jar, for example, may sometimes be seen to discharge over the surface of the glass from the inner to the outer coatings. By covering the surface of a long glass tube with a metallic powder or little pieces of foil, a surface discharge, accompanied by beautiful luminous effects, may be passed along it. When the discharge takes place through the human body as a conductor, the physiological effects which accompany the *shock* are found to depend upon the energy of the discharge, that is, upon the quantity discharged as well as upon the difference of potential, and also to an important degree upon the time rate of discharge. The larger the quantity, and the potential difference, and the shorter the time of discharge, the greater the shock.

55. Disruptive Discharge. Where the gradient of potential in an insulating medium is raised sufficiently high by the approach of the positive and negative boundaries of the electric field to each other the medium breaks down under the stress, and the energy of the field is liberated by *disruptive discharge* between the boundaries. This discharge is usually accompanied by a *spark*, which absorbs the discharged energy of the field, but it may under certain conditions take the form of a *brush* discharge or a glow, which varies in character and effects with the conditions under which it is produced.

When a disruptive discharge of any form is examined by means of a rapidly rotating mirror it is found that it is of an oscillatory character. Thus the image of a short straight spark seen in a revolving mirror is a number of short straight parallel lines of light separated by narrow intervals corresponding to the period of oscillation of the discharge.

The path of the spark when formed is a line of comparatively small resistance, and acts as a conductor through which the oscillatory discharge takes place.

The discharge takes the spark or brush form according as the quantity of electricity to be discharged is large or small. When the quantity is large and the distance small the spark is short, straight, and intense. As the distance increases the spark line ceases to be straight, and takes a

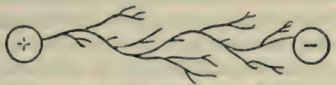


Fig. 76.

branching form similar to that shown in Fig. 76. It should be noted that in a spark of this kind the tips of the branches point from the positive to the negative pole. With a limited quantity of electricity the discharge takes the *brush* form. The characteristics of this form are well known. A bright luminous brush of a light violet colour branches from the positive terminal in the way shown in Fig. 77. The negative terminal is covered with a soft luminous glow showing here and there small bright starlike points where any small irregularities break the smoothness of its surface. Fig. 78 shows, according to Faraday, the difference between the brush from a positive and negative pole.

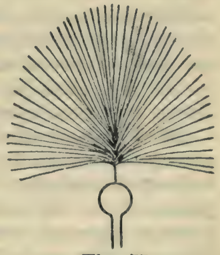


Fig. 77.

Fig. 78.

The brush discharge is accompanied by a sharp hissing sound readily distinguished from the sharp crackle of spark discharge. When it takes place between two terminals, such as the poles of a Wimshurst machine, which are supplied with electricity at a uniform rate, the difference of potential between the terminals remains

practically constant during the discharge. In the case of a spark under the same conditions, the difference of potential rises to a maximum at the instant of discharge and at once falls to zero to rise again to a maximum for the next spark. When a Wimshurst machine is worked without the Leyden jars the discharge between the poles is, for anything but short distances, of the brush form, but when the jars are used the capacity of the terminals is increased and a sufficient quantity accumulates when the maximum difference of potential is reached to give a spark discharge. Experiments with a Wimshurst machine show that for a given distance between the poles the difference of potential necessary to produce brush discharge between terminals of low capacity is practically the same as that necessary to give spark discharge between terminals of large capacity.

The colour of a spark is due chiefly to the volatilisation of particles of the metal terminals in the path of the spark. When the spark is very intense the colour is mainly due to this, but in long zigzag branching sparks the colour is also largely influenced by the medium itself and by particles of any foreign matter that may be present in it. The *glow* is one of the accompaniments of discharge in partial vacuum.

56. Passage of Electricity through a Gas. The passage of electricity between electrodes in a gas or from a charged conductor into the surrounding gas involves the passage from the metal electrode to the gas as well as the passage through the gas itself. When a charged conductor is surrounded by a gas at the ordinary temperature it, in general, loses charge by leakage into the surrounding medium. The results of recent experiments strongly support the view that the molecules of the gas do not themselves carry off the charge from the conductor, but that the small particles of dust in the gas are the active agents in causing the leakage. It is found that the more dust-free the gas surrounding the conductor is made the less the leakage becomes. It is also found that the vapour from a strongly electrified liquid is quite uncharged, thus proving that the molecules of a vapour do not take a charge. The leakage from a charged body surrounded by

a gas is therefore of a convective nature, the charge being carried away by the particles of dust which first come into contact with the conductor and then leave it.

The temperature of a gas is found in some cases to have an important influence on its conducting power. If the gas is one which dissociates into *atoms* on heating its conductivity, after dissociation takes place, is very great. It is assumed that in the dissociated state the *atoms* take part in the convective conduction into which only the dust particles enter at ordinary temperatures. If the gas is one which on heating dissociates into other molecules then there is no marked change in the conductivity. Chlorine, bromine, iodine, hydrochloric acid gas, hydrobromic acid gas, hydriodic acid gas dissociate into atoms on heating and conduct by atomic convection; while air, nitrogen, carbonic acid gas, steam, ammonia are examples of gases which dissociate into molecules, and conduction through them is due to dust convection. As would be expected conduction by convection is a somewhat irregular process, and Ohm's law does not apply to it.

The material of the conductor is found to have an influence on the leakage of the charge from it, and this influence is most apparent, or probably only apparent, when the conductor is negatively charged. This effect is probably due to the disintegration of the surface of the conductor, which, particularly when negatively charged, gives off minute particles of the metal. The "spluttering" of the negative electrode of a vacuum tube supplies direct evidence of this disintegration. The metallic particles from the electrode are deposited on the glass of the tube round the electrode as a thin metallic film. This disintegration of the surface is found to be greatly intensified by the incidence of ultra violet light. An uncharged metal surface on which ultra violet light falls is found to give off particles of the metal. If the metal is negatively electrified the disintegration is much more active but if positively charged it practically ceases.

It is also found that the disintegration of a metallic surface is accompanied by electrical separation, for when a neutral metal surface disintegrates under the action of

ultra violet radiation the metal is left positively charged, the dust being supposed to be negatively charged.

These results are closely connected with the important fact that the incidence of ultra violet light on a negatively charged conductor causes a rapid discharge of the conductor. The explanation seems to be that the incidence of the ultra violet light on the negatively charged conductor causes disintegration of the surface, and the positive electrification resulting from the emission of the negatively charged metallic dust neutralises the negative charge on the conductor. Further, the metallic dust from the surface permeates the medium and facilitates leakage by convection. It is also probable that the gas in the neighbourhood of the disintegrating electrode suffers dissociation, and the dissociated atoms take part in the convection.

The metals differ in their liability to disintegration, and this evidently explains the influence of the material of the electrodes on the passage of electricity through the gas by which the electrodes are surrounded. By determining the loss of weight under exactly similar conditions from the cathode of a vacuum tube Crookes gave the following table, showing the relative disintegration for the metals named:

Palladium	108·00	Brass	51·85
Gold	100·00	Platinum	44·00
Silver	82·68	Copper	40·24
Lead	75·04	Nickel	10·99
Tin	56·96	Iron	5·50

For aluminium and magnesium the loss by disintegration was found to be negligible.

The influence of ultra violet radiation in promoting disintegration also varies with the metal. The order of sensitiveness as given by Elster and Geitel, who investigated this subject for a few metals, is:—potassium, sodium, magnesium, aluminium, zinc, tin. This order seems to agree with the series usually given in connection with Volta's contact theory.

In the experiments on the passage of electricity through heated gases, made by Prof. J. J. Thomson, it was found that the facility with which the passage took place depended upon the temperature of the electrodes. If the

electrodes were cold the electricity passed with great difficulty from the cold metal to the hot gas, and it was necessary to make them glow brightly before this difficulty disappeared. The temperature of a charged conductor or of an electrode has a recognised influence on the passage of electricity from it to the surrounding gas, but the details are complicated and not fully understood. The disintegration of the surface, the emission of gas and vapour from the heated metal, and the nature of the surrounding gas all enter into the question, but it is probable that under normal conditions the incandescence of the conductor tends to facilitate the escape of negative electricity. It is found, for example, that an incandescent wire, *which has been glowing for some time*, charges a plate near it negatively, whatever the nature of the surrounding gas. It was stated by Guthrie that an iron ball heated to whiteness could not retain a charge of any kind, but that as it cooled it could retain first a negative charge, and, at a lower temperature, a positive one, but these results have not been confirmed.

57. Electromotive Intensity. Maxwell defined the electric strength of a gas as the greatest *electromotive intensity*, or potential difference per unit length, which the gas can bear before spark discharge takes place. It is found, however, that the electromotive intensity does not depend upon the gas only but is influenced more or less by the material, shape, sign, and distance apart of the electrodes, the nature of their surface, and the character of the intervening field. The electric strength of a gas is not therefore a characteristic specific property of the gas.

The relation between spark length and potential difference has been the subject of much research. Lord Kelvin made the first measurements of spark electromotive intensity in 1860. Baille in 1882 and Liebig in 1887 also published the results of very elaborate and careful experiments on this point. The simplest conditions for the determination of the electromotive intensity obtain when the electrodes are large and the field between them is approximately uniform. Under these conditions Liebig's results given in the table below, for air at atmospheric pressure, are typical of the general outcome of research on this point.

Potential Difference and Spark Length.

[*C.G.S. Electrostatic units and Centimetres* (1.*C.G.S. es. unit* = 300 *volts*).]

Spark Length in Centimetres.	Potential Difference.	Electro-motive Intensity.	Spark Length.	Potential Difference.	Electro-motive Intensity.
·0066	2·630	398·5	·2398	30·622	127·7
·0105	3·357	319·7	·2800	35·196	125·7
·0143	4·017	280·9	·3245	39·816	122·7
·0194	4·573	235·7	·3920	47·001	119·9
·0245	5·057	206·4	·4715	55·165	117·0
·0348	7·190	206·6	·5588	63·703	114·0
·0438	8·863	195·5	·6226	69·980	112·4
·0604	10·866	179·9	·7405	82·195	111·0
·0841	13·548	161·1	·8830	95·540	108·2
·0903	13·816	153·0	·9576	102·463	107·0
·1000	15·000	150·0	1·0672	110·775	103·8
·1520	20·946	137·8	1·1440	117·489	102·7
·1860	24·775	133·2			

The curves shown in Fig. 79 exhibit these results graphically. The nearly straight line shows the relation

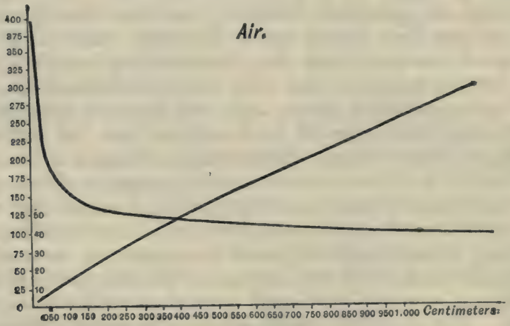


Fig. 79.

between spark length and potential difference, and the other curve that between electromotive intensity and spark length.

It has been shown from the data of Baille's and Liebig's experiments that the relation between spark length and

potential difference may, for sparks more than 2 mms. long, be given by the relation

$$V = a + b\,l,$$

where V is the potential difference, l the spark length, and a and b constants. Baille's results for air gave $a = 4{\cdot}997$ and $b = 99{\cdot}593$, with l in centimetres and V in electrostatic units.

It will be seen from these results that the electromotive intensity varies with the length of the spark and is much

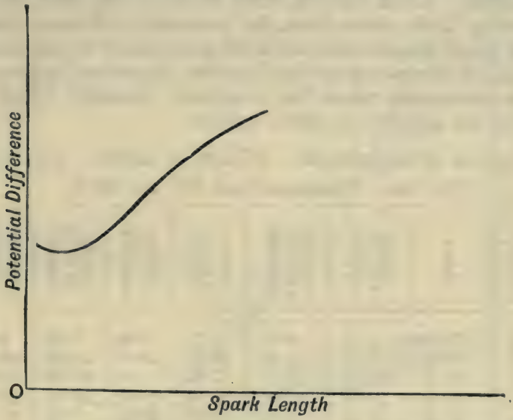


Fig. 80.

greater for short than for long sparks. This result was first discovered by Lord Kelvin in 1860. There is also evidence from experiments conducted by Mr. Peace at the Cavendish Laboratory that the potential difference decreases with the spark length down to a *minimum* value for a very short length and then *increases* with decrease of the spark length. The curve shown in Fig. 80 indicates generally the nature of the relation between spark length and potential difference. At atmospheric pressure the minimum potential difference occurs at such a small value of the spark length that accurate measures of the relation in the

neighbourhood of this critical value cannot well be made. At pressures below atmospheric pressure, however, the minimum potential difference corresponds to longer spark lengths and the existence of a minimum value can be clearly established. When the electrodes are small, and the electric field between them is not uniform, a number of interesting results are obtained. Baille has published results giving the potential difference and spark length for discharge between two equal spherical electrodes of different diameters. These results, some of which are given in the following table, show that the potential difference for a given spark length varies with the diameter of the spherical electrodes, increasing as the diameter decreases, but reaching a maximum value for a certain diameter which is greater the greater the spark length.

POTENTIAL DIFFERENCES: (*C.G.S. es. units*): PRESSURE 760 MM., TEMPERATURE 15° TO 20° C.

Spark length in cm.	Planes.	Spheres 6 cm. in diameter.	Spheres 3 cm. in diameter.	Spheres 1 cm. in diameter.	Spheres ·6 cm. in diameter.	Spheres ·35 cm. in diameter.	Spheres ·1 cm. in diameter.
·05	8·94	8·96	9·18	9·18	9·26	9·30	**9·63**
·10	14·70	14·78	14·99	15·25	15·53	16·04	**16·10**
·15	20·20	20·31	20·47	21·28	21·24	**21·87**	19·58
·20	25·42	25·59	25·95	26·78	26·82	**27·13**	21·91
·25	30·38	30·99	31·33	32·10	**32·33**	31·96	23·11
·30	35·35	36·12	36·59	37·32	**37·38**	36·29	24·12
·35	40·45	41·45	41·47	**42·48**	42·16	39·39	25·34
·40	45·28	46·34	46·77	**47·62**	46·34	41·77	26·03
·45	50·48	51·46	**51·60**	**51·56**	50·44	43·76	26·62
·40	44·80	45·00	45·00	**45·50**	44·80	41·07	26·58
·45	49·63	50·33	49·63	**52·04**	48·42	43·29	28·49
·50	54·35	**55·06**	54·96	54·66	53·25	47·21	30·00
·60	63·82	**65·23**	**65·23**	**65·23**	59·69	53·75	31·51
·70	74·09	**75·40**	73·79	72·28	64·22	56·47	32·92
·80	84·83	**87·98**	84·76	77·61	67·75	58·79	33·82
·90	94·72	**97·44**	94·62	80·13	70·56	59·09	34·93
1·00	105·49	**112·94**	104·69	83·05	72·38	59·49	36·24

The nature of the influence of the shape of the electrodes which determine the nature of the intervening field is shown by the curves given in Fig. 81. These curves, taken from a paper by De la Rue and Müller, show the variation of striking distance with potential difference for different forms of electrodes. The cells by which the potential

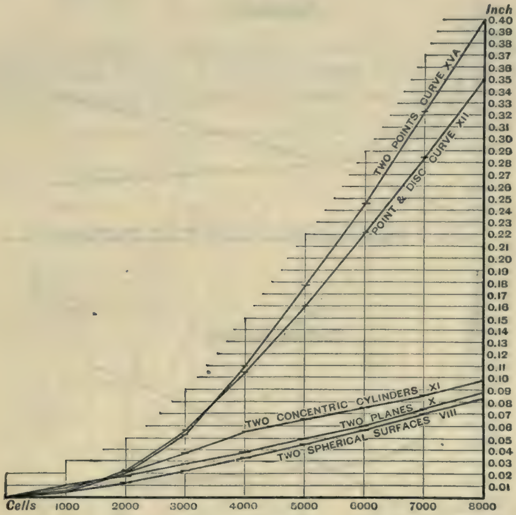


Fig. 81.

difference was measured had an electromotive force of 1·03 volts each.

The relation between spark length and potential difference is, in general character, the same in all gases, but the quantitative details vary with the gas. The curves given in Figs. 82 show Liebig's results for hydrogen and carbon dioxide corresponding to the results for air exhibited in Fig. 79. The potential differences are given in electrostatic units.

The influence of pressure on the relation between spark length and potential difference is of great interest. The general result of change of pressure is that, for a given spark length, the potential difference decreases as the pressure decreases, down to a minimum value and then

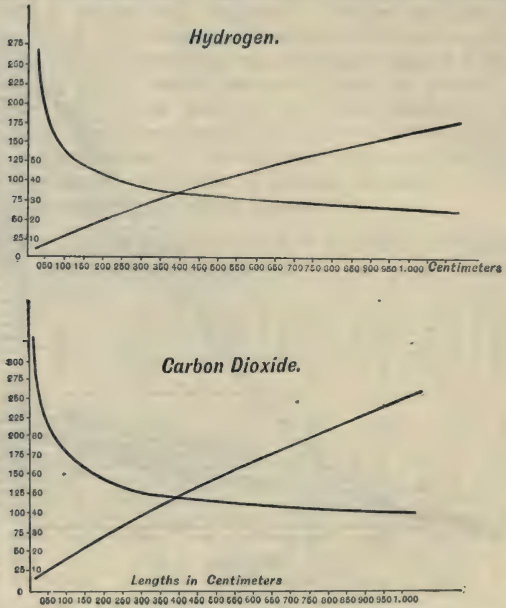


Fig. 82.

increases. The critical pressure at which the potential difference attains its minimum value is lower the greater the spark length, but the minimum potential difference, although it increases slightly as the spark length increases is roughly constant at a little more than 300 volts. The following table gives numerical results for short sparks in air.

Spark Length.	Minimum Potential Difference.	Critical Pressure.
·00100 cm.	326 volts.	250 mm.
·00254 cm.	330 volts.	150 mm.
·00508 cm.	333 volts.	110 mm.
·01016 cm.	354 volts.	55 mm.
·02032 cm.	370 volts.	35 mm.

Fig. 83 exhibits graphically the results of experiments, made by Mr. Peace at the Cavendish Laboratory, on the relation between potential difference and gas pressure at

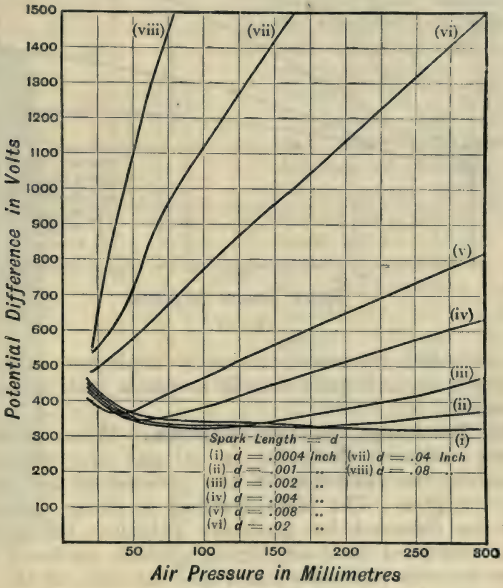


Fig. 83.

given spark lengths. From these results Mr. Peace also gives

curves showing the relation between potential difference and spark length at different pressures.

These curves are given in Fig. 84, and show clearly that at each pressure there is a critical spark length at which

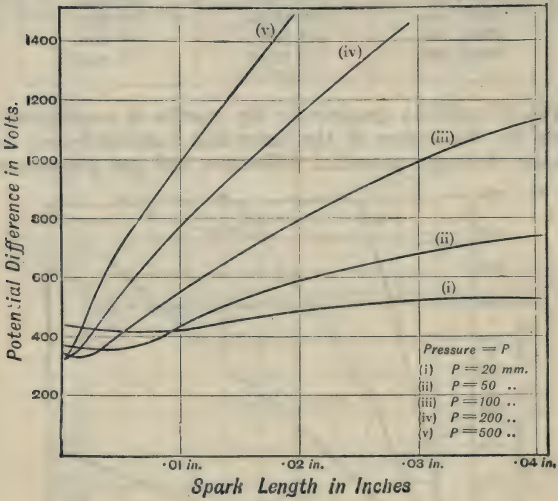


Fig. 84.

the potential difference attains a minimum value, and that this critical spark length increases with *decrease* of pressure.

58. Discharge in low Vacuum. When electricity passes between electrodes in a gas the phenomena attending the reduction of the pressure of the gas are very complex. The main features in these phenomena may be observed by gradually reducing the pressure in a tube filled with air, through which discharge takes place between electrodes fused into the ends of the tube. The tube should be about 20 cms. long, and may conveniently take the form shown in the Fig. 85. The pump used for reducing the pressure should be a good form of

mercury pump, capable of producing a very good vacuum. In order to produce the discharge the electrodes may be connected to the secondary terminals of an induction coil. Provided the distance between the electrodes is greater than the spark length for the coil no discharge takes place when the air in the tube is at atmospheric pressure. As the pressure is decreased, however, a point is ultimately reached at which intermittent flashes or streaks of rosy coloured light begin to pass between the electrodes.

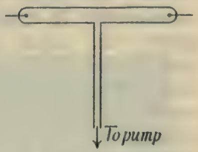


Fig. 85.

As the pressure falls a thin thread of this luminosity apparently persists between the electrodes. It becomes wider, whiter, and more intense with decrease of pressure, and soon appears to fill the tube with a bright luminous glow. When the pressure gets very low this glow loses its apparent uniformity, and is seen to possess different characteristics in different parts of the tube. At a pressure of about 1 mm. of mercury the appearance in the tube is something of the nature shown in Fig. 86. The negative electrode, *k*, is covered with a velvety glow of a violet colour, and next to this is a dark gap known as the *first dark space*, which is at first very narrow, but increases in length as the pressure

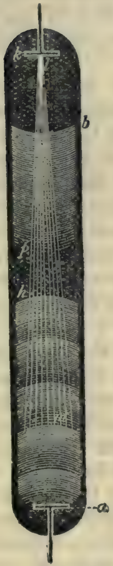


Fig. 86.

Fig. 87.

decreases. Then comes a short luminous column called the negative column or the *negative glow*, which usually differs in colour and intensity from the rest of the luminosity in the tube. These three parts, the violet glow, the first dark space, and the negative glow, are found to be local characteristics of the neighbourhood of the negative electrode, and are probably associated with the transfer of the electricity from the gas to the electrode. They are found to be quite unaffected by the position of the positive electrode, or by the length of the tube. The extent of the negative glow, however, is found to depend upon the size of the tube near the electrode. Thus, in the tube of the shape shown in Fig. 87, if *b* be the negative electrode the glow fills the bulb, but if *a* is made negative only a very slight glow appears at the point of the electrode.

Next to the negative glow is the second dark space, which seems to separate the negative electrode appearances from the *positive column*, which fills the rest of the tube up to the positive electrode. This positive column is the most striking feature of this stage of the discharge, and appears to be the chief agent in the transfer of electricity through the gas. The luminosity of the column is much greater than that of the negative glow, and its colour and spectrum are characteristic of the gas. It passes by the most direct path to the negative electrode, and changes its direction if the position of either electrode is changed. Under good conditions of vacuum and width of tube the positive column is found to be beautifully striated in the manner indicated in Fig. 88. These striæ are often in rapid oscillation backwards and forwards in the tube, and when they are very close together the oscillatory motion hides the striation, which can, however, be observed by means of a rapidly rotating mirror. By using a very long tube, and a rapidly rotating mirror, it has been established by Prof. J. J. Thomson that the luminosity of the positive column travels, in its formation, from the positive towards the negative electrode with a high velocity Through air at a pressure of ·5 mm. of mercury the velocity in a tube 5 mm. in diameter was found to be about half the velocity

of light. Spectroscopic experiments on Döppler's principle prove that the luminous molecules or atoms in the tube are not themselves moving at anything like this rate. The velocity is simply the velocity of propagation of the luminous appearance through the gas of the tube.

The light of the positive column has the power of exciting fluorescence in fluorescent substances, and very beautiful effects can be produced in vacuum tubes made with fluorescent glass or surrounded with fluorescent liquids. These tubes are known as *Geissler vacuum tubes.*

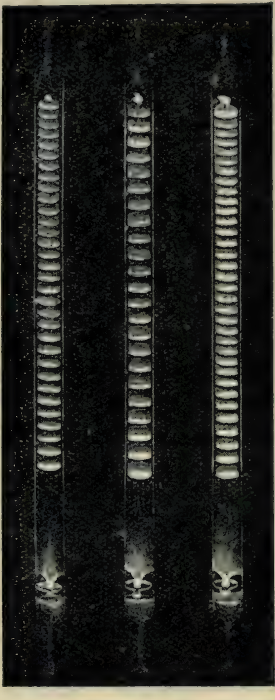


Fig. 88.

59. Discharge in high Vacuum. The phenomena described above characterise the discharge when the vacuum is fairly good, usually less than 1 mm. of mercury, but, as the exhaustion is pushed to a much higher degree, the phenomena change and develop into the highly interesting effects observed by Crookes in high vacuum. As the vacuum improves the discharge takes place with increasing difficulty and the positive column becomes less and less conspicuous, and ultimately disappears. At the same time the first dark space increases and ultimately it practically fills the tube. In very high vacua, approaching a perfect vacuum, discharge takes place only with extreme difficulty, and experimental evidence strongly

supports the view that a perfect vacuum is a perfect non-conductor. The characteristic feature of discharge in high vacuum is the projection of the gaseous matter in the tube from the negative electrode in a direction at right angles to the surface of that electrode. This hail of gaseous particles constitutes what are known as the *negative rays*, or *cathode rays*. The particles travel in straight lines, and their path is, therefore, quite independent of the position of the positive electrode. Among other properties, they have in a marked degree the power of producing fluorescence where they impinge on an obstacle whose surface is capable of fluorescence. The glass of the tube opposite the negative electrode usually fluoresces brilliantly under the impact of the particles projected from the electrode and very beautiful effects can be produced by interposing minerals, and other substances exhibiting fluorescence, in the path of the negative rays. One of Crookes' experiments, illustrating this power of producing fluorescence and also proving that the rays travel in straight lines is indicated in Fig. 89. The Maltese cross shown can be raised or lowered by means of a hinge attachment to its stand, the positive electrode. When lowered so as to be out of the path of the rays the wide end of the tube fluoresces strongly, but when the cross is raised, a shadow, defined by non-fluorescence of the glass, appears on the end of the tube. The outline of the shadow is seen to be the projection of the boundary of the cross by straight lines from the negative electrode. The negative rays are deflected in a magnetic field in

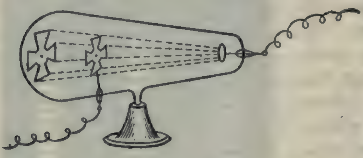


Fig. 89.

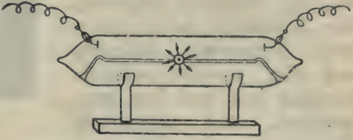


Fig. 90.

the same direction as a stream of negatively charged particles would be deflected.

The tube shown in Fig. 90 is intended to show that the stream of particles which constitute the negative rays can produce mechanical effect. The impact of the particles on the vanes of the wheel shown between the electrodes causes the wheel to rotate.

60. Rontgen Radiation. If the negative electrode in a high vacuum tube be made concave, the rays can be focussed at a point on a piece of platinum foil placed at the centre of curvature of the electrode. It is possible in this way to heat the foil at the focus of the rays to bright incandescence. Vacuum tubes, such as those described above for illustrating the phenomena of discharge in high vacuum, are known as *Crookes tubes*. When the negative rays are focussed on a small plate of platinum it is found that the small focal area of impact not only becomes under the impact a source of ordinary heat and light radiation, but that radiation of very short wave length, which is now known as Röntgen radiation or X-ray radiation, also has its origin at this area. Fig. 91 shows a focus X-ray tube. The negative electrode is concave, and the thin platinum plate, shown in section, is placed at the centre of curvature of the electrode with its plane at an angle of 45° to the axis of the electrode. The X-rays are given off in the direction shown.

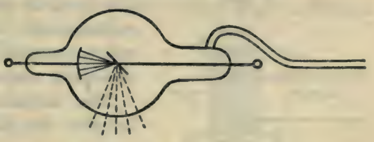


Fig. 91.

They have, to a high degree, the power of producing fluorescence. They also act on a photographic plate and pass through matter with a facility determined by its density. The lower the density of a substance the more transparent it is to these rays. Hence, if the hand is placed between a source of X-rays and a surface capable of bright fluorescence, the bones of the hand being more dense, and therefore less transparent, than the flesh, throw a distinct shadow of less brilliant fluorescence on the surface, acting as a

screen. If a photographic plate be substituted for this screen, and a suitable exposure given, a clear photograph of this shadow can be obtained on the plate. If the hand contained a bullet or a needle, the shadow of the dense material would be clearly differentiated from the less marked shadows of the bones and the flesh.

61. Influence of Magnetic Field. The effect of a magnetic field on the matter in a vacuum tube during discharge depends to some extent on the part the matter takes in the discharge. The positive column is deflected in the same way as a flexible wire carrying a current. This is well shown by an experiment due to De la Rive. The vacuum tube, as shown in Fig. 92, is made so as to fit over a bar magnet or an extension of the iron core of an electromagnet. The electrodes are rings, or a point and a ring, so arranged that the line of discharge is parallel to the length of the magnet. When discharge takes place under suitable conditions the line of luminosity, which, when the vacuum is not too low, is practically the positive column, is quite free to move and rotate round the magnet in the direction required by Ampère's rule.

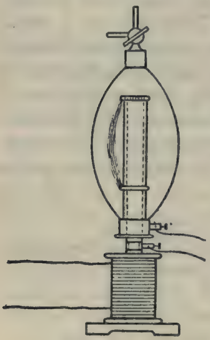


Fig. 92.

The negative glow is found to behave in the magnetic field as a paramagnetic substance of no weight and quite free to move.

The negative rays are deflected in a magnetic field, as already stated. Hittorf found that, when the directions of the rays at their starting point coincides with the direction of the lines of force, the rays continue to travel along the lines of force. When the rays were at right angles to the lines of force they were found to curve round into rings, and when the angle between the rays and the lines of force was less than a right angle they twisted into spirals.

The striation of the positive column is also found to be affected by a magnetic field. The most important effect is

that described by Spottiswoode and Moulton in these words, "If a magnet be applied to a striated column, it will be found that the column is not simply thrown up and down as a whole, as would be the case if the discharge passed in direct lines from terminal to terminal, threading the striæ in its passage. On the contrary, each stria is subjected to a rotation or deformation of exactly the same character as would be caused if the stria marked the termination of flexible currents radiating from the bright head of the stria behind it, and terminating in the hazy inner surface of the stria in question. An examination of several cases has led the authors of this paper to conclude that the currents do thus radiate from the bright head of a stria to the inner surface of the next, and that there is no direct passage from one terminal of the tube to the other."

62. Electrodeless Discharge. The phenomena so far described are those which characterise discharge in a vacuum tube between electrodes. It is possible, however, to produce discharge in a vacuum tube without the use of electrodes. If a coil of wire, round which a rapidly alternating current is passed, is placed round a bulb or ring tube suitably exhausted, induced eddy currents are set up in the gas and the path of these closed currents is marked by the luminosity characteristic of discharge in partial vacuum. For this purpose the best arrangement is to allow the successive discharges of a Leyden jar arrangement to pass through a coil of a few turn. If the conditions are such as to give oscillatory discharge the alternating currents through the coil will be of very high frequency, and the inductive effect will be strong enough to set up induced currents in the gas.

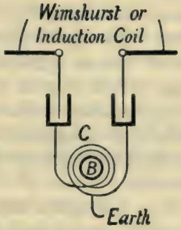


Fig. 93.

By the arrangement shown in Fig. 93 a steady sequence of discharges may be passed through the coil C, and an apparently continuous circular discharge produced in the bulb B in the zone of the coil. The coil should be earthed

to prevent currents being set up in the tube by the differences of potential that would be produced by electrostatic induction between points inside the bulb, if the coil were allowed to acquire a high potential. The appearances seen in a bulb acted on in this way, as the pressure is reduced from the ordinary atmospheric pressure down to a very good vacuum, are very similar to those observed with electrodes. At first no discharge takes place, but when the pressure falls to about 1 mm. of mercury a ring of rosy light appears in the tube, indicating the passage of a current round the luminous path. As exhaustion proceeds the ring thickens and strengthens. Its colour becomes whiter, and the intensity soon attains a maximum. After attaining its maximum the discharge gets fainter and fainter, and ultimately ceases. This shows that the cessation of the discharge in a high vacuum is not due to the difficulty of transfer from the electrode to the gas but is really due to the resistance of the gas itself. The assumption that a perfect vacuum does not conduct is thus confirmed by direct experiment. The fact that a perfect vacuum is transparent is conclusive but indirect evidence on this point. It is found that the pressure at which the luminosity attains a maximum in a tube without electrodes is much lower than that at which the same effect takes place under comparable conditions when electrodes are used. In air, for example, the maximum with electrodes has been observed at a pressure of about ·5 mm. while without electrodes the pressure of maximum effect was as low as ·005 mm.

In discharge without electrodes the appearances connected with the negative electrode are absent and the only luminosity present is that corresponding to the positive column.

This induction method of producing currents in a rarefied gas has been used by Prof. J. J. Thomson to obtain comparative measurements of the conductivity of the gas. If a layer of any material is placed between the primary and secondary of an induction system, the interposed material shields the secondary circuit from the inductive action of the primary to an extent determined by the conductivity of the material. Hence, by comparing the thicknesses of

layers of different materials which give the same shielding effect, the conductivities of the materials can be compared, for their specific resistances are directly proportional to the thicknesses of the layers. When the currents in the primary circuit are of such high frequency as obtain in the discharge circuit of a Leyden jar the thinnest sheet of metal or any fairly good conductor gives complete shielding so that no comparison of good conductors can be made by this method. With feebly conducting electrolytes, however, the shielding effect is found to be sufficiently reduced to allow of fairly accurate comparison of their conductivities. By this method, then, it is evidently possible to compare the conductivities of different gases with each other or with electrolytes of known conductivity. The method of making the comparison is indicated in Fig. 94.

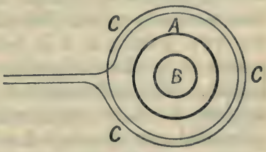


Fig. 94.

The central circle, B, represents the section of a vacuum tube adjusted to critical pressure of maximum luminosity. This tube is enclosed in a shield tube shown in section at A, and in the space between A and B the shield layer of gas or electrolyte is placed. The primary coil, C, lies outside the shield tube A. When gas at atmospheric pressure is placed in A luminosity is readily induced in B, but as the pressure is reduced the conductivity of the shield layer increases, and ultimately the discharge takes place in the space between A and B, and B is completely shielded. The gas in the tube, B, being used as an indicator, it is easy to find in this way the concentration of an electrolyte or the pressure of a gas at which a given shielding effect takes place. The conductivity of a rarefied gas is found by this method to be unexpectedly high; equal, for example, in a typical case to a solution of dilute sulphuric acid containing 25 per cent. of acid. It is found that in a magnetic field the conductivity of a rarefied gas varies with the direction, and is least in a direction at right angles to the lines of force.

63. Lodge's Experiments. The phenomena of oscillating discharge include a number of apparently anomalous effects resulting from the very high frequency of the oscillations. When the discharge is oscillatory the difference of potential between two points in the conductor carrying the discharge may be very great, and will vary very rapidly from instant to instant. Also the inductive effect of the electric oscillations in the discharge circuit, on surrounding conductors, may be very intense and may produce sparks between adjacent conductors even when they are earth connected.

Professor Lodge devised a number of experiments illustrative of these effects. He showed that a conductor through which a Leyden jar is being discharged gives a strong *lateral discharge* through any conductor brought near it at any point. Also if a part of the conductor between any two points a, b is bent into a loop so as to bring the points a, b, close together, sparks pass between the two points. The difference of potential between the points is subject to such rapid variation, and large differences of potential are developed so suddenly, that discharge takes place across the air gap rather than through the conductor joining the points. If the conductor is a long thick wire carried round a large room, not only can a lateral discharge be obtained from any point on it, but the inductive effect on conductors near it is strong enough to cause short sparks to pass between them.

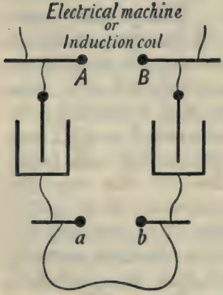


Fig. 95.

One of Prof. Lodge's experiments illustrates in a striking way the possibility of obtaining a spark between two points in a conducting circuit in which oscillatory discharge is taking place. Two Leyden jars are connected in series between the terminals of an influence machine, the two outer coatings being connected, as shown in Fig. 95, by a continuous

conductor, and a spark gap ab arranged in parallel. When the machine is worked it is found that every time a spark passes between the terminals of the machine, a spark also passes in the gap ab, even when the length of this gap is greater than that between the machine terminals. This shows that, in the case of an instantaneous difference of potential between two points, a spark between the points is not necessarily prevented by the points being in metallic connection, that is, the discharge does not necessarily follow the path of least electrical resistance. As explained later, when the current is one of very rapid alternation the opposition to its passage between any two points on a conductor is not determined by the resistance of the conductor but by its inductance and by the fact that, for very rapid alternations, the current is confined to a very thin surface layer or skin of the conductor.

64. Arc Discharge. Gassiot found that when the current through a vacuum tube was increased the character of the discharge changed from the general type of discharge with a striated positive column, and the negative electrode accompaniments to the arc discharge in which there is very little difference of appearance at the two electrodes. In this form of discharge the electrodes themselves become brightly incandescent while the gas is comparatively dark. The temperature of the electrodes is very high. Probably the highest temperatures that can be produced are obtained at the electrodes of a heavy arc discharge. With carbon electrodes the positive electrode is much hotter than the negative, and the disintegration it suffers is much more marked and gives it the characteristic crater-like form. The temperature of the electrodes being so high, the disintegration is probably due not only to the effects of heat but also to molecular and atomic processes corresponding to electrolysis. There will, therefore, be no simple relation between the amount of disintegration and the quantity of electricity passing through the arc. Between the electrodes the discharge is probably carried by the matter mechanically disintegrated, volatilised, or electrolysed from the electrodes, and by the gaseous medium surrounding the

arc, but there is no exact knowledge as to how the discharge is divided between these several elements.

The connection between the length of the arc and the potential difference producing it is usually expressed by the formula $V = a + b\,l$, where V denotes the difference of potential, l the length of the arc, and a and b constants. The value of the constants are found to vary with the material of the electrodes. Lecher gives the following data on this point. The length of the arc, l, is to be taken in centimetres and V is given in volts.

Carbon electrodes		(horizontal)	$V = 33 + 45\,l$
„	„	(vertical)	$V = 33{\cdot}5 + 57\,l$
Platinum	„	(5 mm. diam.)	$V = 28 + 41\,l$
Iron	„	(5·5 mm. diam.)	$V = 20 + 50\,l$
Silver	„	(4·9 mm. diam.)	$V = 8 + 60\,l$.

The form of this formula shows that a potential difference of a units is necessary to give an arc, no matter how short it may be, for when $l = 0$ we get $V = a$.

Fig. 96.

It is found that, in the arc, the gradient of potential is greatest near the electrodes, and is much greater near the anode than near the cathode. Fleming showed, for example, that if a spare electrode be placed in the arc between the electrodes the difference of potential between it and the anode is sufficient to give a strong current in a wire joining them, whereas the difference of potential between it and the cathode was inappreciable. In arc discharge the disintegration of the electrodes is not confined to the cathode, but takes place at both electrodes. The anode, we have seen, is at the higher temperature, and when carbon electrodes are used the loss of the anode by disintegration is nearly twice as great as that of the cathode.

The arc discharge is deflected in a magnetic field in the same way as a flexible current.

The appearance of carbon electrodes used for arc

discharge is shown in Fig. 96. The source of light in an arc light is mainly the incandescent crater of the positive electrode.

The electric furnace is practically a chamber or cell of lime, enclosing the space in which a powerful arc discharge takes place between carbon electrodes at the high temperature produced; in this space the most refractory substances can be melted.

65. Lichtenberg's Figures. The difference between discharge from a positive and from a negative elec-

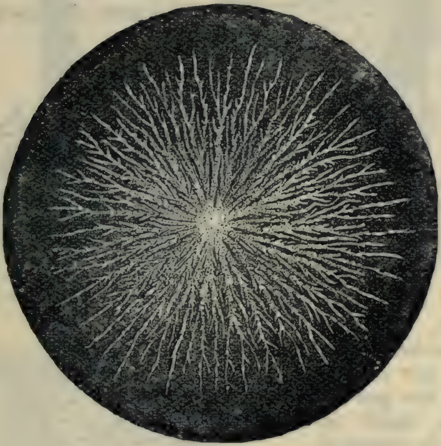


Fig. 97.

trode is illustrated by the figures obtained when the discharge from the electrode spreads over a plate of insulating material covered with a badly conducting powder. Thus, if we cover a plate with a finely-powdered mixture of red lead and sulphur and take the discharge from a positively charged electrode to the

plate, the sulphur, which by friction with the red lead is negatively electrified, is attracted to the lines of the positive discharge over the plate, and marks out a pattern similar to that shown in Fig. 97. Similarly the lines of a negative discharge over the plate are indicated by the positively-charged red lead, and a pattern of the form shown in Fig. 98 is obtained.

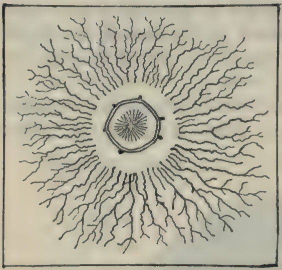


Fig. 98.

If a plate of glass or other insulating material is covered with a fine powder, such as lycopodium powder, and a discharge passed over the surface, between two pointed electrodes in contact with it, the powder arranges itself in beautifully branched moss-like figures, showing a distinct difference between the positive and negative centres. Fig. 99, due to Joly, shows this effect.

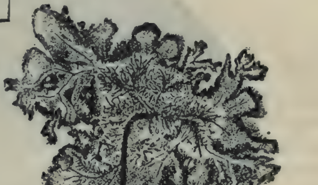


Fig. 99.

Fig. 100, also due to Joly, shows the effect of passing a discharge in the

thin layer of air between two plates of glass, close enough together to show Newton's rings, the electrodes

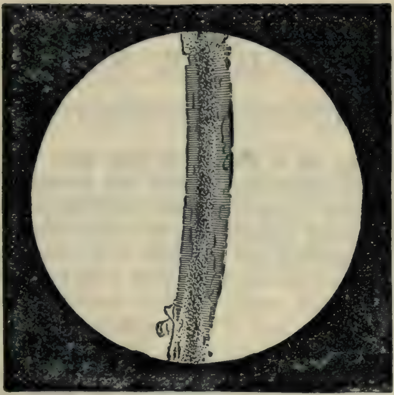


Fig. 100.

being pieces of thin platinum foil fixed between the plates. The line of discharge shows fine transverse furrows on the glass.

CHAPTER XII.

ATMOSPHERIC ELECTRICITY.

66. Potential at a Point in the Air. It has been found by direct measurement that the potential at a point in the open air is always different from the potential of the earth, and usually higher than it. The most satisfactory method of measuring this potential is by means of a *water dropper*. A metal cistern, fitted with a tap having a very fine nozzle, is filled with water, carefully insulated, and fixed in position so that the end of the nozzle is at the point in the air at which the potential is to be measured. Water is allowed to drop rapidly from the nozzle, and owing to the inductive action on the drops of water as they are detached from the nozzle, the cistern gradually becomes charged, in the same way as the surrounding air, up to the potential at the point of the nozzle. If the cistern is connected to an absolute electrometer, or to one pair of quadrants of a quadrant electrometer, the potential it attains when equilibrium is set up can be directly measured, and this gives the potential of the air at the point selected. The potential at a point in the air is, in fine weather, always positive, and increases with height above the ground. The rate of increase with height is very variable. Measurements made by Lord Kelvin in Scotland gave results varying from 20 to 40 volts per foot, but the rate of increase may be much greater or much less than this. During wet and changeable weather the potential at a point in the air may be negative, and is always very variable in value.

The electrification of the air varies, not only with the state of the weather, but, under settled conditions of weather, varies with the season and the hour of the day. The

electrification is stronger in winter than in summer, and the diurnal variation is associated with the variations of temperature, the electrification being a maximum at the times of greatest variation of temperature, and a minimum during the hours of constant temperature.

The equipotential surfaces in the air are usually planes parallel to the surface of the earth. Inequalities on the earth's surface influence, the form of the lower equipotential surfaces; but the irregularities due to this cause disappear at a comparatively low height.

The potential at any point in the air near the earth's surface may be considered as due to the electrification of the air or the electrification of the earth. The increase of positive potential with increase of height is, for example, consistent either with positive electrification of the air or negative electrification of the earth. The fact that a water dropper gives the same indication in the open as when enclosed in wire netting, or in an enclosure of perforated zinc, where it is free from external influence, supports the theory that the mass of the air is electrified.

The cause of the electrification of the atmosphere is doubtful. Experiment gives no satisfactory support to the theory that it is due to electrical separation during the evaporation of water. This theory supposes that the water vapour rising from water on the surface of the earth carries a positive charge with it, leaving the water and the earth negatively charged, but this hypothesis has not been confirmed by experiment.

67. The Aurora. The *Aurora Borealis,* visible in the polar regions, is probably an electric discharge, similar to discharge in low vacuum, in the upper regions of rarefied air. This discharge may be caused by differences of potential between the cold air near the poles and the currents of warmer air and vapour from the equator. This was the explanation given by Franklin. It may, however, be due to the differences of potential which may conceivably be set up in the upper regions of the atmosphere by the inductive action of the earth, supposed to be rotating within a less rapidly rotating shell of outer atmosphere. Spectroscopic examination shows that the light of the

Aurora is essentially similar to that of the glow in low vacuum tubes, and confirms the theory that the phenomenon is due to this form of discharge in the upper regions of the air. The lines of the spectrum correspond to those seen in the spectra of vacuum tubes, with the addition of one or two lines not yet recognised as characteristic of any known substance. The direction of positive discharge appears to be from the upper air down towards the earth.

The appearance of the Aurora is very varied and depends upon the latitude at which it is seen. It usually consists of an arch of pale light in the magnetic north. The arch usually has a characteristic quivering appearance and may consist largely of streamers of a light rose colour radiating from a polar centre. These streamers flicker and vary continually in colour and intensity, but they are normally very faint and only appear distinctly when the Aurora is specially active.

The intensity of the Aurora is closely associated with magnetic storms, but it seems to have little or no effect on the general electrical state of the lower atmosphere.

68. Thunderstorms. The phenomena of a thunderstorm are supposed to result from the intense electrification of the clouds. It is probable that an electrically charged cloud is made up of a very large number of isolated charged drops of water, and is thus charged throughout its mass and not on its surface only. It is conceivable that the condensation of water vapour in positively charged air may give rise to a cloud of positively charged drops of water. As the cloud grows and darkens these drops unite and form larger drops and in doing so the potential of each drop must evidently rise. For, if eight small drops unite to form one larger one the charge on this drop will be eight times that of one of the smaller drops, but its radius will be only twice as great, and, therefore, the potential of each drop will be four times as great and the surface density twice as great as before coalescence. The potential of a heavy cloud made up of comparatively large droplets may in this way rise to a very high value. Different clouds formed under widely different atmospheric conditions may

thus become positively or negatively charged to very high potentials, and in an assemblage of such clouds the equalisation of general potential by disruptive discharge between the clouds or between the clouds and the earth may give rise to all the phenomena of a thunderstorm. The whole question of atmospheric electricity, its origin, the relation of its phenomena, the constitution of thunder-clouds, the causes of their electrification, and other important details of the subject are at present, to a large extent, matters of speculation rather than of exact knowledge.

69. Lightning. Lightning is disruptive spark discharge on a large scale between clouds charged to widely different potentials or between a charged cloud and the earth or an earth connected object. The discharge may be simple or oscillatory in character. It is intensely bright, and is usually of the forked or branching character similar to that shown in Fig. 76. The sound accompanying the discharge, known as thunder, is due to the sudden and violent disturbance of the air along the lines of discharge, and may be short and sharp or prolonged according to the nature of this path. Sheet lightning is probably ordinary discharge at too great a distance for the thunder to be heard, or it may be due to partial brush-like discharges between adjacent clouds. Globe lightning, if it really exists as described, has not been explained. It is said to consist of balls of fire which move slowly along, and ultimately burst with a loud explosion. Since lightning is simply spark discharge of a very intense character, its effects are those of spark discharge highly intensified. All the effects described in the last chapter have been noticed. The physiological effects are generally so intense as to produce death or temporary paralysis.

Professor Lodge has shown that apparently similar lightning discharges may originate in two essentially distinct ways. In the experiment to which Fig. 95 refers, the sparks at A B between the terminals of the machine and those at the spark gap, *a b*, are similar in character, but take place under quite different conditions. The sparks at A B could be at once prevented by joining the terminals by a conducting wire, or by

partially bridging the gap by a short pointed conductor, whereas those at $a\,b$ take place notwithstanding the presence of the conductor joining the points. The difference of potential at A B gradually rises to the sparking value, then falls to zero and rises again. At $a\,b$ the difference of potential is normally zero, but when a spark passes at A B it rises to the sparking value too rapidly for the conductor connecting the points to prevent the spark. When a lightning discharge tends to take place between a cloud and the earth under the conditions which obtain at A B, it is usually possible to prevent it by a lightning conductor; but when the conditions are similar to those which obtain at $a\,b$, an ordinary lightning conductor is of no service. If, for example, a positively charged cloud, A, Fig. 101, hangs over the earth B, and if the charge on A increases, or the distance between A and B decreases, the difference of potential between A and B may rise to sparking value, and ultimately a spark may pass between the cloud and the earth under the same conditions as between the terminals A, B of the machine. A metal conductor L, well connected to earth, would prevent this discharge by inductive action, or if discharge took place between A and the top of the conductor the discharge would be carried by the conductor without the damage that would result to a tree or building of badly conducting material which might in the absence of the conductor be struck by the discharge.

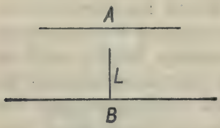


Fig. 101.

Suppose, however, that a positively charged cloud, A, Fig. 102, hangs over the earth with an uncharged cloud, a, between it and the earth, and that a long conductor, L, serves to equalise potential between a and the earth. Then, a lightning flash or spark may pass between A and a, and when it does pass the electrical equilibrium between a and B is suddenly disturbed, and

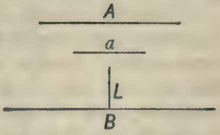


Fig. 102.

a difference of potential sufficient to cause a flash to pass between a and the earth may be suddenly set up. A flash produced in this way resembles, in the conditions of its production, that which takes place at $a\,b$ in the experiment of Fig. 95, and may strike through the air or through any non-conducting mass between a and the earth, in spite of the presence of the conductor L.

70. Lightning Conductors. The lightning conductor still in common use as a protection from lightning was suggested by Franklin over a hundred years ago. It consists of an iron or copper rod or flat strip of about one-quarter of a square inch in section. It runs from the top to the bottom of the building to be protected. At its upper end it is sharp pointed and, in order to resist the action of the atmosphere, the point-piece should be of copper, thickly gilt at the point. or of platinum. At its lower end it should be in good connection with the earth, and to secure this it should be attached to a large earth-plate sunk in the earth to a depth sufficient to be always in wet soil. All conductors and large pieces of conducting material inside and outside the building should be connected to the conductor.

Experience has shown that a conductor of this type is on the whole a satisfactory protection from lightning, although there are no doubt cases in which it is of no service. Under conditions such as those described in the preceding article with reference to Fig. 101 the action of Franklin's conductor may be such as to prevent the lightning flash altogether, or to preserve the building from damage even when it does take place. Under the inductive influence of the charged cloud a stream of charged particles is given off from the point of the conductor. A mass of particles charged with electricity of sign opposite to the charge on the cloud thus collects over the conductor, and may be supposed to neutralise the charge on the cloud directly, or to counteract its effects indirectly. If a flash is not prevented by this action it will, under normal conditions, pass between the cloud and the point of the conductor, and the discharge will be carried by the conductor without damage to the building, provided the contact with

the earth at the foot of the conductor is good. As the discharge current in a lightning rod is usually of very short duration the considerations dealt with in the preceding article are of importance and merit more consideration than they have usually received.

When a lightning flash takes place under the conditions of Fig. 102 above, the lightning rod, as usually set up, is of no service.

To secure full protection under all conditions the single rod suggested by Franklin should be replaced by a network or skeleton of rods or wire enclosing the building. All parts of the network should be in good connection with metal work, pipes, and all masses of metal inside and outside the building should be connected to it, and it should be in good connection with the earth at as many points as possible. In all cases it is of the utmost importance that the earth connection should be good, and to secure this the earth-plates should be at such a depth as to be in contact with water or wet earth all the year round.

PART II.

MAGNETISM.

CHAPTER XIII.

FUNDAMENTAL PHENOMENA.

71. Natural Magnets. The name *magnet* was applied at a very early date to pieces of a mineral found in Magnesia in Asia Minor. These specimens of what is now known as *magnetite*, or magnetic iron ore (F_3O_4), were found to possess the property of attracting pieces of iron and steel, and at a later date it was found that when properly suspended by a fine thread they took up a definite position, always pointing approximately north and south.

Magnetite is found in abundance in Spain, Sweden, and various parts of North America, but not always in the magnetic condition. If a specimen exhibiting magnetic properties be examined, it will be noticed that its power of attracting pieces of iron or steel varies for different portions of its surface, being most marked at two regions loosely called the *poles* of the magnet. These poles are generally placed at two extreme points, and the region midway between them is found to be devoid of the power of attraction exercised by the other portions of the surface. Thus, if a magnet be rolled in iron filings and then lifted out, the filings will be found adhering thickly to the poles and the neighbouring portions of the surface (Fig. 103); but they gradually thin off as the distance from the poles

increases, and midway between the poles no filings are found.

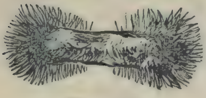


Fig. 103.

Dr. Gilbert, who first carefully observed these properties of the magnet, called the region midway between the poles the equator of the magnet, and the imaginary line joining the poles the *magnetic axis.* This latter term is now frequently used, but the former is seldom seen.

If a magnet is freely suspended, it is found that it sets itself so that the plane containing its *magnetic axis* and the point of suspension lies approximately north and south. Hence, in order that this phenomenon may be most advantageously exhibited, the magnet should be suspended so that its magnetic axis may be horizontal, that is, the point of suspension should be near the "equator" of the magnet; if the point of suspension were near either pole, the magnetic axis might be vertical, or nearly so, and the magnet would give no indication of polarity.

If two magnets be suspended at a distance from each other, so as to set themselves north and south, it will be found that if the north-pointing end of one be brought near the north-pointing end of the other, mutual repulsion will be exhibited. Similarly, if the south-pointing pole of one be brought near the south-pointing pole of another, repulsion ensues; that is, *similar poles repel one another.* If, however, the north-pointing end of one be brought near the south-pointing end of the other, mutual attraction results; that is, *dissimilar poles attract one another.* It will be noticed that these laws of magnetic attraction and repulsion are perfectly general, and similar in nature to the corresponding laws of electrostatic attraction and repulsion.

Iron and steel have been mentioned above as being attracted by magnets, and they are the only substances on which magnetic attraction is at all appreciable. Many other substances, notably nickel and cobalt, are feebly attracted, while others, such as copper and bismuth, are feebly repelled. In fact, Faraday has shown that all

substances may be divided into two classes—the *paramagnetic*, or *magnetic*, which are attracted, and the *diamagnetic*, which are very feebly repelled.

From what has been said it will be seen that a magnet possesses the following properties:—

(1) *Marked attraction for the magnetic substances iron and steel.* This is the most important case of the general law, that all substances are affected by magnetic force—*paramagnetic* substances being attracted and *diamagnetic* substances repelled. In all cases except iron and steel, and, in a less degree, nickel and cobalt, the forces of attraction and repulsion are very feeble.

(2) *Polarity.* That is, a magnet possesses poles, or regions where the power of attraction (or repulsion) is a maximum, and which, when the magnet is freely suspended, tend to point north and south. Further, these poles are not identical in properties—similar poles, that is, poles which point in the same direction, repel one another, whereas dissimilar poles attract one another.

This magnetic quality constitutes what is called *polarity*. In addition to possessing these properties, a magnet is able to communicate them to a piece of hard steel. For example, if a piece of watch-spring be rubbed with the pole of a magnet, it will be found to possess all the properties of a magnet, and will even be able to communicate them to another piece of steel.

Hence, if we wish to test whether a given piece of ore or steel is a magnet or not, we may proceed in the following way:—

(1) Dip the piece in iron filings. If it be a magnet the filings will adhere to it, and the points where they are most thickly clustered will be poles of the magnet. If it be not a magnet, the filings will not adhere to it.

(2) If the piece be found to be a magnet, suspend it by a single silk fibre so that its magnetic axis is horizontal. It will then set magnetically north and south, and the end pointing towards the north will be the north-pointing pole. This pole is sometimes marked with a file mark so as to be readily recognised.

Instead of suspending the magnet as here described, it is

often more convenient to use a piece of magnetised watch-spring as a test magnet. This magnet is readily suspended by a silk fibre, and by simply presenting either pole of the magnet under examination to, say, the north-pointing pole of the test magnet, the nature of the former can be at once determined; if it be also a north-pointing pole, the two poles will repel one another, but if it be a south-pointing pole, then mutual attraction will result. This use of a test magnet or needle to determine the poles of another magnet is very common, but the pole of the magnet under test must not be held too near the pole of the needle, for, even with similar poles, the attraction of the magnet for the steel may be greater than the repulsion between the poles, and attraction instead of repulsion will result.

Natural magnets are, more or less, curiosities. They are seldom or never seen in experimental work, artificial steel magnets being used instead

In explanation of the fact that a magnet, when freely suspended or pivoted, points north and south, Dr. Gilbert assumed that the earth itself must act as a large magnet, having its poles near its geographical north and south poles. The attraction of these poles for the dissimilar poles of the suspended magnet causes the latter to set in a north and south direction, the north-pointing end being dissimilar to the magnetic north pole of the earth by which it is attracted, and the south-pointing pole dissimilar to the magnetic south pole of the earth. Subsequent investigation has confirmed this explanation of Dr. Gilbert's, and terrestrial magnetism, or the magnetism of the earth, is now an important branch of the science of magnetism. The north-pointing pole of a magnet is usually for shortness called the *north pole*, and the south-pointing pole the *south pole*: thus the so-called north and south poles of a magnet are respectively *dissimilar* to the magnetic north and south poles of the earth.

72. Magnetic Induction. If the pole of a magnet be brought near a short piece of steel wire, the latter is at once attracted by the magnet, and may be lifted in the way shown at *a* in Fig. 104. Let another short piece of steel be now brought near the lower end of the piece

clinging to the magnet; this second piece is also attracted, and, if the magnet be strong enough, it will remain hanging to the first piece as shown at *b*. If the first piece be now gently detached from the magnet, it will be found that the

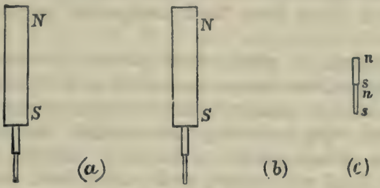


Fig. 104.

lower piece still clings to it, and, if the two pieces of steel be tested, each will be found to be a magnet with poles as shown at *c*. It will be noticed that these poles are arranged relative to one another and to the magnet N S so that dissimilar poles are in contact, and therefore exert mutual attraction.

If this experiment be modified, as indicated in Fig. 105, by arranging the magnet and the pieces of steel on a table so that they are not allowed to come into contact with one another, it will be found that the result is exactly the same as before, only that the magnetisation produced in the pieces of steel is somewhat more feeble than in the first case. Thus, in the cases illustrated by *a* and *b* of Fig. 105, if iron

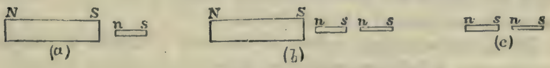


Fig. 105.

filings be sprinkled over the pieces of steel, they will adhere in tufts to the ends of the pieces, and behave in precisely the same way as when sprinkled over a small magnet obtained by the arrangement of Fig. 104. On removing the magnet N S, each piece of steel will be found to possess all the properties of a magnet.

If the same experiments be performed with pieces of soft iron instead of steel, a very important difference is noticed in the results. In the cases illustrated by *a* and *b* of Figs. 104, 105, the *same* effects are produced: the pieces of iron become magnetised, but immediately the magnet N S is removed, they lose their magnetic properties almost entirely.

Thus a piece of steel, when placed in contact with or near a magnet, becomes *permanently* magnetised, but a piece of iron under the same conditions becomes only *temporarily* magnetised. On removing the magnet the iron loses its magnetism almost entirely; the slight amount remaining is called *residual magnetism*, and varies with the quality of the iron, from a very small percentage for the best soft iron to quite an appreciable quantity for harder qualities.

These effects are due to what is called *magnetic induction*. A magnet acts inductively on magnetic bodies (steel or iron) placed near it, and the attraction exercised by magnets on magnetic bodies is a direct result of this inductive action. Thus, in Figs. 104, 105 (*a*), the south-pointing pole, S, of the magnet N S, acts inductively on the piece of steel or iron, and induces a *dissimilar* pole, *n*, in the *nearer end* of the piece, and a *similar* pole, *s*, at the *farther end*. As a result of the attraction between S and *n*, the steel or iron is attracted to the magnet, and the force of attraction may be sufficient to overcome the weight of the piece of metal. In steel the effect of induction is permanent and the induced poles persist, but in iron the effect is more or less transient, and the induced poles persist only while the metal is actually subjected to induction.

Thus we see that, when a magnet attracts a magnetic substance, induction precedes attraction, which is really the attraction between two dissimilar poles—the pole which is presented to the substance, and the unlike pole which it induces at the nearest point of that substance. This fact may be emphasised by the following experiment. Take two magnets similar in size and strength, and place them with their dissimilar poles together as shown in Fig. 106. If either end of the combined magnets be presented to a piece of iron or steel, little or no attraction is exhibited.

The inductive effect of one pole, N, is *neutralised* by the opposite inductive effect of the pole S, and the piece of magnetic substance, I, experiences no attraction. This result may also be explained by saying that the pole N of one magnet is *neutralised* by the equal and opposite pole S of the other, and thus the combined magnets have no external inductive effect at all.

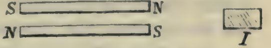


Fig. 106.

Magnetic induction, like electrostatic induction, is due to the action of the medium, but the property of the medium on which it depends is evidently different in the two cases. Thus in electrostatics we have to do with that property of the medium which determines the action of two electrically charged bodies on one another, but in magnetism we have to do with that property which determines the action of two magnetic poles on one another.

It is possible to obtain some idea of the mechanism of magnetisation by induction by considering the following experiment.

Let a thin glass tube a few inches in length be loosely filled with steel filings and closed at both ends. If the tube be now laid on a table, and the pole of a magnet drawn over it from end to end, it will be noticed that the superficial layer of filings immediately under the pole experiences magnetic induction, and under its influence the filings arrange themselves regularly with their longer axes parallel to the length of the tube. Thus, before being subjected to the action of the magnet, the filings are arranged irregularly with their axes in any direction, as shown diagrammatically in Fig. 107 (a).

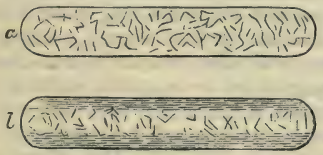


Fig. 107.

Under the influence of the magnet, however, each individual filing becomes magnetised, and, as a result of the mutual attraction and repulsion between their poles and

the pole of the magnet, they arrange themselves as shown in Fig. 107 (*b*).

The depth of the layer of filings affected will vary with the strength of the magnet, but even with a very strong magnet the disturbed layer is comparatively shallow.

If the magnet be drawn along the tube several times, always in the same direction, and so as to bring every portion of the surface of the enclosed filings under the influence of the pole, it will be found on testing that the tube of filings possesses all the properties of a feeble magnet. If, however, the tube be shaken so as to disarrange the particles, it loses its magnetic properties even although the individual steel filings retain their magnetism.

If the molecules in a bar of steel be supposed to act in the same way as the individual filings in the tube, we have a partial explanation of the fact, stated above, that a piece of steel may be magnetised by the inductive action of a magnet, and we also see why a bar of steel may be magnetised by rubbing it with a magnet. From this it appears that magnetisation may be associated with molecular arrangement, and that the magnetic properties of the body magnetised may be due to this arrangement; but the magnetisation of individual molecules is a question on which we cannot here enter. From what has been said it is clear that the difference between steel and iron as magnetic bodies may be explained on the assumption that in steel the molecular arrangement accompanying magnetisation may be permanent, while in iron it persists only when constrained by the inducing magnet, and when the latter is removed most of the molecules take up their normal positions.

Another point of difference between iron and steel is that, while soft iron is easily magnetised by magnetic induction, steel, and especially hard steel, is influenced with difficulty. For example, if a light magnet be suspended by a fibre, and a piece of hard steel be held near one pole, the attraction resulting from the inductive action is not very strong; but if a piece of soft iron be substituted for the steel, the attraction is much more marked. This effect is very conveniently shown by arranging a piece of soft iron and a piece of hard steel on each side of the pole of the magnet

as shown in Fig. 108. If the iron and steel be placed at equal distances from the pole, the attraction for the former will be the greater, and the magnet will be deflected towards it. By removing the iron to a greater distance from the pole, the two attractions may be balanced against one another, and the needle will remain in its original position. This shows that the inductive attraction for the steel is less than for the iron, that is, soft iron is more susceptible to magnetic induction than hard steel. The difference is, however, practically the same as the one already mentioned, and both are included in the statement that in soft iron the molecular arrangement constituting magnetisation is easily set up, and as easily lost when the constraining force is removed.

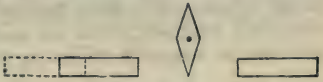


Fig. 108.

73. Artificial Magnets. In the preceding articles we have noticed that when steel is subjected to the inductive action of a magnet it becomes permanently magnetised.

Advantage may be taken of this fact to convert a bar of hard steel into a magnet. Thus, if a strip of steel be rubbed from end to end, always in the same direction, with the pole of a magnet, it acquires magnetic properties identical with those of the magnet. For example, if A B (Fig. 109) represent the steel strip, and if it be rubbed always in the direction A B with the north pole of a magnet, it will also become a magnet having a north pole at the end A and a south pole at the end B. This method of magnetisation, where the piece of steel to be magnetised is rubbed from end to end with one pole of a magnet, is called magnetisation by *single touch*. The magnetising magnet must be drawn over the steel always in the same direction, the rubbing end following the path indicated by the dotted line and arrows of Fig. 109. If the piece of steel be thick, both sides should be rubbed, first one side, then

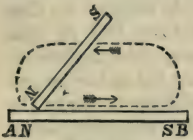


Fig. 109.

the other. A similar but more effectual method of magnetisation is by the method of *divided touch* indicated in Fig. 110. The steel bar to be magnetised is laid with its extremities on the dissimilar poles of two magnets arranged as shown in the figure. It is then rubbed by the dissimilar poles of two other magnets, from the middle outwards, the rubbing ends of the magnets following the paths indicated by the

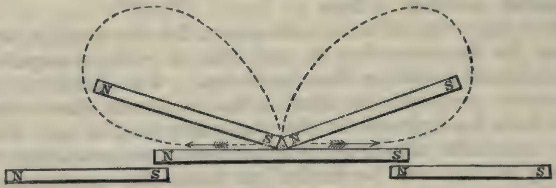


Fig. 110.

dotted lines and arrows. A modification of this method is known as magnetisation by *double touch*. A piece of wood or cork is placed between the rubbing ends of the magnets, and these ends are then moved together from the middle to either end, back to the other end, and so on backwards and forwards, finishing at the middle.

It will be noticed from the figures that in all the methods each pole of the magnet produced is unlike that of the magnetising magnet with which it was last touched. This result is evidently due to the effect of magnetic induction on the molecules of the steel; for example, in Fig. 109, as the magnet is drawn over the bar, a result similar to that illustrated by Fig. 107 in the case of a tube full of iron filings is produced; the molecules arrange themselves in regular order with their magnetic axes parallel to the length of the bar, and with their south poles pointing towards the north pole of the magnetising magnet which has just passed over them. The final result of this is, that most of the molecules in the surface layer of the steel strip are arranged with their south poles pointing towards B, and their north poles pointing towards A. This may be represented diagrammatically by Fig. 111.

It will be noticed that the molecular magnets are arranged with dissimilar poles in contact, and, if these poles are equal, there will be mutual neutralisation, except at the ends A and B, where the free north poles of the end molecules constitute a north pole, N, at A, and the free south poles at B constitute the south pole S. With this arrangement there would be exact neutralisation at all points in the length of A B, and the magnet would exhibit magnetic properties only at the ends; for example, if dipped into iron filings these would adhere only at the ends. This, however, is never the case with an actual magnet—the filings adhere most thickly at, or near, the ends, but they also adhere to the sides at all points except near the middle of the magnet, the arrangement of the filings indicating that the attraction diminishes from the poles to the equator. This is due to the fact that in a magnetised rod or bar the molecules cannot take up a strictly linear arrangement, such as is indicated in Fig. 111. For reasons that will be appreciated later the arrangement is more of the nature shown in Fig. 112. The lines of molecules are slightly curved away from the axis of the bar and the ends of the lines, which are unneutralised poles, appear not only at the ends but at the sides of the bar. The distribution of free ends over the surface at each end of the bar explains why the properties of the magnet included under polarity are exhibited over a more or less extended region at each end of the bar. If the bar is long and thin the lines of molecules are very slightly curved, and the free pole regions extend only a very short distance from the ends. With short thick bars the curving of the lines is more marked, and free poles are distributed in gradually

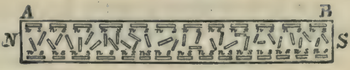


Fig. 111.

Fig. 112.

increasing numbers from the middle of the bar to each of its ends, the north poles being found along one half and the south poles along the other half.

It is thus evident that the amount of *free* or unneutralised magnetism at any section of the magnet varies with the position of the section considered, being greater the nearer the section is to either end of the magnet. In Fig. 113, let N S represent a magnet, and at a number of points taken at equal distances along its length let perpendiculars be erected, each proportional to the amount of free magnetism per unit length at the section corresponding to its position on the magnet. If the perpendiculars are drawn to the left of the magnet for north magnetism, and to the right of it for south magnetism, then the curve *n o s* drawn through their extremities may be said to indicate the *distribution* of free magnetism on the magnet. Further the *areas N o n* and *S o s* indicate the *amounts* of free magnetism distributed on the corresponding halves of the magnet.

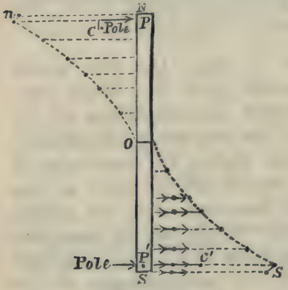


Fig. 113.

The *poles* of a magnet are sometimes defined as the points at which the free north and south magnetism of the magnet may be supposed to be concentrated, so as to produce the same external magnetic field, but such points cannot, in general, be found. In Fig. 113, if C and C′ denote the centroids of the figures *N o n* and *S o s*, then the poles of the magnet may for some purposes be assumed to be at P and P′ the feet of the perpendiculars from C to the axis of the magnet. In the case of thin bar magnets, these points are very near the ends of the magnet, so that for general purposes the poles of a long bar magnet are assumed to be at the ends of the magnet, though in reality they are so placed that the actual length of the magnet is slightly greater than the distance between its poles.

The molecular theory of magnetisation described above is further supported by the fact that if a steel magnet be broken in two, each piece is a complete magnet. For example, if the magnet AB (Fig. 114) be broken into the two pieces AC, C′B, then each piece is a magnet with poles as indicated. That is evidently just what we should expect. At the end, C, of the piece AC all the south poles of the end molecules are freed from the neutralisation of the north poles of the end molecules at C′ of the piece BC′. Hence C becomes a free south pole, and C′ a free north pole. Similarly, if a magnet be broken into any number of pieces, each piece is found to be a complete magnet.

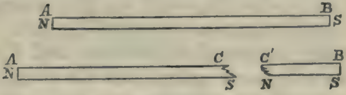


Fig. 114.

When a magnet is irregularly magnetised, it may exhibit what are known as *consequent poles*, that is to say, poles lying between the terminal ones of the magnet. Fig. 115 shows a magnet with consequent poles at n and s. The magnet practically consists of three smaller magnets joined

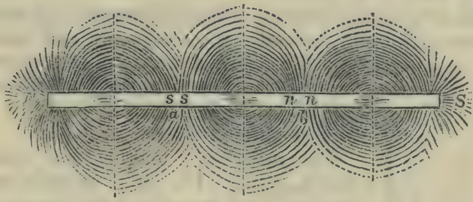


Fig. 115.

end to end, so that similar poles are united at n and s. Magnets with consequent poles may be produced either by accidental irregularity in magnetisation, or purposely. For example, by touching a bar of steel at several points with the poles of a strong magnet, consequent poles of polarity opposite to that of the magnet pole are produced at the points touched.

To show that the magnetisation may affect only the surface layer of molecules, let a piece of steel be magnetised by lightly stroking with another magnet, and then placed for a short time in strong nitric acid. The acid eats away the outer layers of molecules, and the steel will be found to have to some extent lost its magnetic properties.

Artificial magnets are made in various forms, the bar magnet and horse-shoe magnet being the commonest. Bar

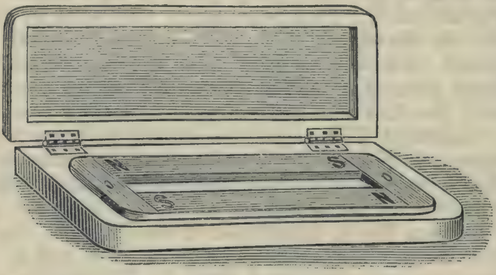


Fig. 116.

magnets are usually in pairs; Fig. 116 shows a pair arranged in a box with their dissimilar poles adjacent and united by loose pieces of soft iron called *keepers*. These keepers serve to maintain the magnetic state of the magnets; their function will be more fully explained later.

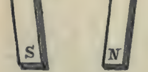


Fig. 117.

The *horse-shoe* magnet has the form shown in Fig. 117. It is practically a bar magnet, bent so as to bring the two poles near to one another; thus constructed, it is more convenient for attracting pieces of iron, for the attraction of both poles can be utilised simultaneously. When not in use a keeper of soft iron is placed across its poles.

Since the surface layers of the steel are generally the more highly magnetised, there is evidently very little advantage in using steel bars of great thickness; and, with

a given mass of steel, more powerful magnets can be made by making several thin bar magnets of the required length, and uniting them as a *compound magnet*, than in magnetising the steel as one thick bar of the same length. Fig. 118 shows the simple forms of a compound bar magnet, and and a compound horse-shoe magnet; all the north poles of the component magnets are at N, and constitute the north pole of the compound magnet. Similarly, all the south poles of the component magnets at S constitute the south pole of the compound magnet.

Fig. 118.

A useful piece of apparatus, called the magnetic needle, consists of a strip of carefully magnetised hard steel pivoted at its centre on a hard steel point. The strip is usually of the shape shown in Fig. 119, and at its centre is inserted a conical cap of agate or glass, which serves to pivot it on the steel point, and allows it to turn freely with little or no friction. When properly balanced and pivoted, the needle sets itself north and south, that is, it sets in what is called the *magnetic meridian* of the place where it is suspended. The *compasses* sold by opticians are simple forms of this

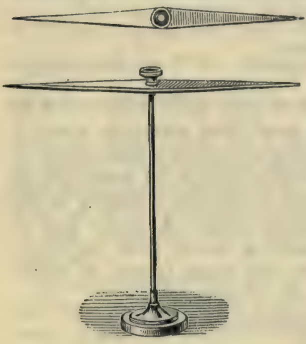


Fig. 119.

instrument, the needle being balanced over a card indicating the points of the compass. The *mariner's compass* is a more elaborate instrument, in which the compass card is attached to the needle, so that the north point on the card always indicates the magnetic north.

The properties of a magnet or of a magnetic body are greatly influenced by heat. Thus, if a strip of steel be magnetised and slowly raised to a red heat, it gradually loses it magnetic properties, and at a bright red heat it entirely loses them, and does not recover them on cooling. Similarly, if a piece of iron be heated to a bright red heat, it is not attracted by a magnet. These facts support the molecular theory of magnetisation, and are evidently due to the molecular changes produced by the influence of heat.

74. Laws of Magnetic Attraction and Repulsion. The general law that like poles repel, and unlike poles attract, has already been given, and we have now to consider the quantitative expression of this law. This is exactly similar to the electrostatic law given in Art. 6. Thus, if two magnetic north poles of strengths m and m' are separated in air by a distance d, the force of repulsion exerted betweeen them is given by

$$f = \frac{m\,m'}{d^2}.$$

This involves the definition of unit pole, which may be given thus: A magnetic pole is of unit strength when it repels another similar equal pole placed at unit distance from it in air with unit force. If the distance be measured in centimetres and the force in dynes, then the strength of the pole is given in C.G.S. magnetic units. A north pole is usually considered positive, and a south pole negative; hence when $m\,m' / d^2$ is positive, the force is one of repulsion, and when negative it is one of attraction.

The law is more generally expressed by writing

$$f \propto \frac{m\,m'}{d^2}, \text{ or } f = \frac{1}{\mu} \cdot \frac{m\,m'}{d^2},$$

where μ is a constant of a definite value for any given

medium, but differing for different media. The value of μ for air is assumed to be unity, and the definition of unit pole given above is based on this assumption, for, if $\mu = 1$ and $m = m'$ we get, for air, $f = m^2/d^2$ and therefore for unit values of f and d, m is unity.

75. The Magnetic Field. The term *magnetic field* in magnetism is analogous to *electric field* in electrostatics. The magnetic field due to any magnet is the whole space in which magnetic force due to the magnet is appreciable. Similarly, the magnetic field due to any system of magnets is the whole space in which magnetic force due to the system is appreciable.

The magnetic force at any point in a magnetic field, or the *intensity of the field* at any point, is determined in magnitude and direction by the resultant force which a unit *north* pole would experience if placed at that point. Thus, in Fig. 120, if N S represent a magnet with poles of strength m and $-m$, the intensity of the field at a point P may be determined thus. The force at P due to the pole N is given by $\frac{m \times 1}{\mathrm{N\,P}^2}$ and acts in the direction N P; let it be represented by P R. Similarly, the force at P due to the pole S is given by $\frac{-m \times 1}{\mathrm{S\,P}^2}$ and acts in the direction P S; let it be represented by P Q. Then, by the parallelogram of forces, the *resultant* force at P due to the magnet N S is represented by P T. The magnitude and direction of this resultant force determines the intensity of the magnetic field due to the magnet N S at the point P.

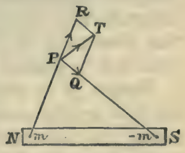


Fig. 120.

By conventions similar to those explained in Art. 7 a magnetic field may be mapped out by lines of magnetic force, a line of magnetic force being, in general, a curve such that the direction of the tangent at any point gives the direction of the resultant magnetic force at that point. The magnitude of the force at any point may also be measured by the number of lines of force per unit area

at the given point in a plane at right angles to the direction of the line of force passing through that point.

In Fig. 120, P T is the direction in which a unit *north* pole would be urged; hence a unit *south* pole would be urged with an equal force in the opposite direction. Similarly, a *north* pole of strength m would be urged in the *direction* P T with a force of magnitude equal to m *times* that represented by P T, and a *south* pole of strength $-m$ would be urged with equal force in an opposite direction.

Hence, if a very short magnet be placed at P, its north pole will be urged along P T, and its south pole will experience a nearly equal force in the opposite direction. That is, the force of translation, tending to move the magnet from the position P, will be very small, but the magnet will set itself in the line P T, that is, parallel to the direction of the resultant magnetic force at the point where it is placed.

Hence, if a *small* magnet be placed at different points in the neighbourhood of a large magnet, it will set itself along the resultant direction of the magnetic force at each point, that is, its length will be a tangent to the line of

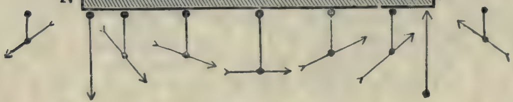


Fig. 121.

force passing through the point where it is placed. This experiment may readily be performed by magnetising a very short strip of steel watch-spring, and suspending it at different points in the neighbourhood of a large magnet. Fig. 121 shows roughly the positions for different points in

the plane of the paper. This experiment may, however, be modified so as to give much more satisfactory results. Let a sheet of thin glass or thin pasteboard be placed over a magnet, and let iron filings be dusted over it. It will be

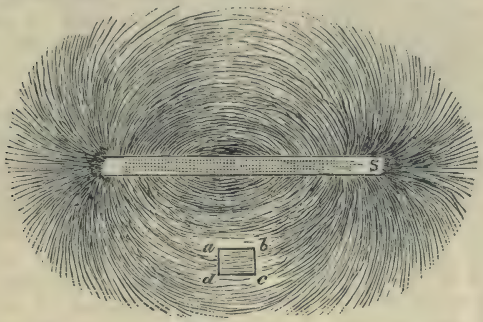


Fig. 122.

noticed that as the filings fall on the card, they at once come under the inductive action of the magnet, and each, becoming a magnet, sets itself parallel to the direction of the resultant force at the point where it is placed. On

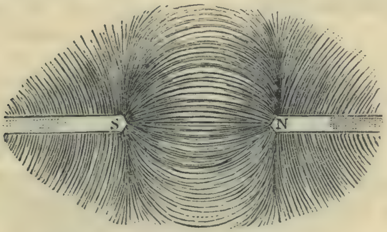


Fig. 123.

gently tapping the card, the filings thus arrange themselves in continuous curves, and roughly map out the lines of force in the plane of the card.

Fig. 122 shows the general arrangement of the filings

over an ordinary bar magnet, and the student may readily verify the fact that the direction of the magnetic force at

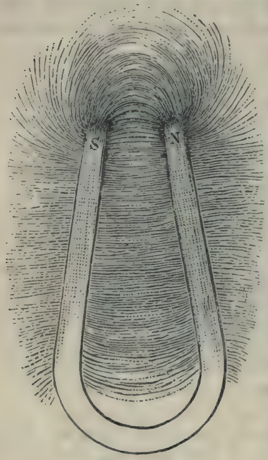


Fig. 124.

any point is tangential to the curve passing through that point. Since the intensity of the magnetic field is defined with reference to a unit *north* pole, the lines of force are supposed to run from N to S, that is, in the direction in which a unit north pole would travel.

Fig. 123 shows the general direction of the lines of force between two dissimilar poles of two bar magnets, and Fig. 124, for a horse-shoe magnet, illustrates the same thing.

The distribution of the lines of force between two similar poles is shown by Fig. 125, and the student will find it instructive to draw the force diagram (cp. Fig. 120) for different points in this field, so as to verify the general directions of the lines.

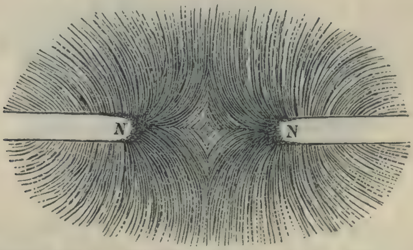


Fig. 125.

It must be remembered that the above figures only show the lines of force *in the plane of the card*, that is,

they represent longitudinal *sections* of the field. All longitudinal sections are, however, exactly similar, so that the figures completely represent the field if we remember that the lines of force surround the magnet on all sides, and that the above figures represent their disposition in *any* longitudinal section. It must further be remembered that lines of force have no real existence—they are merely graphical representations of the direction of the magnetic force at any point in a field of force, and the experiments with the filings are merely magnetic devices for indicating to the eye the general direction of these lines, just as a chain hanging vertically indicates the direction of the force of gravitation.

Magnetic induction may be said to act indifferently through all substances except magnetic substances; that is, for most substances, the value of μ (Art. 74) is much the same, but for magnetic substances it has a higher value. Thus, if a piece of soft iron be placed between two unlike magnetic poles, nearly all the lines of force pass through the iron. This is readily shown by the help of iron filings. If a short piece of soft iron, of about the same breadth and thickness as the magnets, is placed between the unlike poles of two bar magnets, as shown in Fig. 126, the distribution of the lines of force, obtained in the usual way by means of filings, shows that the magnetic induction takes place chiefly through the piece of iron, the lines of force being as it were gathered up by it, so that the intensity of the magnetic field external to the iron is greatly reduced.*

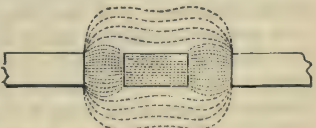


Fig. 126.

* The filings in the space over the piece of iron will not arrange themselves in lines of force; in fact, on tapping the card the space will probably be free from filings. This is because nearly all the lines of force are in the *iron*, and therefore there is no field where the filings lie. These results are more correctly explained by saying that the magnetic field which obtains is the *resultant* field due to the two inducing poles *and the two induced poles* of the piece of iron.

If a piece of wood, brass, glass, etc., be placed between the poles, the normal distribution of Fig. 123 is not sensibly disturbed. If a soft iron *ring* be placed between the poles, then the distribution shown in Fig. 127 is obtained, and it will be noticed that the space within the ring is, as it were, screened from the magnetic induction of the poles. Most of the lines of force pass through the iron of the ring, and in the space within there is no magnetic field, so that if a small magnet were placed there it would set itself indifferently in any position. Hence, if a magnet is suspended inside an iron tube or an iron enclosure of any kind, it is effectively screened from the influence of any magnetic field in which it is placed.

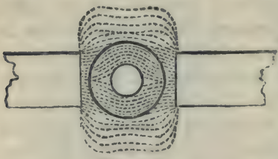


Fig. 127.

This property of soft iron explains why *keepers* of soft iron are placed on magnets to assist in maintaining their magnetic condition. The end molecules of the magnets have their interior extremities fixed by the attraction of adjacent molecules, but their free extremities experience no fixed attraction, and for this reason their equilibrium is not very stable and is easily disturbed. Hence, if a magnet be handled roughly or knocked about in any way, it gradually loses its magnetic properties. When, however, a keeper is placed on a pair of magnets as in Fig. 116, or across the poles of a horse-shoe magnet, the inductive action of each of the poles is concentrated on the other, and the force of attraction between the poles is greatly increased. The free ends of the molecules are fixed by this mutual attraction, and the molecular arrangement of the magnet is not now so easily disturbed. For the same reason, a sheet of iron placed between a magnet and a magnetic needle shields the latter from the inductive action of the magnet—the lines of force take the course indicated diagrammatically in Fig. 128, and do not pass across the iron.

Steel acts in much the same way as iron, but not so

effectively; the lines of force pass much more readily through soft iron than through steel, and the softer the iron the greater is its *permeability*.

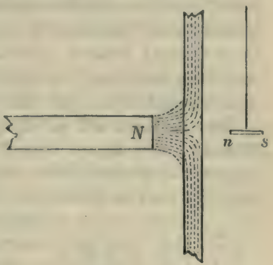


Fig. 128.

The effect produced on the external field of a horse-shoe magnet by placing a keeper across its poles is readily seen by testing the distribution of the lines of force when the keeper is first off and then on. Fig. 124 shows the first case, and Fig. 129, giving the second case, shows that when the keeper is on there is, as we should expect, little external magnetic field. The same thing may be illustrated by holding the poles of a horse-shoe magnet near a suspended test needle. The needle is at first deflected from its normal position, but on putting the keeper across the poles it at once returns to that position.

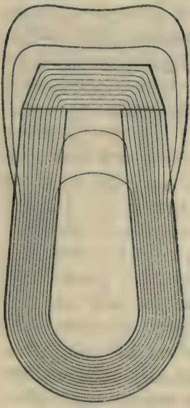


Fig. 129.

From a consideration of Figs. 122-125 it will be seen that in general the intensity of a magnetic field varies, both in magnitude and direction, at different points in the field; that is, the directions of the lines of force and the number crossing unit area are different at different points. If, however, we consider a portion of the field whose dimensions are small compared with its distance from the poles of the magnetic system to which the field is due, the intensity of the field may be considered to be uniform throughout this small area. For example, in the magnetic field due to the magnet N S (Fig. 122), the field in the space *a b c d* may be considered *uniform* in *direction* and *magnitude*, that is, the lines of

force are practically *parallel equidistant straight lines.* For this reason the magnetic field due to the earth at any point on the earth's surface is said to be uniform; the intensity of the earth's magnetic field is of course different at different places, but at any one place it is practically uniform, and may be represented by parallel straight lines equidistant from one another.

76. Action of a Magnet in a Uniform Magnetic Field. We have already learnt in Art. 75 that, when a magnet is placed in a magnetic field, it tends to set itself parallel to the direction of the lines of force at the point where it is placed. Hence, when a magnet is freely suspended in a uniform magnetic field, it at once sets itself with its magnetic axis parallel to the lines of force. Thus, let N S (Fig. 130*) represent a magnet freely suspended in a uniform magnetic field, and let the strength of its poles be denoted by m. Then if I denote the intensity of the field, the north pole N will experience a force $m\text{I}$ in the direction indicated in the figure, and the south pole S will experience an equal force in the opposite direction. Hence for any position in which the magnet is not parallel to the lines of force, it experiences a couple urging it into that position The *moment* of this couple is equal to $m\text{I} \times \text{S T}$, and its magnitude evidently depends upon the position of the magnet—it has a minimum value equal to zero when the magnet is parallel to the lines of force, and a maximum value equal to $m\text{I} \times \text{S N}$ when the magnet is at right angles to the lines of force. For any intermediate position, where the axis of the magnet makes an angle α with the direction of the lines of force, the moment experienced by the magnet is $m\text{I} \times \text{S T}$. But if S N, the length of the magnet, be denoted by l, we have $\text{S T} = l \sin \alpha$.

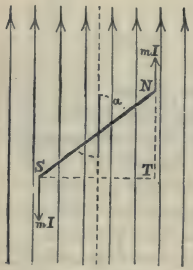


Fig. 130.

* The direction of the lines of force indicated by the arrow-heads is the direction in which a *north pole* would be urged.

The moment of the couple tending to place the magnet parallel to the lines of force is thus given by $m\,l\,\mathrm{I}\sin\alpha$. In this expression m and l are two constants of the magnet, and their product $m\,l$ is known as the *magnetic moment* of the magnet.

This important constant is usually denoted by M, and may be defined briefly as the product of the strength of either pole of the magnet into the distance between the poles, or more satisfactorily as the maximum moment which the magnet can experience in a field of unit intensity. This latter definition is applicable whatever be the form of the magnet or the distribution of its magnetism. It will be learnt later that *the magnetic moment per unit volume* determines *the intensity of magnetisation* of the material.

It is evident from what has been said, that, if a freely suspended magnet be deflected from its position of equilibrium in a magnetic field, it at once experiences a couple urging it back into this position, and as a result it oscillates backwards and forwards, just as a pendulum oscillates when disturbed from its position of rest. This oscillation of the magnet is approximately isochronous for very small vibrations, and it can be shown that, for a given magnet, the time of a single oscillation depends upon the magnetic moment of the magnet, and the intensity of the field in which it oscillates. The moment of the couple resulting from a deflection of the magnet through an angle α is given by $\mathrm{M\,I}\sin\alpha$. If α is small this may be written $\mathrm{M\,I}\,\alpha$, that is, the moment of the couple resulting from the displacement is proportional to the displacement, and the motion of the magnet is simple harmonic motion. Since the moment of the couple for a small displacement α is approximately $\mathrm{M\,I}\,\alpha$ the time of oscillation is given by $t = 2\pi\sqrt{\mathrm{K}/\mathrm{M\,I}}$, where K denotes the moment of inertia of the magnet round the axis of vibration. This shows that the time of oscillation is inversely proportional to the square root of the magnetic moment M, and also to the square root of the field intensity I. Since the number of vibrations executed in a given time is also inversely proportional to the time of one vibration, it follows that the *square* of the number of oscillations, which

the magnet performs in a given time in any magnetic field, is directly proportional to the strength of the field, and to the magnetic moment of the magnet. Hence, if the *same* magnet be caused to oscillate in two different magnetic fields, it is evident that the intensities of the fields will be directly proportional to the square of the number of oscillations performed by the magnet in each field, in a given time. For example, if in one field the magnet make 10 oscillations per minute, and in the other 12 oscillations per minute, then the intensity of the first field is to that of the second as $10^2 : 12^2$, that is, as $100 : 144$ or as $1 : 1{\cdot}44$.

77. Action of one Magnet on Another. In Fig. 131 let N S and *n s* represent two magnets in the same horizontal plane. N S is supposed tc be fixed, and *n s* to be pivoted at O so as to move freely in a horizontal plane. Let m and μ denote the respective strengths of the poles of the magnets. Then, considering the action of the poles of N S on the pole *s* of *n s*, we have that the pole N attracts it with a force represented by *s a* and equal to $\frac{m\,\mu}{(\mathrm{N}\,s)^2}$, and the pole S repels it with a force represented by *s b* and equal to $\frac{m\,\mu}{(\mathrm{S}\,s)^2}$. The resultant action of the magnet N S on the pole *s* is therefore represented by *s c*; and by an exactly similar construction, the resultant action on the pole *n* may be represented by *nf*. Under the action of these forces, the magnet *n s* will take up a position of equilibrium in which the moments of the forces represented by *s c* and *nf* round O are equal, that is, $sc \times \mathrm{O}p = nf \times \mathrm{O}q$.

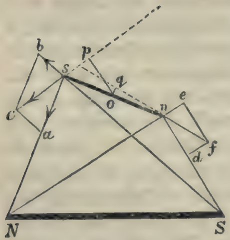


Fig. 131.

If the magnet *n s* is not pivoted at O, but perfectly free to move in the plane of the paper, then it is evident that the action of the forces *s c* and *nf* must be to draw it up to N S, that is, the magnet experiences a force causing motion of

translation, and not a mere *directive force* setting it in a particular position. When pivoted at O, the resistance of the pivot prevents the translatory motion, but allows the magnet to set in the position of equilibrium indicated above. When a magnet with poles of equal strength is placed in a uniform field, it can experience no force causing motion of translation, but only a *directive couple* setting it parallel to the lines of force, for, the field being uniform, the forces acting on the poles are equal and opposite and parallel to each other. Thus, if a magnetised sewing needle be carefully placed on the surface of water, it readily sets itself in the magnetic meridian, but does not move over the surface of the water.

CHAPTER XIV.

FUNDAMENTAL THEORY.

78. Intensity of the Magnetic Field at any Point on the Axial Line of a Bar Magnet. The axial line for any bar magnet is the prolongation of the axis of the magnet

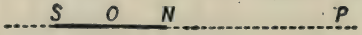


Fig. 132.

in either direction. Let N S, Fig. 132, represent the magnet; then, to determine F, the intensity of the field at any point, P, on the axial line, we have,

$$F = \frac{m}{NP^2} - \frac{m}{SP^2},$$

where m is the strength of the magnet poles. If O be the middle point of N S, and O P be denoted by d, and O N by l, then we have,

$$F = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2},$$

or

$$F = \frac{4\,m\,d\,l}{(d^2-l^2)^2},$$

and as M, the magnetic moment of the magnet, is equal to $2\,m\,l$ this may be written

$$F = M\frac{2\,d}{(d^2-l^2)^2}.$$

If l is very small compared with d this reduces to

$$F = \frac{2\,M}{d^3}.$$

The direction of F is that of O P produced, that is, along the axial line.

79. Intensity of the Magnetic Field at any Point on an Equatorial Line of a Bar Magnet.

An equatorial line is a line in the field at right angles to the magnetic axis of the magnet through the middle point of the axis.

Let N S, Fig. 133, represent the magnet. Then, to determine F, the intensity of the field at any point, P, on the equatorial line, we have, with the same notation as above,

$$F = \frac{2m}{l^2+d^2} \cdot \cos a$$

or

$$F = \frac{2m}{d^2+l^2} \cdot \frac{l}{\sqrt{d^2+l^2}}$$

that is

$$F = \frac{2ml}{(d^2+l^2)^{\frac{3}{2}}} = \frac{M}{(d^2+l^2)^{\frac{3}{2}}}.$$

If l is very small compared with d this reduces to

$$F = \frac{M}{d^3}.$$

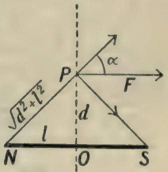


Fig. 133.

The direction of F is at right angles to the equatorial line at P or along the parallel through P to the axial line.

80. Magnetic Potential. The idea of potential in the magnetic field corresponds exactly to that of potential in electrostatics. The law for the action between two poles or two point charges of magnetism is the same as that for two point charges of electricity. Hence the results established in Art. 24 for electrostatics apply in the magnetic field. The magnetic force at any point at a distance d, in air, from a north (positive) pole of strength m is m/d^2 and the work done in bringing a unit north pole from an infinite distance up to that point is m/d, that is, the magnetic potential at the point, due to the pole is m/d. Similarly, as in Art. 24, the difference of potential between any two points at distances d and d' from the pole is given by

$$\frac{m}{d} - \frac{m}{d'}.$$

The potential at any point due to a very small magnet can readily be deduced from this.

Let N S (Fig. 134) represent the short magnet. If the strength of its poles be m and $-m$, then the potential at P, due to the north and south poles N and S, is $\frac{m}{\mathrm{N\,P}} - \frac{m}{\mathrm{S\,P}}$. Since N S is supposed to be very small compared with the distances N P and S P, this may be written as $\frac{m}{n\,\mathrm{P}} - \frac{m}{s\,\mathrm{P}}$. That is, if O be the middle point of N S, we have the potential at P given by—

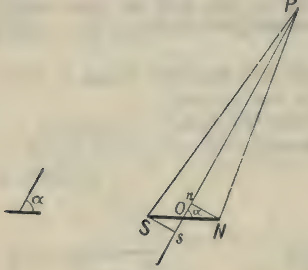


Fig. 134.

$$\frac{m}{(\mathrm{OP} - \mathrm{O}\,n)} - \frac{m}{(\mathrm{O\,P} + \mathrm{O}\,n)} \text{ or } \frac{2\,m\,.\,\mathrm{O}\,n}{\mathrm{O\,P}^2 - \mathrm{O}\,n^2},$$

and, as $\mathrm{O}\,n^2$ is negligibly small compared with $\mathrm{O\,P}^2$, this result reduces approximately to $\frac{2\,m\,.\,\mathrm{O}\,n}{\mathrm{O\,P}^2}$ or $\frac{m\,.\,n\,s}{\mathrm{O\,P}^2}$. But if the angle between O P and O N be denoted by a, $n\,s = \mathrm{N\,S}\cos a$, and we get $\frac{m\,.\,n\,s}{\mathrm{O\,P}^2} = \frac{m\,.\,\mathrm{N\,S}\cos a}{\mathrm{O\,P}^2}$. The quantity $m\,.\,\mathrm{N\,S}$ is evidently the magnetic moment of the magnet, and if this be denoted by M, we get the result that the potential at P, due to the small magnet N S, is given by $\mathrm{M}\cos a/\mathrm{O\,P}^2$, or if O P by denoted by d, by $\mathrm{M}\cos a/d^2$. It should be noted that a is the angle between O P, and the *positive* direction of the magnetic axis of N S.

The quantity $\mathrm{M}\cos a/d^2$ is here shown to be the potential at P, due to a very small bar magnet N S. It is also the potential energy of the small magnet in the magnetic field at O due to a unit north pole at P, for it expresses both the work done in bringing a unit north pole from infinity up to P in the field of the magnet, and the work done in

bringing the magnet from infinity up to the position N S at O, in the field due to a unit north pole at P. That is $M \cos \alpha/d^2$ expresses the potential energy of either member of a system made up of a unit north pole at P and a very small bar magnet in the position N S at O. The field at O due to a unit north pole at P is $1/d^2$ in the direction O s, and the magnet being very small we may assume that it is in a uniform field of this intensity. The potential energy of the magnet in this field is $M \cos \alpha \,.\, 1/d^2$, where α is the *supplement* of the angle between the axis of the magnet and the direction of the field. That is, the potential energy of a magnet of moment M in a field of strength H, whose direction makes an angle β with the axis of the magnet, is $-M H \cos \beta$. When $\beta = 0$ this potential energy has its minimum value $-M H$, when $\beta = 180$ we get the maximum value M H, and when $\beta = 90$ the energy is zero. It is evident from these results that the work done in rotating a magnet through 90° or 180° from its position of rest in a uniform field is M H or 2 M H. This result is more simply obtained by determining directly the work done on each pole during the rotation, or the work done against the couple opposing the rotation. As the opposing couple is a variable one, while the force on each pole is constant, the determination of the work done on the poles is the simpler one. For a rotation of 180° each pole, of strength m, is displaced through a distance $2\,b$ in a field of strength H. The work done is therefore given by $2 m H \,.\, 2 l$ or 2 M H.

81. Resolution of Magnetic Moment. The moment of a magnet may be resolved in accordance with the rules that apply to the resolution of a force into components. Just as a force F may be resolved at any point into components F_1, F_2, F_3 . . . making angles α_1, α_2, α_3 . . . with its direction, so the moment of a magnet, assumed to have the direction of the axis of the magnet, may be resolved, at the mid-point of the axis, into components M_1, M_2, M_3 . . . associated with axes making angles α_1, α_2, α_3 . . . with the axis of M. The truth of this may be proved as follows. If we resolve F and F_1, F_2, F_3 . . . along a fixed direction with which their directions make angles

$\beta, \beta_1, \beta_2, \beta_3 \ldots$, then, by the principles of mechanics, we have

$$F \cos \beta = F_1 \cos \beta_1 + F_2 \cos \beta_2 + F_3 \cos B_3 + \ldots$$

or

$$\mathbf{F} \cos \boldsymbol{\beta} = \Sigma F \cos \beta.$$

Similarly, if the law of resolution be assumed to apply to magnetic moments, and if $M, M_1, M_2, M_3 \ldots$ be supposed to be resolved along a fixed direction, making angles $\beta, \beta_1, \beta_2, \beta_3 \ldots$, with their axes, then, we have

$$M \cos \beta = M_1 \cos \beta_1 + M_2 \cos \beta_2 + M_3 \cos \beta_3 + \ldots$$

or

$$\mathbf{M} \cos \boldsymbol{\beta} = \Sigma M \cos \beta.$$

If P be a point at a distance d from the common mid-point of intersection of these axes along the given fixed direction, then the potential at P due to magnets of moments $M, M_1, M_2, M_3 \ldots$ is given by

$$\frac{M \cos \beta}{d^2}, \frac{M_1 \cos \beta_1}{d^2}, \frac{M_2 \cos {}_2\beta}{d^2}, \frac{M_3 \cos \beta_3}{d^2}, \ldots;$$

and since, as shown above,

$$M \cos \beta = M_1 \cos \beta_1 + M_2 \cos \beta_2 + M_3 \cos B_3 + \ldots$$

we have,

$$\frac{M \cos \beta}{d^2} = \frac{M_1 \cos \beta_1}{d^2} + \frac{M_2 \cos \beta_2}{d^2} + \frac{M_3 \cos \beta_3}{d^2} + \ldots$$

that is,

$$\frac{\mathbf{M} \cos \boldsymbol{\beta}}{d^2} = \Sigma \frac{M \cos \beta}{d^2}$$

This means that the potential at P, due to the magnet of moment, M, is the algebraic sum of the potentials due to the components $M_1, M_2, M_3 \ldots$, into which it has been resolved. This result is in accord with the principles of magnetic potential. Hence the assumption that the rules for the resolution and composition of forces may be applied to magnetic moments leads to results in accord with magnetic theory and may therefore be accepted.

82. Intensity of the Field due to a small Bar Magnet at any Point. Let N S (Fig. 135) represent the small magnet. Its moment M may be resolved into two components, M $\cos\alpha$ along O P, and M $\sin\alpha$ at right angles to O P, the angle N O n being denoted by α. The force at P due to M $\cos\alpha$ is $\frac{2\,M\cos\alpha}{OP^3}$ along Pa, and that due to M $\sin\alpha$ at the same point is $\frac{M\sin\alpha}{OP^3}$

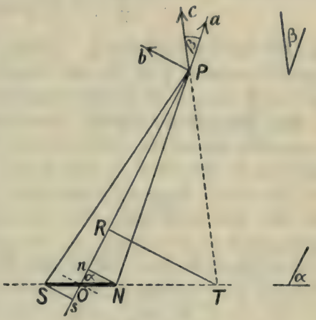


Fig. 135.

along P b. If the resultant of these forces acts along P c and the angle a P c be denoted by β, then

$$\tan\beta = \frac{M\sin\alpha}{OP^3}\bigg/\frac{2\,M\cos\alpha}{OP^3} = \tfrac{1}{2}\tan\alpha.$$

That is $\tan\alpha = 2\tan\beta$. This result determines the *direction* of the resultant force, and a simple geometrical construction for finding this direction is readily deduced. From the figure $\tan\alpha = TR/OR$, and $\tan\beta = TR/RP$, and therefore $\frac{TR}{OR} = 2\frac{TR}{RP}$ or $RP = 2\,OR$, that is O R is $\frac{1}{3}$ O P. Hence, to find the direction P c the construction is as follows. On O P take a point R, such that O R is one third of O P. Through R draw R T at right angles to O P and cutting S N produced at T. Join T P and produce T P to c, thus obtaining the required direction P c. The *magnitude* of the resultant force is, from Arts. 78 and 79, evidently given by

$$\frac{M}{OP^3}\sqrt{4\cos^2\alpha+\sin^2\alpha},$$

or

$$\frac{M}{d^3}\sqrt{1+3\cos^2\alpha}$$

if O P be denoted by d. When α is o or π this general result becomes $2\,M/d^3$, the result of Art. 78, and if $\alpha = \pi/2$ or $3\,\pi/2$ we get M/d^3 the result of Art. 79.

83. Magnetic Shell. A magnetic shell is a thin sheet of magnetic material, magnetised at every point in a direction normal to the shell at the point considered. The *strength* of the shell depends upon the intensity of magnetisation of the material and the thickness of the shell, and is measured at any point by the product of the intensity of magnetisation and thickness at that point. That is, if I denote the intensity of magnetisation and t the thickness of the shell at any point; the strength of the shell ϕ is given by

$$\phi = \mathrm{I}\,t.$$

The *potential* due to a shell at any point may be determined as follows. Consider (Fig. 136) a small element of the shell at O. The length of the element taken parallel to the direction of magnetisation, that is, normal to the shell, is equal to t, the thickness of the shell at O. Let the area of the ends of the element be denoted by a, where a is very small. Then the magnetic moment of the element is equal to I $a t$, where I denotes the intensity of magnetisation. By Art. 80 the potential at P due to this element is $\frac{\mathrm{I}\,a\,t\cos\alpha}{\mathrm{O\,P}^2}$, where α is the angle between O P and the normal at O in the direction of the magnetic axis of the element. But It is the strength of the shell, Φ, and $\frac{a\cos\alpha}{\mathrm{O\,P}^2}$ is the measure of the solid angle subtended by the small area a at P. If this small solid angle be denoted by $\delta\omega$, then the potential at P, due to the element of the shell at O, is given by $\Phi\,(\delta\omega)$. Hence if Φ be the same at all points

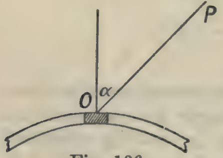


Fig. 136.

of the shell, the potential at any point is measured by $\Phi\,\omega$, where Φ denotes the strength of the shell and ω the solid angle subtended by the shell at the point. If the shell is not a closed surface the value of ω evidently depends only on its boundary. If the shell is closed, then for a point outside the shell $\omega = 0$, and the potential due to it is zero, and for a point inside the shell $\omega = 4\pi$, and the potential is $4\pi\Phi$. This is the value of the potential for *any* point inside the shell, and therefore there is no difference of potential for points inside the shell, that is, there is no magnetic force there. Fig. 137 shows, for a nearly closed shell, the values of ω for points outside and inside the shell, and indicates how the value is zero in one case and 4π in the other for a completely closed shell. It will be evident that the difference of potential for two points on the same normal, one infinitely close to the positive surface, and the other to the negative surface, of the shell is constant and equal to $4\pi\Phi$, where Φ is the strength of the shell. Let A, A′ (Fig. 138) be the two points, A on the positive side and A′ on the negative side. The potential due to the shell at A is $\Phi\,(4\pi - \omega)$, where ω is the solid angle subtended by the aperture of the shell S S at A. Similarly the potential at A′ is $-\Phi\,\omega$, and the difference between the potential at A and that at A′ is $4\pi\Phi$, that is the work done in conveying unit pole from A′ to A or from the negative to the positive side of the shell is $4\pi\Phi$, where Φ is the strength of the shell.

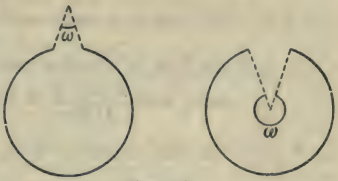


Fig. 137.

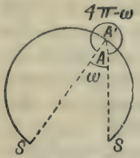


Fig. 138.

The potential at P (Fig. 139), due to the element of a shell at O, has been shown to be $\dfrac{\mathrm{I}\,a\,t\cos\alpha}{\mathrm{O\,P}^2}$. That is, the potential energy of the element of the shell in the field

due to a unit north pole at O is given by $\frac{I a t \cos \alpha}{O P^2}$. But the field at O, due to a unit north pole at P, is $\frac{1}{O P^2}$ along O P′, and the flow of force across the area a at O is given by $\frac{1}{O P^2} \cdot \cos \alpha \cdot a$ or $\frac{a \cos \alpha}{O P^2}$ in the direction O S, that is, by $-\frac{a \cos \alpha}{O P^2}$ in the direction O N. Now the potential energy of the element may be written as $I t \cdot \frac{a \cos \alpha}{O P^2}$ and $I t = \Phi$,

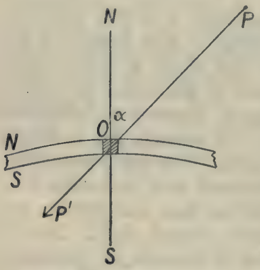


Fig. 139.

the strength of the shell, and $-\frac{a \cos \alpha}{O P^2} = f$, or $\frac{a \cos \alpha}{O P^2} = -f$, where f is the flow of force across the surface area of the element in the direction of the magnetic axis of the element. The potential energy of the element is therefore equal to $-\Phi f$. This argument applies to every element of the shell, and, therefore, the potential energy of the shell as a whole is given by $-\Sigma \Phi f$ or $-\Phi \Sigma f$. But Σf is the total flow of force across the shell, that is, the total flow of force through the boundary of the shell in the direction of magnetisation, that is, from the south face to the north face. If this flow of force be denoted by $\mathbf{F}$, then the potential of a shell in a magnetic field is given by $-\Phi\, \mathbf{F}$, where Φ is the strength of the shell and $\mathbf{F}$ the total flow of force through the boundary of the shell from the negative to the positive side.

If we consider a magnetic field due to two magnetic shells in the field, the mutual energy of the shells is readily deduced. Let Φ and Φ' be the strengths of the shells. The flow of force $\mathbf{F}$, as defined above, through the shell of strength Φ, is proportional to Φ' and may be taken as equal to $M \Phi'$, where M is a constant. The potential energy of

one shell in the field of the other is $-\Phi \,.\, M\,\Phi'$. Similarly, the flow of force through the shell of strength Φ', due to the other shell, is proportional to Φ and may be taken as equal to $M'\,\Phi$, where M' is a constant. The potential energy of this shell is therefore given by $-\Phi' \,.\, M'\,\Phi$. But the potential energy of either shell in presence of the other is the same and, therefore, $\Phi\, M\, \Phi' = \Phi'\, M'\, \Phi$ or $M = M'$, and the mutual energy of the system is $-M\,\Phi\,\Phi'$, where Φ and Φ' are the strengths of the shells. The constant M is called the *coefficient of mutual induction* for the two shells, and the fact that $M = M'$ means that the flow of force from one shell through the contour of the other *per unit strength of shell* is the same *for* both shells.

The theory of magnetic shells derives its importance from the fact that a closed circuit carrying a current gives rise to the same magnetic field, and is subject to the same action when placed in a magnetic field, as a shell of the same contour and of strength numerically equal to the strength of the current in the circuit. This subject is dealt with in a later chapter.

CHAPTER XV.

SIMPLE MAGNETIC MEASUREMENTS.

84. The Torsion Balance. The torsion balance described in Art. 8 was applied by Coulomb to the purposes of magnetic measurement. Fig. 140 shows the instrument arranged for magnetic work. The shellac lever, *a b*, shown in Fig. 8 is replaced by the magnet N S, and the insulating rod, *g*, by the magnet, M.

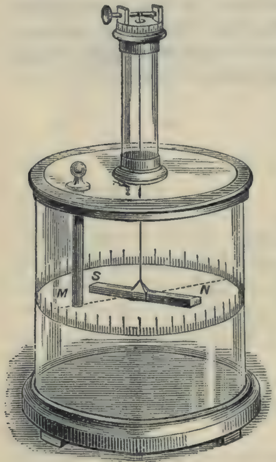


Fig. 140.

The general method of work is exactly similar to that described in Art. 8, but, owing to the directive action of the earth's field on the magnet, N S, the following additional adjustments have to be considered.

(1) The suspension wire must be free from torsion when the magnet lies in the magnetic meridian. This may be effected by suspending, in the place of the magnet, a *copper* bar of about the same size and weight, and then adjusting the torsion head until this bar lies in the magnetic meridian. When the magnet is now replaced in its stirrup, it at once sets in the meridian without putting any twist on the wire.

(2) When the magnet is deflected out of the meridian by the repulsion of the lower pole of M, it is urged back by *two* couples, one due to torsion, as explained in Art. 8, and the other due to the horizontal field of the earth. The moment of the former couple is proportional to T, the twist on the wire: the moment of the latter is, according to Art. 76, given by $m\,l\,\mathrm{H}\sin\delta$, where m is the strength of the pole of the magnet, l its length, H the intensity of the horizontal component of the earth's field, and δ the angle of deflection from the meridian. Coulomb, however, for the sake of simplicity, did not apply this relation, but determined by experiment the value of this moment in terms of the torsion moment. This he did by first removing M and adjusting N S in the meridian, so that the suspension wire was free from torsion. He then turned the torsion head so as to deflect N S out of the meridian, and noted the amount of twist that had to be put *on the wire* in order to deflect the magnet 1° out of the meridian. Thus, in one experiment, it was found that the *torsion head* had to be turned through 360° in order to deflect the magnet (in the same direction) 10° out of the meridian, that is, a twist of 350° $(360-10)$ had to be put *on the wire* to produce 10° deflection from the meridian. Hence, *assuming the moment due to* H *to be proportional to the angle of deflection,** this shows that 1° deflection from the magnetic meridian is equivalent to 35° of torsion on the wire.

These adjustments being made, the balance may be used to verify the laws of magnetic attraction and repulsion in precisely the same way as described in connection with the verification of the corresponding laws of electrostatics. For example, in proof of the law of inverse squares, Coulomb made the following experiment. He first found, in the way explained above, that 1° deflection of the magnet out of the magnetic meridian corresponded to 35° of torsion of the wire. The magnet M was then placed in position, and

* Since this moment is expressed by $m\,l\,\mathrm{H}\sin\delta$, it is evidently proportional, not to the angle of deflection, but to the *sine* of that angle. For small angles, however, the sine of an angle is proportional to the angle; so that, unless the deflection exceeds 10 or 20 degrees Coulomb's method is sufficiently exact.

was found to repel the suspended magnet through 24°. The twist on the wire was thus 24°, and adding to this the $24 \times 35°$ of torsion to which the 24° deflection out of the magnetic meridian is equivalent, the total *torsion equivalent* of the deflection was taken as $(24 + 24 \times 35)°$ or $(36 \times 24)° = 864°$. The torsion head was then turned so as to reduce the deflection from 24° to 12°, and it was found that eight complete revolutions were necessary to effect the desired result. The actual twist on the wire is now equal to $8 \times 360° + 12° = 2892°$, and adding to this the torsion equivalent of the 12° deflection out of the meridian, we get a total of $2892° + (12 \times 35°) = 2892° + 420° = 3312°$ This proves that, on halving the distance between the two magnetic poles, the force of repulsion between them is increased fourfold, for 3312 is approximately four times 864, and the force of repulsion is roughly proportional to the torsion equivalents of the deflections compared.

As already explained, the torsion balance is a comparatively rough instrument, and is now but little used. It has, however, played an important part in the development of magnetic measurements, and the principles of its construction are still of some practical importance.

85. The Magnetometer. The magnetometer consists essentially of a small magnetic needle pivoted or suspended so as to move freely in a horizontal plane. When the needle is pivoted, as in the form shown in Fig. 141, the

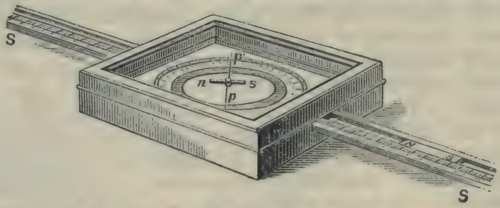


Fig. 141.

deflection of the needle can be read off on a circular scale by means of a light pointer $p\,p'$, attached to the needle $n\,s$, with its length at right angles to the axis of the needle.

To eliminate error due to parallax in reading the position of the needle, the circular scale should be on mirror glass, or on a ring of paper or other substance lying on a sheet of mirror glass.

Fig. 142 shows a simple form of mirror magnetometer. In this form the needle is either fixed, with its axis horizontal, at the back of a light concave mirror, or rigidly attached, as shown in the same figure, to a light frame

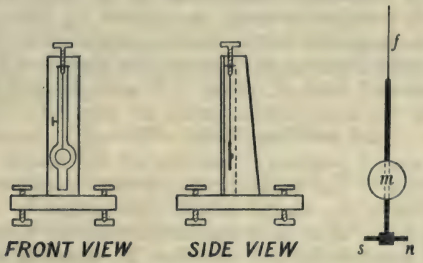


Fig. 142.

carrying the mirror. The needle and mirror system is suspended by a single silk fibre, *f*, in the simple wooden stand shown in the figure. The deflection of the needle of a mirror magnetometer is measuredby the scale and mirror method of Art. 43. By this method a very small deflection of the needle can be accurately measured, but deflections exceeding a few degrees cannot be observed as, under ordinary conditions, the *spot* goes off the scale for a very small angular deflection.

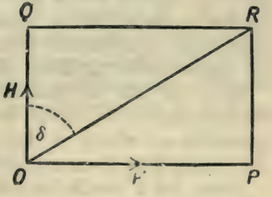


Fig. 143.

86. The Deflection Method. If at any point two magnetic fields exist at right angles to each other, the magnitude and direction of the resultant field is readily determined.

Thus, if at O (Fig. 143) there exist a field of intensity F along O P, and also a field of intensity H along O Q, then,

by the usual parallelogram construction, the resultant field is of magnitude $\sqrt{F^2+H^2}$ along O R. If the angle Q O R be denoted by δ, we have

$$\frac{F}{H} = \tan\delta \quad \text{or} \quad F = H\tan\delta.$$

This result gives a relation between the magnitudes of the two fields, which is of special importance in its application to practical methods of measurement. If we suppose first the field H to exist by itself at a point O, then a *small* magnet or magnetic needle freely suspended at O would set itself with its axis parallel to the direction O Q. If now the second field, F, along O P at O, be added to H along O Q, the small needle would set parallel to O R, the direction of the resultant field. The angle between the first and second position of the axis of the needle, that is, the *deflection* of the needle from its first position by the introduction of the second field at right angles to the first, is the angle δ. Here H is assumed to be known, and δ is evidently measurable, and therefore F, which is equal to $H\tan\delta$, can be determined. This is the principle of the *deflection method* of measuring a field, F, by superposing it at right angles to a *known* field, H (usually the horizontal component of the earth's magnetic field), and measuring δ, the angle of deflection of a small needle freely suspended at the point where the known field, and the field to be determined, have the respective values H and F. The suspended needle must be small, for unless both fields are uniform, the circle in which the needle rotates about its axis must be so small that it may be assumed that there is no appreciable variation of F or H within its limit.

Since F can be measured practically in this way it is evident that if F can be expressed in terms of some other quantity on which it depends then this quantity can be determined by measuring F.

Thus, let a needle be suspended in a uniform field, H, at any point P (Fig. 132), and let the field due to the magnet N S placed "end-on," so that P is a point on its axial line, be introduced at P at right angles to H. Then the needle will be deflected through an angle δ, which can be measured, and

$F = H \tan \delta$. But by Art. 78 $F = \frac{2M}{d^3}$, where M is the magnetic moment of the magnet N S and d the distance from its middle point to O. Hence, we get

$$\frac{2M}{d^3} = H \tan \delta \quad \text{and therefore} \quad M = \frac{d^3 H \tan \delta}{2}.$$

This shows how M, the magnetic moment of a magnet, may be determined by direct measurement.

If the magnet, N S, be so placed that P is a point on its equatorial line, as shown in Fig. 133, the magnet is said to be placed "broadside-on" and from the result of Art. 79 we get, as above,

$$\frac{M}{d^3} = H \tan \delta \quad \text{or} \quad M = d^3 H \tan \delta.$$

The two special cases here considered, where the magnet is placed "end-on" and "broadside on" to the suspended needle, are known as the two positions of Gauss. It should be noted that in both positions the magnet, N S, and the needle, before deflection, are at right angles, that is, the field H and the field due to the magnet are, in both positions, at right angles to each other.

87. The Method of Oscillations. As already explained, the time of oscillation of a freely suspended magnetic needle varies inversely as the square root of the intensity of the field in which it is suspended, that is, the square of the number of oscillations executed by the needle in a given time is directly proportional to the intensity of the field. Hence, if a needle be caused to oscillate at two places on the earth's surface where the horizontal intensities are, say, H_1 and H_2, and the number of oscillations performed in a given time at these places be respectively n_1 and n_2, then we have

$$\frac{H_1}{H_2} = \left(\frac{n_1}{n_2}\right)^2$$

In this way the intensity of the horizontal component of the earth's magnetism at any place may be compared with its intensity at any other place.

If we wish to compare the intensity of the field, due to a

given magnetic system, with the horizontal component of the earth's field, it will in general be necessary to eliminate from the former the effect of the latter. Thus, let a bar magnet be adjusted in the neighbourhood of the needle, so that the latter still sets in the magnetic meridian, and let it be required to compare the field *due to the magnet* in the neighbourhood of the needle with the horizontal field of the earth. Let I denote the intensity of the field due to the magnet, and H the horizontal field of the earth, then the total intensity of the field around the needle when the magnet is in position is $(I+H)$; and if n_1 denote the number of oscillations of the magnet in a given time in this field, and n the number in the same time in the earth's field, we have

$$\frac{I+H}{H}=\frac{n_1^2}{n^2},$$

or

$$\frac{I}{H}=\frac{n_1^2-n^2}{n^2}.$$

Similarly, if any two magnetic fields, from which the field of the earth is to be excluded, are to be compared, the influence of the latter must be eliminated from each. This is done by noting the number of oscillations of the needle, first when under the influence of the earth's field only, and then when under the combined influence of the earth's field and each of the two fields considered, these latter being so arranged that their lines of force are parallel and concurrent with those of the earth. Let n denote the number of oscillations executed by the needle in a given time under the influence of the earth's field only, and n_1 and n_2 the number of oscillations in the compound fields made up respectively of I_1 and H, and I_2 and H, where I_1 and I_2 are the intensities of the fields to be compared. Here then we have—

$$\frac{I_1}{H}=\frac{n_1^2-n^2}{n^2},$$

and

$$\frac{I_2}{H}=\frac{n_2^2-n^2}{n^2}.$$

Therefore

$$\frac{I_1}{I_2} = \frac{n_1^2 - n^2}{n_2^2 - n^2}.$$

This method may be applied to determine quantitatively the distribution of free magnetism along a bar magnet. The normal component of the intensity of the magnetic field near any point on a magnet is directly proportional to the amount of free magnetism at that point. Hence, to determine the distribution of the magnetism it is only necessary to oscillate a small needle* opposite successive points in the length of the bar, and to compare the magnetic fields, *due to the magnet*, at these points. For this purpose the magnet, as shown in Fig. 144, is fixed vertically to the north of the needle, with its south pole pointing up, so that the needle still sets in the magnetic meridian. The number of oscillations of the needle in a given time, when placed at different distances from the end of the magnet, is now noted; and the number of oscillations executed when under the influence of the earth's field alone being known, the necessary comparisons can be made and the distribution of magnetism along the bar determined. The table given below gives the result of an experiment of this nature, and the student should apply the data supplied to draw the curve of distribution and compare it with that given in Fig. 113.

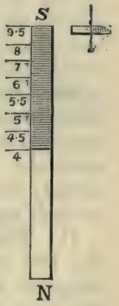


Fig. 144.

* In this case the needle must be weighted, to prevent it being attracted up to the magnet. It is to be noticed that in all these methods account must be taken of the variation of the magnetisation of the needle by the inductive action. This last method especially is vitiated by this cause. The magnet acts inductively on the needle, and the amount of this induction is different at different points in the length of the magnet, diminishing from the poles towards the centre—hence the magnetic moment of the test-needle is not constant, but diminishes as it is moved from the poles to the centre of the magnet.

Determination of the Distribution of "Free" Magnetism on the North Half of a Bar Magnet.

Needle makes four vibrations per minute when under the influence of the earth's field only.

Distance of needle from centre of magnet.	Number of oscillations of needle.	Relative intensities of fields due to magnet.	Distribution of magnetism from centre towards N. pole.
0 cm.	4	$4^2-4^2=$	0
2 ,,	4·5	$4{\cdot}5^2-4^2=$	4·25
4 ,,	5	$5^2-4^2=$	9
6 ,,	5·5	$5{\cdot}5^2-4^2=$	14·25
8 ,,	6	$6^2-4^2=$	20
10 ,,	7	$7^2-4^2=$	33
12 ,,	8	$8^2-4^2=$	48
14 ,,	9.5	$9{\cdot}5^2-4^2=$	74·25

88. Comparison of Magnetic Moments of Magnets. The comparison of the magnetic moments of two given magnets may be effected by three convenient and reliable methods. The principles involved in these methods have already been explained; we shall therefore proceed at once to their applications.

(1) *The Torsion Balance Method.* Let A and B denote the two magnets whose moments are to be compared. Suspend A in the stirrup of the balance, so that it sets in the magnetic meridian without putting any twist on the wire. Now turn the torsion head so as to deflect the magnet, through an angle δ, out of the meridian. Let T_1 denote the twist on the wire when this adjustment is made.

Repeat these operations with the magnet B, *taking care that the final deflection from the magnetic meridian is exactly the same for each magnet.* Let T_2 denote the final twist on the wire in this case. Then, if M_1 denote the moment of A, and M_2 the moment of B, we evidently have

$$\frac{M_1 H \sin\delta}{M_2 H \sin\delta} = \frac{T_1}{T_2}$$

for the moment of the couple due to the earth's field is in each case proportional to the twist on the wire.

In this relation, however, $H \sin \delta$ is the same for the two magnets, and therefore

$$\frac{M_1}{M_2} = \frac{T_1}{T_2},$$

that is, the magnetic moments of the magnets are directly proportional to the respective torsions necessary to deflect them through the same angle out of the magnetic meridian.

The magnetic moment of a magnet, as defined in Art. 76, involves two quantities—the strength of the poles of the magnet and the distance between them. In general this latter quantity is uncertain, but for thin bar magnets it is approximately equal to the length of the magnet. Hence, with thin bar magnets of known absolute or relative length, the strength of their poles can be compared by this method. For example, let magnet B be, say, three times as long as A; then if l denote the length of A, and m_1 and m_2 the strengths of the poles of A and B respectively,

$$M_1 = m_1 l,$$

and

$$M_2 = m_2 \,.\, 3\,l.$$

Therefore

$$\frac{M_1}{M_2} = \frac{m_1\,l}{3\,m_2\,l} = \frac{m_1}{3\,m_2}.$$

Hence,

$$\frac{m_1}{3\,m_2} = \frac{T_1}{T_2},$$

that is,

$$\frac{m_1}{m_2} = \frac{3\,T_1}{T_2}.$$

(2) *The Deflection Method.* To compare the magnetic moments of two magnets by this method, it is only necessary to note the deflection produced by each magnet separately when placed *at the same distance* from a magnetometer needle, the arrangement of apparatus being exactly that described in Art. 86. Then, if M_1 and M_2 denote the

magnetic moments to be compared, and δ_1 and δ_2 the angular deflections produced, we have—

$$\frac{M_1}{H} = \frac{d^3}{2} \tan \delta_1,$$

and

$$\frac{M_2}{H} = \frac{d^3}{2} \tan \delta_2,$$

where d denotes the distance from the centre of the magnet to the magnetometer needle in each case.

Hence we get—

$$\frac{M_1}{M_2} = \frac{\tan \delta_1}{\tan \delta_2},$$

that is, the magnetic moments are directly proportional to the tangents of the angles of deflection. As in the preceding method, if M_1 and M_2 be expressed as $m_1 l_1$ and $m_2 l_2$, then m_1 and m_2 may be compared if l_1 and l_2 or their ratio be known.

The time of oscillation of a magnet of magnetic moment M, in a magnetic field of intensity H, has been shown to be given by

$$t = 2\pi \sqrt{\frac{\kappa}{MH}},$$

where κ is the moment of inertia of the oscillating system.

Hence, if two magnets of magnetic moments M_1 and M_2 and moments of inertia κ_1 and κ_2 are made to oscillate in the same field, the horizontal component of the earth's field, and the times of oscillation are t_1 and t_2, then

$$t_1 = 2\pi \sqrt{\frac{\kappa_1}{M_1 H}} \text{ and } t_2 = 2\pi \sqrt{\frac{\kappa_2}{M_2 H}}.$$

This gives

$$\frac{t_1}{t_2} = \sqrt{\frac{\kappa_1}{\kappa_2} \cdot \frac{M_2}{M_1}},$$

or

$$\frac{M_1}{M_2} = \frac{\kappa_1}{\kappa_2} \left(\frac{t_2}{t_1}\right)^2.$$

If, as in Art. 87, n_1 and n_2 denote the number of oscillations performed in the same time by the two magnets we have

$$\frac{M_1}{M_2} = \frac{\kappa_1}{\kappa_2} \cdot \left(\frac{n_1}{n_2}\right)^2.$$

From this general result it is evident that if the moments of inertia of the magnets are equal, or $\kappa_1 = \kappa_2$, we have

$$\frac{M_1}{M_2} = \left(\frac{n_1}{n_2}\right)^2.$$

This method has the disadvantage that it necessitates the determination of κ_1 and κ_2. This may be avoided by causing the two magnets to oscillate together as one system, first with like poles pointing in the same direction, and then with unlike poles pointing in the same direction. The magnets are arranged with parallel axes in the same vertical plane, each magnet being symmetrical to the axis of oscillation, so that when one is reversed in direction the moment of inertia of the oscillating system is unchanged. If t_1 and t_2 are the times of oscillation and κ the moment of inertia of the compound system we have,

$$t_1 = 2\pi\sqrt{\frac{\kappa}{(M_1+M_2)H}} \quad \text{and } t_2 = 2\pi\sqrt{\frac{\kappa}{(M_1-M_2)H}}$$

This gives

$$\left(\frac{t_1}{t_2}\right)^2 = \frac{M_1-M_2}{M_1+M_2} = \left(\frac{n_2}{n_1}\right)^2$$

or

$$\frac{M_1}{M_2} = \frac{t_1^2+t_2^2}{t_2^2-t_1^2} = \frac{n_1^2+n_2^2}{n_1^2-n_2^2}.$$

89. Measurement of a Magnetic Field. It will be found that most magnetic measurements involve the measurement of a magnetic field. There are three simple methods of measuring the intensity of a magnetic field.

1. *The Deflection Method.* By this method the field to be measured is compared with a known standard field, such as the horizontal component of the earth's field. The magnetometer is set up so that the needle is controlled by the known standard field and the conditions of the experiment are so adjusted that the field to be measured is produced, at the point where the magnetometer needle is placed, as explained in Art. 86, *in a direction at right angles to the control field* taken as a standard. Then, if F denote

the unknown field and H the standard field, we have, as in Art. 86,

$$F = H \cdot \tan \delta,$$

where δ is the deflection of the magnetometer needle.

F is thus determined when H and δ are known.

2. *The Oscillation Method.* The unknown field is compared with the known standard field by the method explained above.

3. *The Induction Method.* This method is described in detail later in the chapters on Electromagnetic Induction.

CHAPTER XVI.

TERRESTRIAL MAGNETISM.

90. The Magnetic Field of the Earth. We have already learnt that, when a magnetic needle is freely suspended at any point on the earth's surface, it invariably sets in a particular position, with its magnetic axis pointing approximately north and south. This shows that at all points on the earth's surface a magnetic field exists, in which the lines of force are approximately parallel to the geographical meridian of the place. This magnetic field is supposed to be due to the earth's magnetism, but whether the earth is itself a permanent magnet, or whether its magnetism is due to some unknown cause, is, at present, an open question. The simplest explanation is to consider the earth as a permanent magnet, with its poles not far distant from the north and south poles of the earth. Assuming the distribution of magnetism to be somewhat irregular, the magnetic field that should result agrees pretty closely with observations of the actual field of the earth.

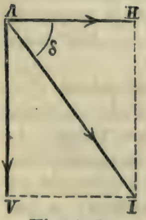


Fig. 145.

In order to quantitatively determine the magnetic field of the earth at any point, the direction and magnitude of the magnetic force at that point must be determined. The direction of the force may be found by determining (1) the vertical plane in which it acts, and (2) its direction in that that plane. The intensity of the field may be determined either by measuring this magnetic force directly, or by measuring a component making a known angle with it, For example, if the magnetic force at any point is found to act in a plane

represented by the plane of the paper in a direction A I (Fig. 145), then the intensity of the field may be determined by measuring A I directly, or *either* of the components A H or A V. For, supposing A H to be measured and the angle H A I known, then—

$$\frac{\mathrm{A\,H}}{\mathrm{A\,I}} = \cos \mathrm{H\,A\,I}, \text{ that is, } \mathrm{A\,I} = \frac{\mathrm{A\,H}}{\cos \mathrm{H\,A\,I}}.$$

We shall now proceed to consider the practical determination of the earth's magnetic field.

The quantities which completely determine the magnetic field of the earth are sometimes called the *magnetic elements.* The *direction* of the magnetic force is determined by the *Declination* and the *Inclination* or *Dip.* The *Declination* at any place is the angle between the magnetic and geographical meridians of the place, the *magnetic meridian* at any place being the great circle whose plane contains the direction of the terrestrial magnetic force at that place. When the direction of the geographical meridian is known, a knowledge of the angle of declination enables us at once to find the magnetic meridian of a place, or the vertical plane in which the magnetic axis of a freely suspended magnet would set itself.

The *Inclination,* or *Dip,* at any place, is the angle which the direction of the magnetic force makes *in the magnetic meridian* with the horizontal at that place. Thus in Fig. 145, if the plane of the paper represent the magnetic meridian and A I the direction of the magnetic force at A, then the angle H A I represents the inclination or dip at A.

The *intensity* of the earth's field at any place is measured by determining the *horizontal component* of the total magnetic force at that place.

91. Declination. To determine the declination it is necessary to determine the geographical and magnetic meridians, and to measure the angle between them. In this article we shall consider only a very simple method of effecting these measurements. Suppose, for example, we wish to mark on a large table in the laboratory a line showing the positions of the two meridians, and including the angle of declination between them. Let a straight

piece of wire about a foot long be fastened vertically into the table so that the sun casts its shadow on the table. The direction of this shadow when *it is shortest*, which will be about noon, will approximately give the geographical meridian of the place. Let this direction be marked by a line, N S (Fig. 146), drawn on the table.

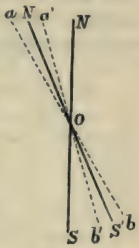


Fig. 146.

To determine the magnetic meridian, let a light bar magnet be suspended over the table by a single silk fibre. To each end of this magnet let a short piece of fine brass wire be attached by shellac or sealing-wax, in such a position that, when the magnet is properly suspended, the pieces of wire are, as accurately as possible, vertical (Fig. 147). Suppose the magnet to be suspended over the table. Let a needle or piece of wire be stuck into the table at any point, a, through which the line, $w\,w$, joining the pieces of wire at the ends of the magnet passes. This point is readily determined by one observer sighting along the lower points of the wire and directing another observer where to place the needle. In the same way another needle can be placed at b, so that $a\,o\,b$ represents the direction of the line joining the wires fixed at the ends of the magnet. If $w\,w$ coincided with the magnetic axis of the magnet, then the line $a\,b$ would represent the trace of the magnetic meridian on the table, and the angle N $o\,a$ would be the required angle of declination. In all probability, however, $w\,w$ is not the magnetic axis of the magnet; but whatever the position of this axis, we can eliminate the error due to its non-coincidence with $w\,w$ by turning the magnet upside down, so that the top face becomes the bottom face, and then determining the line $a'\,o\,b'$ in exactly the same way as

Fig. 147.

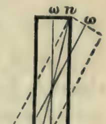


Fig. 148.

$a\,o\,b$ was found. The true direction of the trace of the magnetic meridian will now lie midway between the lines $a\,o\,b$ and $a'\,o\,b'$, and may be drawn by bisecting the angle between them. The line N′ S′ represents this direction, and the angle N O N′ measures the declination. To prove

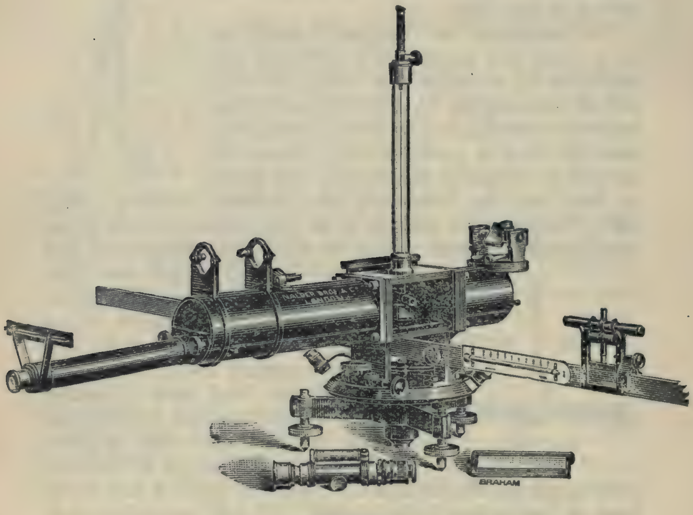


Fig. 148.

that N′ S′, bisecting the angle between $a\,b$ and $a'\,b'$, gives the true direction in which the magnetic axis of the magnet points, consider Fig. 148. Taking an extreme case, let $n\,s$ represent the position of the magnetic axis of the magnet. Then if the full outline represent the position of the magnet in the first case, on inverting the magnet it will take up the position indicated by the dotted outline, for in this position the magnetic axis has the same direction as

before.* From the figure it is evident that the magnetic axis $n\,s$, and, therefore, the magnetic meridian, lies midway between the two positions of $w\,w$.

For the exact determination of declination, a Declination Compass, one form of which is shown in Fig. 149, is used. By suitable astronomical observations the line of collimation of the telescope can be set exactly in the geographical meridian of the place, and the reading on the horizontal scale corresponding to this position is noted. The telescope is then set by observation of the position of a freely suspended bar magnet, so that its line of collimation is parallel to the magnetic axis of the magnet. The angle between the two positions of the telescope gives the declination.

The magnitude of the angle of declination will necessarily vary for different positions on the earth's surface. Suppose, in Fig. 150, that we are looking down on the northern hemisphere of the earth. Let G represent the geographical north pole, and M the magnetic north pole; then, if the distribution of the earth's magnetism were perfectly regular, the full lines would represent the magnetic meridians, and the dotted lines the geographical meridians, of the northern hemisphere. The angle at any place between the two meridians for that place gives the declination there, and it is evident from the figure that that angle varies for different places. For any place on the lines a M, G b the angle of declination is zero, for the magnetic and geographical meridians are here coincident, for any place on M G the angle of declination is evidently 180°, and for places not on these lines the declination will have values between 0°

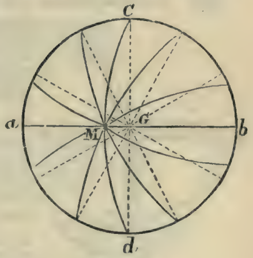


Fig. 150.

* The magnetic axis must lie in the magnetic meridian, and therefore, on inversion of the needle it must either be coincident with, or parallel to, its first position. A line fulfilling this condition may be defined as the magnetic axis of a magnet.

and 180°. Further, in the quadrant *a b d* the declination is to the west of the geographical meridian, and in the quadrant *a c b* to the east.

Magnetic observation fixes the magnetic north pole in Boothia Felix in latitude 70° 5′ N., and longitude 96° 45′ W. The position of the south magnetic pole has been determined recently (1908) by Prof. David, of the Shackleton expedition, to be in 72° 25′ S. latitude and 154° E. longitude, but it is possible that there is more than one south polar region. In England the angle of declination is about $16\frac{1}{2}$° W., that is, the magnetic meridian is about $16\frac{1}{2}$° West of the geographical meridian. (See the map on p. 280.)

In Fig. 158 the thick lines diverging from M show roughly the true course of the magnetic meridians; they are called *isogonic* lines, indicating that at all points on any one of them the angle of declination is the same.

92. Inclination, or Dip. If the magnetic meridian be found as in the preceding article, and a magnetic needle free to move in a *vertical plane* (Fig. 151) be placed with its axis in the meridian, it will be found that it does not set horizontally, but dips, in England, at an angle of about 68° with the horizontal. This angle is the angle of Dip, and a magnetic needle mounted so as to indicate it is called a *Dipping Needle.*

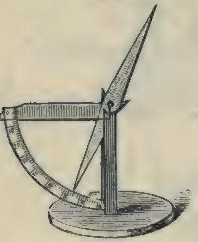


Fig. 151.

To determine the Inclination or Dip at any place it is therefore only necessary to have a magnetic needle mounted so as to move freely in a vertical plane in front of a circular scale on which the angle of Dip may be read off. A proper instrument for measuring the angle of Dip is known as a *Dip Circle.* It consists, as shown in Fig. 152, of three essential parts—the needle mounted so as to move with as little friction as possible in a vertical plane, the vertical circular scale in front of which the needle moves, and the horizontal circular scale on which the rotation of the frame carrying the vertical scale and needle can be read. The needle is mounted by means of

a short horizontal axis of hard polished steel passing, as accurately as possible, through the centre of gravity of the needle. When mounted on its bearings, this axis should pass through the centre of the vertical scale at right angles to the plane of the scale. The scale itself is

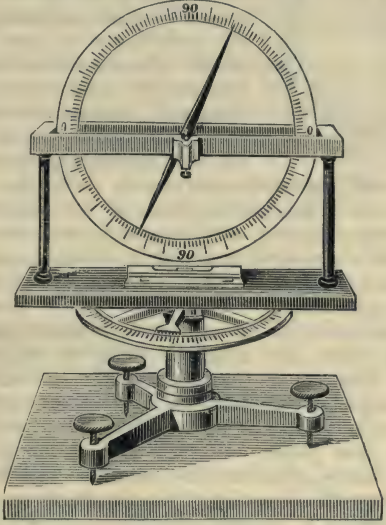


Fig. 152.

usually graduated so that both ends of the needle can be read, and, since the Dip is measured from the horizontal, the *zero line* is horizontal. Fig. 153 shows generally how the scale is graduated: in the best forms of the instru ment, the positions of the ends of the needle are read by small reading microscopes, which carry verniers round the vertical scale. The horizontal scale may be graduated continuously from 0° to 360°, commencing at any point, or it may conveniently be graduated in the way indicated in Fig. 153—its use will appear in the sequel.

To determine the Dip by a Dip Circle, it is obviously necessary, first, to set the plane of the needle in the magnetic meridian—this might be done approximately by determining the meridian by means of a magnetic needle, but it is more conveniently and accurately done by the indications of the Dip Circle itself. The instrument is carefully levelled by means of the screw feet, and the spirit-level attached to it. The needle and vertical scale are then moved round the horizontal scale until the needle sets exactly vertically.

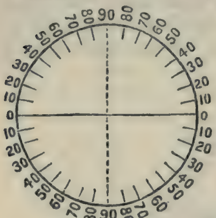


Fig. 153.

When this is the case the plane of the needle is *at right angles* to the magnetic meridian, and hence on turning it through 90°, measured by the help of the vernier attached to the horizontal scale, the needle is set exactly in the magnetic meridian.

To understand this last adjustment let us consider what we know about the magnetic field of the earth. The resultant direction of the magnetic force at any place lies in the magnetic meridian for that place, and makes a definite angle, known as the angle of Dip, with the horizontal in that plane. In Fig. 145 let the plane of the paper, as before, represent the magnetic meridian, and let A I represent the direction of the resultant magnetic force at A. Imagine a magnet, moving in a vertical plane *at right angles* to the plane of the paper, to have its north pole at A—the force acting on the pole will be in the direction A I, *but only the vertical* component of this force will have any effect on the needle, for the horizontal component, A H, being at right angles to the plane of motion of the needle, can have no effect on it, except to produce a slight pressure on its bearings. Hence, when the plane of motion of the needle is at right angles to the magnetic meridian, only the *vertical* component of the earth's magnetic field is effective, and consequently the needle, being entirely under the influence of this component, sets vertically.

The needle being thus set in the magnetic meridian, it only remains to read off the angle of Dip on the vertical scale, and if the needle be accurately centred and suspended, and the scale accurately levelled and engraved, the direct reading of either end of the needle gives the true Dip. In practice, however, the needle and scale are never in accurate adjustment, and a number of corrections are necessary to eliminate :—

(1) The error due to eccentricity, that is, the error which results if the axis of suspension of the needle should not pass through the centre of the vertical scale.

(2) The error which results if the zero line of the vertical scale is not truly horizontal.

(3) The error due to non-coincidence of the magnetic axis of the needle with its geometrical axis.

(4) The error which results from non-coincidence of the point of suspension of the needle with its centre of gravity.

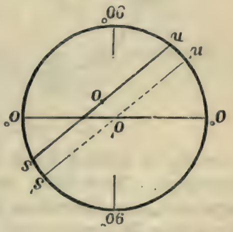


Fig. 154.

Error (1) is eliminated by reading both ends of the needle and taking the mean of the two readings. This is evident from Fig. 154, for if O represent the point of suspension of the needle and O′ the centre of the circular scale, then the true Dip o *n*′ or o *s*′ is evidently the arithmetical mean of o *n* and o *s*, the actual readings of the two ends of the needle.

Error (2) is eliminated by taking the mean reading for two positions of the needle in the magnetic meridian. When the needle is first set at right angles to the magnetic meridian, it can evidently be put in the meridian by turning through 90°, either to the right or to the left of the observer. Let it be turned first to the right, say, and let the position of the needle be read; then let it be turned through 180° in either direction, and it will be again in the magnetic meridian, but reversed relative to the observer. The mean of the readings of both ends of the needle, taken in these two positions, gives the Dip corrected for the sources of

error (1) and (2). For in Fig. 155, where the right-hand drawing represents the reverse (or obverse) view of the left-hand one, if o′o′ represent the true horizontal, and o o the actual position of the zero-line, then the true Dip o′*n* or o′*s* is evidently the arithmetical mean of the reading o *n* or o *s* in the left-hand drawing, and o *n* or o *s* in the right-hand drawing.

The source of error indicated in (3) has already been considered in the determination of Declination. It is eliminated in the same way by reversing the needle relative to the scale, that is, by lifting the needle off its bearings,

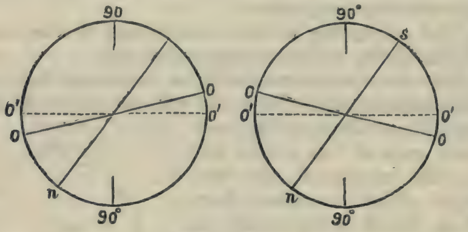


Fig. 155.

turning it round back for front, and replacing it on the bearings. The four readings indicated above are now repeated, and the mean of the eight readings thus obtained gives the Dip corrected for errors (1), (2), and (3).

To correct for error (4) it is necessary to remagnetise the needle, so as to reverse its poles. The eight readings mentioned above are now repeated, and the mean of the sixteen readings gives the true Dip. The theory of this correction may be seen in a general way thus: suppose the centre of gravity of the needle to be nearer the north pole of needle than the point of suspension is; then it is evident that the observed angle of Dip will be too great, but on reversing the magnetism of the needle the observed angle will, for the same reason, be too small, and hence, by taking the mean of the two readings, the error is approximately eliminated.

The angle of Dip is sometimes determined from observations of the apparent dip taken in *any* two vertical planes at right angles. Let P A and P B (Fig. 156) be the traces in a horizontal plane of the two planes, making angle α_1 and α_2 with the magnetic meridian P M. The components of H, the horizontal component of the earth's field, along P A and P B are evidently H cos α_1 and H cos α_2 and V, the vertical component at P, is the same for both planes. Hence if δ_1 and δ_2 denote the apparent dip in the planes of P A and P B we have

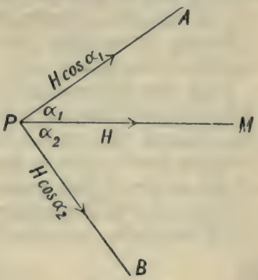


Fig. 156.

$$\frac{V}{H\cos\alpha_1} = \tan\delta_1 \quad\text{and}\quad \frac{V}{H\cos\alpha_2} = \tan\delta_2.$$

Since α_1 and α_2 are complementary, we may write $\sin\alpha_1$ for $\cos\alpha_2$ and the result may be put in the form

$$\frac{H\cos\alpha_1}{V} = \cot\delta_1, \text{ and } \frac{H\sin\alpha_1}{V} = \cot\delta_2.$$

This gives

$$\frac{H^2\cos^2\alpha_1}{V^2} + \frac{H_2\sin^2\alpha_1}{V^2} = \cot^2\delta_1 + \cot^2\delta_2,$$

that is,

$$\frac{H^2}{V^2}(\cos^2\alpha_1 + \sin^2\alpha_1) = \cot^2\delta_1 + \cot^2\delta_2.$$

or

$$\frac{H^2}{V^2} = \cot^2\delta_1 + \cot^2\delta_2.$$

Hence, since

$$\frac{H}{V} = \cot\delta,$$

where δ is the true dip in the plane of P M, the magnetic meridian, we get

$$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2.$$

This result determines δ in terms of δ_1 and δ_2, and to determine δ_1 and δ_2 it is only necessary to observe the apparent dip in *any* two planes at right angles. In this way the difficulty of setting the needle exactly in the magnetic meridian is avoided.

Like the declination, the angle of Dip is different for different positions on the earth's surface. In the northern hemisphere, the north pole—that is, the north-seeking pole—of the needle always dips downwards, and the magnitude of the angle of Dip varies with change of position on the surface of the hemisphere. At one point the needle sets itself vertically, and the Dip has its maximum value of 90°. This point is the magnetic north pole. Similarly, in the southern hemisphere the south-seeking pole of the needle dips downwards, and at the south pole the needle is vertical. From this it follows that, at a series of points in the neighbourhood of the equator, the needle is horizontal, and dips neither towards the north nor the south pole of the earth—at these points, therefore, the angle of Dip is zero, and the locus of all such points is called the *magnetic equator*, or more properly the *aclinic line*, or line of no dip.

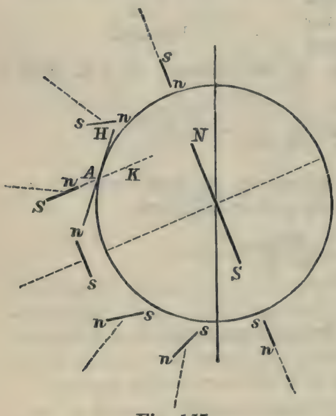


Fig. 157.

Hence, if a dipping needle were carried by any path, say along a magnetic meridian, from the magnetic north pole to the magnetic south pole, the position of the needle would change through 180°. At the magnetic north pole the needle is vertical, with its north-seeking pole dipping down, but as it is carried south the angle of Dip gradually decreases from its initial value of 90° at the pole to 0° at the *aclinic line*. On crossing this line, the south-seeking

pole of the needle begins to dip, and the inclination gradually increases from 0° to 90° in passing from the equator to the south pole. Fig. 157 shows these changes diagrammatically: in studying the figure it must be remembered that the Dip at any place is measured by the angle between the horizontal at the place and the direction of the magnetic axis of the dipping needle at that place; also that the horizontal at any place is the line drawn tangential to the earth's surface at that place. Thus at A the line A H is the horizontal, and the angle H A K is the angle of Dip. The position of equilibrium of the dipping

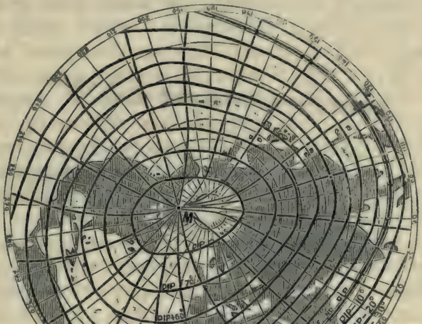


Fig. 158.

needle at any place is determined by the principles illustrated in Fig. 131, where N S may represent the earth (as a magnet) and *n s* the dipping needle.

It is evident that a number of places at which the angle of Dip is the same can be found—thus, roughly speaking, all places at equal distances from either magnetic pole should have the same Dip. Hence, if lines be drawn through all places at which the Dip is the same, we shall

have a series of *isoclinic lines* corresponding on a magnetic map to the lines of latitude on a geographical map—the *aclinic line* is one of these lines and corresponds to the geographical equator. In Fig. 158 the irregular circles concentric with M represent the isoclinic lines for every 10° of Dip—the aclinic line or magnetic equator being the irregular curve of no Dip.

93. Cause of the Earth's Magnetic Field. The magnetic field at the surface of the earth is, in general detail and neglecting local irregularities, similar to that due to a very small magnet with its mid point at the centre of the earth, its north pole pointing to the earth's south pole, and its axis making an angle of about 15° with the earth's axis. From the theory of a small magnet it is possible on this supposition to give a general expression for the magnetic force at any point on the surface of the earth taken as a sphere of radius r.

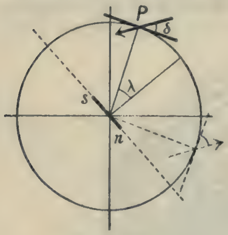


Fig. 159.

For any point, P, on the earth's surface let λ denote the latitude of the place from the magnetic equator and δ the angle of Dip. Then $(90 - \lambda)$ and $(90 - \delta)$ correspond as indicated in Fig. 159 to α and β of Art. 82, and we get

$$\tan \delta = 2 \tan \lambda.$$

Also the magnetic force at a point on the magnetic equator is given by M/r^3 and, by Art. 82, the force at any point of latitude λ from the magnetic equator is $\frac{M}{r^3}(1 + 3\sin^2\lambda)$.

That is, if I_o denote the intensity at the equator, then I, the intensity at any other point, is given by

$$I = I_o (1 + 3 \sin^2 \lambda).$$

From the relation $I_o = M/r^3$ we get $M = I_o r^3$, a result from which M can be calculated. If we pass from the supposition that M is the magnetic moment of a small imaginary magnet equivalent magnetically to the earth as

a magnet, and consider M to be the moment of the earth as a uniformly magnetised sphere, we get the intensity of magnetisation given by $\frac{I_o\, r^3}{\frac{4}{3}\pi r^3}$ or $\frac{3\, I_o}{4\pi}$. From the known values of I_o this gives the intensity of magnetisation as roughly equal to ·08 units.

Gauss in describing the magnetisation of the earth indicated a general method of determining the distribution of magnetism to which its magnetic field is due. Whatever this distribution may be, equipotential surfaces may be imagined as drawn in the field resulting from it. These surfaces would cut the earth's surface in lines which would at every point be at right angles to the magnetic meridian for the point. The lines, which Gauss called *magnetic parallels,* would therefore be equipotential lines for the horizontal component of the earth's field, and their distance apart would therefore vary at any point inversely as the intensity of the horizontal component at that point. At any point or points at which the equipotential surfaces touch but do not cut the earth's surface the direction of the force, being at these points normal to both surfaces, is vertical, and the points correspond to the magnetic poles of the simpler symmetrical distribution considered above. From a magnetic survey of the earth's surface it is possible to fix the position of Gauss' magnetic parallels and to determine with a degree of approximation depending upon the number and accuracy of observation data and the magnitude of local disturbances, the nature of the magnetic distribution to which the earth's field is due. Gauss came to the conclusion that the magnetic system is probably altogether within the surface of the globe, but by an extension of Gauss' method Schuster has shown that at least the diurnal variations of the magnetic elements are undoubtedly due to some source of magnetisation external to the earth.

94. The Horizontal Component of the Earth's Magnetic Field. To determine the total intensity of the earth's magnetic field it is usual to measure the *horizontal component* of that force. The reason for this is that in measuring a magnetic field it is necessary to work with pivoted or suspended magnets free to move in the field.

Now, owing to action of gravity, it is most convenient to work in a horizontal plane, in which the magnetic field is that due to the horizontal component of the earth's total field.

In Fig. 145, let A I represent the total intensity of the earth's field; then, resolving horizontally and vertically, A H represents the horizontal component, A V the vertical component, and the angle H A I the angle of Dip. These quantities are usually represented by I, H, V, and δ respectively, and from Euc. I. 47 and Art. 90 we have

$$I^2 = H^2 + V^2.$$

$$I = \frac{H}{\cos\delta}, \text{ or } H = I\cos\delta.$$

$$I = \frac{V}{\sin\delta}, \text{ or } V = I\sin\delta.$$

Also, from figure

$$\frac{IH}{AH} = \tan\delta, \text{ that is, } \frac{V}{H} = \tan\delta.$$

These relations between the magnetic elements are important; from them it is evident that if any two of the quantities I, H, V, and δ are known, then the others can be found. The two usually determined are H and δ: we have already detailed how δ is measured, and we have now to show how H can be measured. This is a somewhat difficult operation, and we can only indicate generally how it is performed. The determination consists of two steps.

(1) *A Deflection Experiment.* In this experiment the mirror magnetometer is used. When left to itself the needle sets itself in the magnetic meridian under the influence of the horizontal component, H, which is the measure of the intensity of the field in which it lies. A magnet, N S, is placed "end on" with its *centre* at a distance d to the east or west of the centre of the needle $n\,s$. As a result the latter is deflected through an angle a, and, by Art. 86, we know that

$$\frac{M}{H} = \frac{d^3}{2}\tan a,$$

where M denotes the magnetic moment of the magnet N S.

For the more exact determination of the angle of deflection, four observations for the same value of d are taken in the way indicated in Fig. 160, the true value of a being the mean of the eight readings obtained by reading both ends of the needle in the four positions of the figure.

The object of the experiment is the determination of M/H from observation of d and a by means of the relation

$$\frac{M}{H}=\frac{d^3}{2}\tan a,$$

or, when required by means of the more accurate relations of Art. 78.

(2) *An Oscillation Experiment.* The magnet N S is suspended by a silk fibre inside a box with glass sides, and the position of the box is adjusted until the magnet sets in the

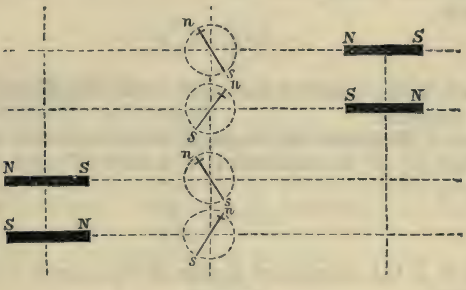


Fig. 160.

magnetic meridian with its axis in the vertical plane passing through two vertical lines ruled on opposite faces of the box. When this adjustment is made the magnet is slightly deflected by bringing the pole of a magnet, held at right angles to it, near one of its poles, and is then allowed to oscillate backwards and forwards in a horizontal plane under the influence of the horizontal component of the earth's magnetic field.

The time of each oscillation of the magnet is then carefully determined—most readily by noting the time (in minutes and seconds) occupied by, say, 50 or 100 oscillations, and dividing this time by 50 or 100 as the case may be. This determination is more easily made by two observers—one counting the oscillations commencing at 0, and counting an oscillation every time the magnet passes through its initial position of rest; the other noting on a chronometer, or watch with a seconds hand, the exact time of every fifth transit of the magnet through it position of rest. From a table of these times taken, say up to the 105th oscillation, the average time of 50 oscillations, and finally of one oscillation may be accurately found.

The oscillations are isochronous only for infinitely small amplitudes, so that for accuracy it is necessary to correct the observed time of oscillation for the average amplitude of vibration during observation. This may be done by the formula

$$t = t_o \left(1 - \frac{a^2}{16}\right),$$

where t_o is the observed time and a the amplitude in circular measure.

Now, if M denote the magnetic moment of the magnet, the time (t) of its oscillation in a field of intensity H is given by

$$t = 2\,\pi \sqrt{\frac{\mathrm{K}}{\mathrm{M\,H}}},$$

where K is the moment of inertia of the magnet.* Hence we have

$$t^2 = \frac{4\,\pi^2\,\mathrm{K}}{\mathrm{M\,H}},$$

---

* For a bar magnet of rectangular section of length l, breadth d, and mass W

$$\mathrm{K} = \mathrm{W}\left(\frac{l^2 + b^2}{12}\right),$$

and for a magnet of cylindrical section of length l, radius r, and mass W

$$\mathrm{K} = \mathrm{W}\left(\frac{l^2}{12} + \frac{r^2}{4}\right).$$

or

$$MH = \frac{4\pi^2 K}{t^2}.$$

Hence, by determining t with a magnet of regular form, for which K can be calculated, we are able to determine the *product* of the two quantities M and H, for which the deflection experiment gives the ratio.

Thus we have

$$\frac{M}{H} = \frac{d^3}{2}\tan a, \tag{1}$$

and

$$MH = \frac{4\pi^2 K}{t^2} \tag{2}$$

Now, by dividing (2) by (1) we get—

$$H^2 = \frac{8\pi^2 K}{d^3 t^2 \tan a},$$

or

$$H = \frac{2\pi}{t}\sqrt{\frac{2K}{d^3 \tan a}},$$

and thus H, the horizontal component of the earth's magnetic field, is determined.

If the units involved in K are stated in grammes and centimetres, t in seconds, and d in centimetres, then H gives in *dynes* the horizontal component of the force which a unit magnetic pole would experience in the field of the earth.

In an exact determination by this method the length of the deflecting magnet cannot be neglected. If the formula of Art. 78, involving l, be taken, we have

$$F = M\frac{2d}{(d^2 - l^2)^2},$$

where F denotes the field at the point where the magnetometer needle hangs. If $2l$ be taken as the length of the magnet this gives

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d}\tan a,$$

and

$$H = \frac{2\pi}{(d^2 - l^2)t}\sqrt{\frac{2Kd}{\tan a}}.$$

The length l really denotes half of the distance between the poles of the magnet, and is approximately equal to half the length of the magnet, only for a very thin magnet. Where l cannot be determined directly with sufficient accuracy it has to be eliminated from the result by the following method. We have

$$F = 2M \frac{d}{(d^2 - l^2)^2}.$$

This may be expanded and written—

$$F = \frac{2M}{d^3}\left\{1+2\left(\frac{l}{d}\right)^2+3\left(\frac{l}{d}\right)^4+\ldots\right\}.$$

As $\frac{l}{d}$ is a small quantity, the higher powers of it may be neglected, and we get as a sufficiently accurate result

$$F = \frac{2M}{d^3}\left(1+2\frac{l^2}{d^2}\right).$$

Since l^2 is unknown we may write x^2 for $2\ l^2$ and we get

$$F = \frac{2M}{d^3}\left(1+\frac{x^2}{d^2}\right).$$

And from the relation $F = H \tan a$ we get—

$$H \tan a = \frac{2M}{d^3}\left(1+\frac{x^2}{d^2}\right),$$

or

$$\tan a = \frac{2M}{H}\left(\frac{1}{d^3}+\frac{x^2}{d^5}\right).$$

Now if two observations of a are made for distance d_1 and d_2, giving deflections a_1 and a_2, we have—

$$\tan a_1 = \frac{2M}{H}\left(\frac{1}{d_1^3}+\frac{x^2}{d_1^5}\right),$$

$$\tan a_2 = \frac{2M}{H}\left(\frac{1}{d_2^3}+\frac{x^2}{d_2^5}\right),$$

and eliminating x from these results, we get

$$\frac{M}{H} = \frac{d_1^5 \tan \alpha_1 - d_2^5 \tan \alpha_2}{2(d_1^2 - d_2^2)}$$

It should be noticed that the magnetic moment of the magnet N S can also be determined from equations (1) and (2), for, by multiplication, we have

$$M^2 = \frac{2\pi^2 K d^3 \tan \alpha}{t^2}$$

or

$$M = \frac{\pi}{t}\sqrt{K d^3 \tan \alpha}.$$

Thus, the experimental method which gives H also serves to determine M. The value of H varies with a change of position on the surface of the earth. We have seen that $H = I \cos \delta$; hence, H increases as I increases, and decreases as δ increases. In travelling from the equator* to either pole the value of I increases, but that of $\cos \delta$ decreases, and more rapidly than I increases; therefore, on the whole, H decreases in passing from the equator to the poles. Thus, at Quito, near the equator, the value of H is about ·32 C.G.S. units; in Great Britain it varies from ·18 units in London to ·16 units in Glasgow, and at the north magnetic pole it is zero.

95. Magnetic Variations. The magnetic elements vary not only from place to place on the earth's surface but also from time to time at the same place. These variations may be classified as—

1. *Secular variations.* The secular variations are such as would result if the magnetic axis rotated about the earth's axis from west to east in a period of about 500

* Owing to the irregular distribution of the earth's magnetism, the *aclinic line* is not, as it should be, the line of minimum intensity. If a line were drawn through all points where I is a minimum, we should obtain another curve very close to the aclinic line with a claim to be called the *magnetic equator*. Theoretically, the magnetic equator should be a line of zero dip and minimum intensity, and at all points on it the direction of the magnetic axis of a compass needle should be at right angles to its plane.

years. The nature and extent of the variation are indicated by the values of declination and dip at London given below for different dates.

Date.	Declination.	Dip.
1580	11° 15′ E.	71° 52′
1657	0° 0′	73° 16′
1680	4° 6′ W.	73° 35′
1723	14° 17′ W.	74° 42′
1780	22° 14′ W.	72° 14′
1820	24° 31½′ W.	70° 3′
1860	21° 40′ W.	68° 19′
1900	16° 53′ W.	67° 12′
1910	16° 0′ W.	67° 0′

2. *Diurnal Variations.* The elements, particularly the declination, vary in a fairly definite periodic manner during every twenty-four hours. The details of the variation are

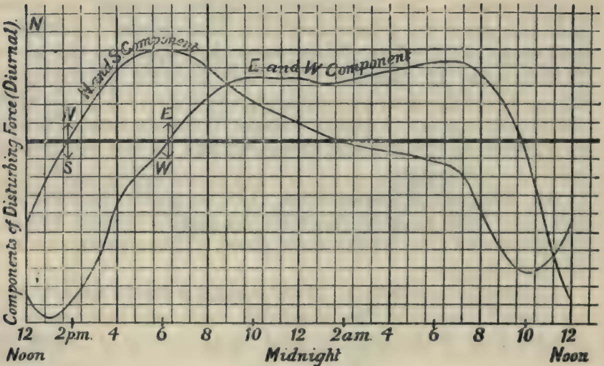


Fig. 161.

different at different places but its general nature is indicated by the curves given in Fig. 161. The curves show, for a typical day, the components, in and at right angles to the meridian, of the disturbing force affecting the horizontal component of the earth's field from hour to hour

during the day. It will be noticed that the component disturbances are periodic and that when one is a maximum the other is a minimum. The vertical component is also subject to diurnal variation of a similar character. The extent of the diurnal variations vary with the season of the year, and have also an eleven year period corresponding to the eleven year period for sun-spots.

In addition to these periodic variations there are irregular disturbances associated with magnetic storms.

The variations in the several magnetic elements are measured and registered by suitable magnetometers. The variations usually observed are those of the inclination and the horizontal and vertical components of the earth's field. The inclination variation is indicated by the change in position of a freely suspended bar magnet. The variation of the horizontal component is indicated by means of a bar magnet suspended by a bifilar suspension and set *at right angles* to the meridian. In this position the moment due to the earth's field tending to set the magnet in the meridian is measured by MH, where M is the magnetic moment of the magnet. This moment is exactly balanced by the controlling couple due to the bifilar suspension, and, as this couple is under proper conditions perfectly constant, it is evident that any deflection of the magnet from its position of rest must, if we assume M to be constant, be due to variation of H and the sense and magnitude of the deflection indicate the direction and extent of the variation. The variation in the vertical component is indicated by a magnet balanced delicately in a horizontal plane about a horizontal axis at right angles to its length. The magnet, acting like the beam of a balance, adjusts its position of equilibrium to the value of the vertical component and the deflection of the magnet in a vertical plane from a given position of rest indicates the variation of the component from a known value corresponding to the position of rest.

The variations of the dip may be deduced from the recorded values of the horizontal and vertical components.

The magnetic elements of Kew for the year 1908 were Declination 16° 22′ W., Dip 67° 0′, Horizontal Component, ·1846 dynes per unit pole. (See also the Map on p. 280.)

96. Magnetisation by the Inductive Action of the Earth's Magnetic Field. We have learnt in Art. 72 that when a piece of iron or steel is placed in a magnetic field it becomes magnetised, and that in steel the magnetisation is in some degree permanent, but in iron the residual magnetism varies with the softness of the iron, being practically *nil* for very soft wrought iron.

It follows from this that, since the earth possesses a magnetic field, every piece of iron or steel on it is more or less a magnet; but as the earth's field is, for purposes of magnetisation, extremely feeble, special precautions are necessary in order to experimentally verify the truth of this fact. Further, a piece of steel, being somewhat difficult to magnetise, may not possess magnetic properties unless it has lain for a very long time in one fixed position, or unless the arrangement of its molecules is facilitated by jarring or hammering while in a favourable position in the field. Thus, if a rod of steel or hard iron, for example, an ordinary poker, be placed with its length along the line of Dip, that is, parallel to the lines of force in the earth's field, then, on smartly tapping its upper end with a hammer, it at once becomes a magnet, the lower end being a north pole, so that on presenting this end to the north pole of a compass needle repulsion is distinctly exhibited. Similarly, if the poker were allowed to remain in this position for some weeks or months, it would, under the continued inductive action of the field, gradually acquire magnetic properties without being struck by a hammer.

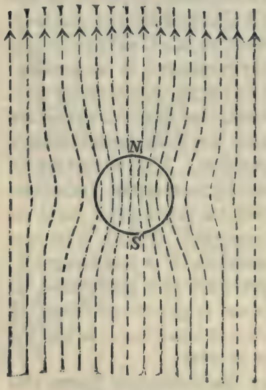


Fig. 162.

The poker is placed with its length parallel to lines of force, because in this position it is acted on inductively by the total intensity of the earth's magnetic field—if placed

in the magnetic meridian with its length horizontal, its induced magnetism may be said to be due to the inductive action of H, the horizontal component of the earth's field; and if placed vertical, the inductive effect is said to be due to the vertical component V.

Similarly, a rod of soft iron under the same conditions is acted on in precisely the same way, but the effect is temporary, and too feeble to be readily exhibited.

To determine the magnetic effect produced when a piece of magnetic substance is placed in *any* position in a magnetic field, it must be remembered that magnetic induction acts more readily through magnetic substances than through non-magnetic substances. Now the effect on a magnetic substance placed in the field depends upon the distribution of the lines of force through it; it is magnetised by induction, its magnetic axis being parallel to the general direction of the lines of force in it and its poles at the points where the lines enter and leave it. Thus, if an iron or steel rod be placed with its length at right angles to the direction of the lines of force, then, at any point in its length, Fig. 162 shows the polarity and general distribution of the lines of force.

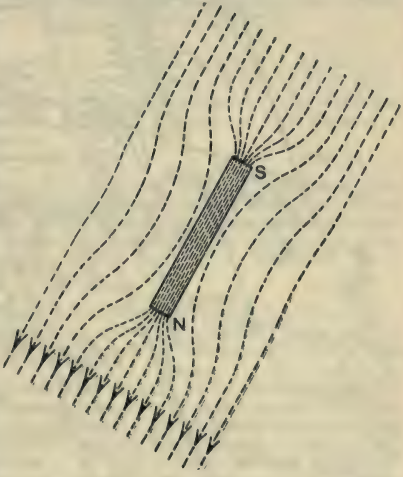


Fig. 163.

Similarly, if placed, as in the case of the poker considered above, with its length parallel to the lines of force, then Fig. 163 indicates the polarity and also the general distribution of the lines of force.


THE
BRITISH ISLANDS
ISOGONIC AND ISOCLINIC
LINES FOR THE
YEAR 1910
72°
71°
70°
69°
68°
67°
21°
20°
19°
18°
17°
16°
Wick
Dingwall
Elgin
Aberdeen
Fort William
Dundee
Oban
Glasgow
EDINBURGH
Berwick
Ayr
Eskdalemuir
Londonderry
Port Patrick
Carlisle
Newcastle
Hartlepool
Belfast
Sligo
Barrow
Lancaster
York
Drogheda
Stonyhurst
Hull
Athlone
Galway
Liverpool
Manchester
Sheffield
DUBLIN
Bangor
Chester
Lincoln
Wicklow
Limerick
Shrewsbury
Hunstanton
Leicester
Lynn
Norwich
Aberdovey
Waterford
Birmingham
Aberystwyth
Cambridge
Valencia
Cork
Bantry
Brecknock
Harwich
Swansea
Tenby
Bristol
Reading
Margate
Cardiff
Taunton
Southampton
Dover
Bude
Lyme Regis
Exeter
Brighton
Ashburton
Portsmouth
Plymouth
Falmouth

CALCULATIONS.

If two magnetic poles of strengths m and m' are separated in air by a distance d, the force of repulsion exerted between them is given by—

$$f = \frac{m\,m'}{d^2}. \qquad \text{(Art. 74.)}$$

If d be expressed in cms., and m and m' in C.G.S. magnetic units, then f is given in dynes. A north pole is considered of positive sign and a south pole negative. Hence a positive value of f indicates a force of repulsion and a negative value a force of attraction. The magnetic force due to any point in a magnetic field at any point is determined in magnitude and direction by the resultant force which a unit north pole would experience if placed at that point. (Art. 75.)

A pole of strength m placed in a field of intensity I experiences a force mI (dynes) in a direction tangential to the line of force passing through the point at which the pole is placed. The magnetic moment of a magnet with poles of strength m and distance l between its poles is given by

$$M = m\,l. \qquad \text{(Art. 76.)}$$

If a magnet of moment M is displaced from its position of rest in a uniform magnetic field of intensity I, so that its axis makes an angle α with the direction of the lines of force, then the moment of the couple tending to replace the magnet in its position of rest parallel to the lines of force is given by

$$M\,I \sin \alpha. \qquad \text{(Art. 76.)}$$

If the *same* magnet be caused to oscillate in *different* magnetic fields, the intensities of the fields will be directly proportional to the square of the number of oscillations performed by the magnet in each field in a given time. (Arts. 76, 87.)

If *different* magnets of the same moment of inertia be caused to oscillate in the same magnetic field, the magnetic moments of the magnets will be directly proportional to the square of the number of oscillations performed by each magnet in a given time. (Art. 88.)

When a magnet of moment M, placed "end on" at a distance d from a magnetic needle freely suspended in a uniform field of intensity H, deflects the needle through an angle α from its position of rest, then the ratio of M to H is given by—

$$\frac{M}{H} = \frac{d^3}{2} \tan \alpha.$$

Similarly, if the magnet is placed "broadside on," we have—

$$\frac{M}{H} = d^3 \tan \alpha.$$

If I denote the total intensity of the earth's magnetic field, H and

V the horizontal and vertical components, and δ the angle of Dip, then we have the following relations—

$$H^2+V^2 = I^2.$$

$$H = I\cos\delta;\ V = I\sin\delta$$

$$\frac{V}{H} = \tan\delta. \qquad \text{(Art. 90.)}$$

EXAMPLES III.

1. The force of attraction between two equal dissimilar magnetic poles at a distance of 10 cm. from each other is 4 dynes. Find the strengths of the poles in C.G.S. units.

Here, in the relation

$$f = \frac{m\,m'}{d^2},$$

we have $m = m'$, $d = 10$ cm., and $f = 4$ dynes.

$$\therefore\ 4 = \frac{m^2}{10^2}.$$

or

$$400 = m^2,$$

or

$$m = 20 \text{ C.G.S. units.}$$

That is, each pole has a strength of 20 C.G.S. units, one being a north pole and the other a south pole.

2. A north pole whose strength is 10 C.G.S. units experiences a force of 20 dynes when placed at a given point in a certain magnetic field; find the intensity of this field at the given point.

Here a north pole of 10 units of strength experiences a force of 20 dynes. Therefore a north pole of *unit strength* would, at the same point in the field, experience a force of $\frac{20}{10} = 2$ dynes; that is, the intensity of the field at the given point is 2 dynes.

3. A magnet 10 cm. long, with poles of unit strength, is freely suspended in a horizontal uniform magnetic field of intensity 0·18 dyne; find the moment of the couple tending to restore the magnet to its position of rest when it is deflected in a horizontal plane through 30° from that position.

The moment of the couple tending to restore the magnet to its position of rest is given by $M\,I\sin\alpha$.

Here $M = ml = 1\times 10 = 10$ C.G.S. units.

$I = 0{\cdot}18$ dyne.

$\alpha = 30°$.

$\therefore$ moment of restoring couple $= 10\times 0{\cdot}18\times\sin 30°$.

$$= 1{\cdot}8\times\frac{1}{2} = 0{\cdot}9 \text{ (dyne cm. or C.G.S. units).}$$

4. Two magnets, A and B, are caused to oscillate in the same magnetic field; A performs 15 vibrations per minute, and B 10 vibrations per minute. The magnet A is then caused to oscillate in one magnetic field and B in another; A now performs 5 vibrations per minute and B 20 vibrations per minute. Compare the intensities of the fields in which A and B now oscillate, and compare also the magnetic moments of these magnets.

From the data of the question the second field in which the magnet A oscillates is to the first field in which both A and B oscillate in the ratio

$$\frac{5^2}{15^2}=\frac{1}{3^2}=\frac{1}{9}.$$

That is, the intensity of the second field is $\frac{1}{9}$th that of the first.

Similarly, the second field, in which B oscillates, is to the first as

$$\frac{20^2}{10^2}=\frac{2^2}{1}=\frac{4}{1}.$$

That is, the intensity of the second field is 4 times that of the first. Therefore the ratio of the intensities of the second fields in which A and B respectively oscillate is given by—

$$\frac{1}{9} : 4 \text{ or } 1 : 36.$$

That is, the intensity of the second field in which B oscillates is 36 times as great as that in which A oscillates a second time.

From the data given in the beginning of the question we have, assuming the magnets to have the same moment of inertia

$$\frac{\text{The magnetic moment of A}}{\text{The magnetic moment of B}}=\frac{15^2}{10^2}=\frac{3^2}{2^2}=\frac{9}{4}.$$

5. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If, on turning the upper end of the wire half round, the magnet is deflected through 30° from the meridian, show how much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively.

Here the magnet lies in the uniform magnetic field due to the horizontal component (H) of the earth's field, and when deflected the moment of the couple tending to restore it to its position of rest in the magnetic meridian is $M H \sin \delta$, where M denotes the magnetic moment of the magnet and δ the angle of deflection from the meridian. (Art. 76.)

In each case the moment of this couple is balanced by the opposing moment due to the *torsion on the wire*; hence we have

$$T_{30} : T_{45} : T_{60} :: \sin 30° : \sin 45° : \sin 60°.$$

From the data given—

$$T_{30} = 180° - 30° = 150°$$

Therefore,

$$150 : T_{45} = \frac{1}{2} : \frac{1}{\sqrt{2}}, \text{ or}$$

$$T_{45} = 150\sqrt{2}.$$

That is, the twist on the wire will be $150\sqrt{2}$ degrees, and the upper end of the wire must be turned through $(150\sqrt{2}+45)$degrees.
Also,

$$150 : T_{60} = \frac{1}{2} : \frac{\sqrt{3}}{2},$$

or

$$T_{60} = 150\sqrt{3},$$

and the upper end of the wire must be turned through $(150\sqrt{3}+60)$ degrees.

6. Compare the intensities of the horizontal components of the earth's magnetic field at two places, A and B, given that at A the dip is 60° and the total intensity ·3 dyne, and at B the dip is 45° and the total intensity ·2 dyne.

Here, since $H = I \cos \delta$, we have at A

$$H = \cdot 3 \cos 60^\circ = \cdot 3 \times \frac{1}{2} = \cdot 15 \text{ dyne};$$

and at B

$$H = \cdot 2 \cos 45^\circ = \cdot 2 \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{10} = \cdot 14 \text{ dyne}.$$

7. A declination needle makes 50 oscillations in one minute at a point on the earth's surface where the dip is 60°, and 57 oscillations in a minute where the dip is 45°. Compare the magnetic intensities at the two places.

8. Show how to compare the magnetic force exerted by the earth on a suspended needle with that exerted by an artificial magnet at a given distance. A *long* bar magnet is placed with its length in the magnetic meridian, and its north pole 20 cm. south of the south pole of a freely suspended needle. If the needle oscillates 25 times per minute when the magnet is not in position, and 100 times per minute when under the influence of the magnet, find the strength of the magnet pole, assuming $H = 0 \cdot 18$ dyne and neglecting the action of the south pole of the magnet.

9. A magnetic needle, the magnetic moment of which remains constant, is suspended so as to move freely in a horizontal plane. When deflected from the magnectic meridian at three different places on the earth's surface, it is observed to oscillate 7·5, 8·3, and 10·4 times respectively in one minute. Compare the intensities of the earth's horizontal magnetic force at the three places.

10. What is meant by the three magnetic elements at any point on the earth's surface? Let the horizontal force be 3·8, and the vertical force 8·5. Find the total magnetic force.

11. Explain how to determine the ratio of the magnetic moment of a magnet to the earth's horizontal magnetic force.

CHAPTER XVII.

MAGNETISATION.

97. Intensity of Magnetisation and Magnetic Induction. If the magnetic state of a magnet is such that the quantities of free magnetism appearing on the opposite surfaces of a cross section taken at right angles to the axis at any point on the magnet are exactly equal but of opposite sign, and uniformly distributed over the surface of the section, then the magnet is said to be *uniformly magnetised.* If such a magnet be divided up into a number of pieces by planes parallel to and at right angles to the axis, then it is evident that the pole strength of each piece will be proportional to its sectional area, and the magnetic moment of each piece will be proportional to its volume, the magnetic moment of any piece being the same fraction of the magnetic moment of the original magnet as the volume of the piece is of the initial volume of the magnet. This proportionality between the magnetic moment and volume is evidently true whatever be the shape of the piece considered, and may be taken as an indication of the magnetic state of the substance of the magnet. In accordance with this view of the matter the magnetic moment per unit volume has been called the *Intensity of Magnetisation* of the substance of the magnet.

If we consider a uniformly magnetised magnet of pole strength m, length l, and uniform cross section a, then the intensity of magnetisation, $\mathbf{I}$, is given by—

$$\mathbf{I} = \frac{ml}{la} = \frac{m}{a};$$

that is, the intensity of magnetisation may be defined as the pole strength per unit area of cross section, but the definition given above is the more general.

From what has been said in Art. 73, it will be understood that uniform magnetisation, as defined above, is not found in the case of actual magnets. In the case of a uniformly magnetised magnet, the free magnetism would appear at the ends only, and the intensity of magnetisation would be the same throughout its substance. Actual magnets, on the other hand, exhibit a distribution of free magnetism along their length, and the intensity of magnetisation varies from point to point in their volume. The intensity of magnetisation at any point in a magnet may be defined as the magnetic moment per unit volume at the given point.

Consider a long thin rod of iron or other magnetic substance placed in a uniform magnetic field in air, with its length parallel to the lines of force in the field. The rod will become magnetised by induction (Art. 72), and its ends will exhibit polarity. The number of lines of force crossing unit area of the surface separating the ends of the rod from the external medium will therefore now be greater than the number crossing unit area at the same point in the medium before the introduction of the rod; for, in addition to the lines due to the existence of the magnetic field, we have the extra lines due to the induced pole on the end of the rod.

The number of lines of force emanating from a pole of strength m in air can readily be determined from the following considerations. Imagine a north pole of strength m isolated at a point in air. The north pole of a long infinitely thin magnet may be considered as such a pole. The magnetic intensity of the field *due to this pole* at any point at a distance r from the pole is $\frac{m}{r^2}$; that is, the number of lines of force crossing *unit area* of a spherical surface of radius r having its centre at the pole is m/r^2. The area of this spherical surface is $4\pi r^2$; hence the total number of lines of force crossing it, that is, the total number of lines emanating from the pole in air, is

$$4\pi r^2 \times m/r^2 = 4\pi m.$$

Hence, in the case considered above, if $\mathbf{H}$ denote the

intensity of the field in which the rod of magnetic material is placed, then the total number of lines of force crossing unit area of the surface separating the ends of the rod from the surrounding air given by $\mathbf{H} + 4\pi\frac{m}{a}$, where m denotes the strength of the induced pole and a the area of cross section of the rod. But we have seen above that $\frac{m}{a} = \mathbf{I}$; therefore $\mathbf{H} + 4\pi . \frac{m}{a}$ may be expressed as $\mathbf{H} + 4\pi\mathbf{I}$, where $\mathbf{I}$ denotes the intensity of magnetisation at the end of the rod. The increase in intensity denoted by $4\pi\mathbf{I}$ is due to free magnetism induced on the end of the rod.

This increase in field intensity due to the presence of magnetic material in a magnetic field is more instructively considered in the following way. Imagine the rod referred to above cut across at any point by a section at right angles to its length in such a way as to leave a very narrow gap filled with air between the two cut faces. These two cut faces will exhibit equal and opposite polarity; and if the rod is uniformly magnetised, the quantity of free magnetism on these faces will be equal to the corresponding quantities on the end faces of the rod. Hence, it follows that the number of lines of force crossing the air gap is given by $\mathbf{H} + 4\pi\mathbf{I}$; for the field in the gap is made up of the initial field, $\mathbf{H}$, and the field due to free magnetism on the cut faces bounding it. These lines of force crossing the gap are supposed to be continued as *lines of magnetic induction* or magnetisation in the material of the rod—that is, the flow of magnetic induction along the rod is measured by the flow of force across the air gap. Further, considering free magnetism as a surface effect, wherever lines of magnetic induction leave the rod to pass into the medium, free magnetism appears at the surface of separation. In the case of a uniformly magnetised rod, the lines of the magnetic induction would pass from one end to the other, and leave it only at the ends, so that free magnetism would appear only on the end surfaces; in the case of a magnet not uniformly magnetised, the lines of induction may be

supposed to leave it laterally as well as at the ends, so that free magnetism appears not only at the ends, but also as a surface distribution along the length of the bar.

The magnetic force in the magnetic material itself is defined as the force in a small cavity in the material, the cavity being so cut that there is *no free magnetism* on its surface. For example, in the case of the rod considered above, if a very narrow hole is bored in it with its length parallel to the lines of magnetisation in the material, there will be no free magnetism on the lateral surface of the hole; and if the section of the hole is infinitely small, the amount of free magnetism on the ends will also be infinitely small, so that there will be no free magnetism on the surface of this infinitely narrow tunnel, and the magnetic force in the space gives what is called the force in the material of the bar. In the case of a very long thin rod this force in the material of the rod is evidently merely the strength of the initial field in which the rod is placed; but when the rod is thick compared with its length, another point has to be considered. It has been shown above that $4\pi m$ lines of force diverge from a pole of strength m in air. If the rod at the end of which the pole of strength m is supposed to exist were infinitely thin, then *all* these lines of force would pass through the air, and curving round to the other pole of opposite sign at the other end of the rod would unite together, and might then be supposed to run through the rod as lines of induction back to their starting-point. When, however, the rod is of finite thickness, only a portion of these $4\pi m$ lines pass round through the external medium in this way; the remainder pass through the rod itself to the opposite pole as lines of force, and the number of lines thus diverted from the external path increases with the thickness of the rod.

Hence it follows that the magnetic force at any point in the material of the rod is less than the intensity of the initial field by the force at that point due to the distribution of free magnetism on the external surface of the rod. This force due to the external distribution of free magnetism on the rod is sometimes called the *demagnetising force*, for it

reduces the magnetic field to which the induced magnetisation of the rod is due—that is, the poles at the ends of the rod tend to magnetise the rod in the opposite sense, and thus produce partial demagnetisation. This demagnetising force is greater the greater the ratio between the thickness and the length of the rod, and experiment shows that it is only when the length is about three hundred times the thickness that the demagnetising effect is negligible.

From what has been said above it will be evident that magnetic induction may be considered in the following way. When a magnetic substance is placed in a magnetic field existing in a given medium, a distribution of free magnetism is determined over the separating surface, and the field both external and internal to that surface is the resultant field due to the initial field, and that due to the distribution of free magnetism on the surface of separation. Further, if we consider all substances as susceptible of magnetisation, we may state the result more generally. If any portion of a medium in which a magnetic field is established is replaced by another medium, a distribution of free magnetism is determined on the surface of separation of the media, and the magnetic field at any point is the resultant of the initial field and that due to the surface distribution of free magnetism. Also, when lines of magnetisation pass from one medium to another, positive (north-seeking) magnetisation appears on the surface of separation, if the lines pass from a more magnetic to a less magnetic medium—for example, from iron to air; and negative (south-seeking) magnetisation when they pass from the less magnetic to the more magnetic, as from air to iron. It follows from this that if a substance less magnetic than air be placed in air in a magnetic field, it will be magnetised by induction with the polarity *opposite* to that which a piece of iron would exhibit. This is the phenomenon of *diamagnetism*, and is evidently only an ordinary case of magnetic induction.

In the quantitative study of magnetic induction three quantities are of first importance. These are Intensity of Magnetic Field, $\mathbf{H}$; Intensity of Magnetisation, $\mathbf{I}$; and Magnetic Induction, $\mathbf{B}$. Each of these three quantities are directed quantities—that is, they are quantities possessing

both magnitude and direction. The directions of **I** and **B** are not necessarily the same as that of **H**. But in what follows we shall consider only the case where all three quantities have the same direction: the results obtained will apply to the more general case if the signs of addition and equality be taken in vectorial sense as referring to composition and resultant effect.

The first two of these quantities have already been explained. By magnetic induction is meant the flow of induction per unit area across any section taken at right angles to the direction of flow: as explained above, this flow of induction at any section of the substance may be measured by the total flow of force in air across a very narrow air gap taken at the point considered at right angles to the direction of magnetisation.

The relation between the three quantities, **H**, **I**, **B**, may evidently be considered in two ways. When a portion of any substance is subjected to magnetic induction, quantitative results may be obtained by determining the intensity of magnetisation, **I**, produced by a given value of **H**, or by determining the value of **B** under definite conditions corresponding to given values of **H**. In both cases it is important to note that **H** denotes, not the initial field in which the substance is placed, but the intensity of the resultant field in the interior of the substance itself—that is, the demagnetising force due to surface distribution has to be considered.

The simplest case of magnetic induction is that of a very long thin rod or wire placed in a uniform magnetic field with its length parallel to the lines of force. And this is a case of great practical importance. In this case, if the length of the wire is more than three hundred times its diameter, there is practically no demagnetising force; and therefore **H**, the strength of the magnetic field acting inductively on the wire, may be taken as that of the uniform field in which it is placed. The intensity of magnetisation, **I**, is given by the strength of pole per unit area of cross section, or by the magnetic moment per unit volume, and is therefore known if the magnetic moment and the dimensions of the wire are known. This is the basis

of the *magnetometer method* of measurement described below.

98. Permeability and Susceptibility. In considering the relation between the intensity of magnetisation, **I**, and the field, **H**, to which it is due, it is convenient to deal with the ratio **I**/**H**. This ratio evidently gives a measure of the susceptibility of the substance to magnetisation, and is commonly called the *magnetic susceptibility* of the substance; it is usually denoted by κ, and we therefore have—

$$\mathbf{I}/\mathbf{H} = \kappa.$$

In the same way the ratio of the magnetic induction, **B**, to **H**, the intensity of the inducing field, is known as the *magnetic permeability* of the substance. This ratio, denoted by μ, is the quantity introduced in Art. 74.

Hence we have

$$\mathbf{B}/\mathbf{H} = \mu.$$

But

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}.^*$$

Therefore

$$\mu = 1 + 4\pi\mathbf{I}/\mathbf{H};$$

and substituting κ for **I**/**H**, we get—

$$\mu = 1 + 4\pi\kappa,^*$$

* This is the usual way of presenting these relations, but it will be seen that they imply that **B**, **H**, and **I** are quantities of the same dimensions, and that μ and κ are mere numbers. A more complete way of presenting the matter is as follows. If μ be defined from the relation $\mathbf{B} = \mu\mathbf{H}$ it is evident that μ can be unity only for an unmagnetisable medium. Hence, if we take air as a magnetisable medium of permeability μ_0, and consider for a field of intensity **H**, the flow of induction across a crevasse, filled with air, in a medium of permeability μ, we have μH as the induction in the medium, and $\mu_0\,[\mathbf{H} + \frac{4\pi}{\mu_0}(\mathbf{I} - \mathbf{i})]$ as the induction in the air-gap, **i** being the intensity of magnetisation of the air. Hence we have

$$\mu\mathbf{H} = \mu_0\left[\mathbf{H} + \frac{4\pi}{\mu_0}(\mathbf{I} - \mathbf{i})\right] \quad \text{or} \quad \mathbf{B} = \mu_0\mathbf{H} + 4\pi(\mathbf{I} - \mathbf{i}),$$

that is, $\mu = \mu_0 + 4\pi(\mathbf{I} - \mathbf{i})/\mathbf{H}$. Here, if μ_0 be taken as unity and $(\mathbf{I} - \mathbf{i})$ as the intensity of magnetisation of the medium, we get $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}$ and $\mu = 1 + 4\pi\kappa$, as above. Article 46 deals with the corresponding relations in electrostatics.

where μ is the magnetic permeability of the substance subjected to induction and κ is the magnetic susceptibility of the substance.

Experiment shows that these quantities, μ and κ, for a given substance, vary with the magnetic state of the substance. Thus κ decreases as **I** and **H** increase, for **I** tends to a maximum as **H** increases; similarly μ decreases as **H** increases, for $\mu = 1 + 4\pi\kappa$. Further, since μ is taken as unity for air, it is evidently greater than unity for substances more magnetic than air (paramagnetic substances), and less than unity for substances less magnetic than air (diamagnetic substances). Hence it follows that κ is positive for paramagnetic substances and negative for diamagnetic substances; also, since μ is taken as unity for air, κ for air must be taken as zero.

99. The Magnetometer Method. It will be shown later that the magnetic field produced in the interior of a long coil wound regularly and uniformly with n turns per unit of length is uniform and of magnitude $4\pi n C$, where C denotes the current in the coil. Hence it is evident that the magnetic field in the interior of the coil can be varied by varying the current in the coil. If, then, we take a long regularly wound coil, and place in it a long thin iron wire, we can by varying the current in the coil subject the wire to the inductive action of magnetic fields of varying intensity, and in this way investigate the magnetic behaviour of iron in magnetising fields of different strengths.

In an investigation of this kind the usual method of work is to determine by experiment the Intensity of Magnetisation, **I**, or the Magnetic Induction, **B** corresponding to known values of **H**, the Magnetic Field. The relation between **I** and **H** or between **B** and **H** can then be exhibited by means of a curve drawn by taking values of **H** for abscissæ, and the corresponding values of **I** or **B** as ordinates. To determine **H** and **I** or **B** the most satisfactory arrangement is that in which the conditions of magnetisation are such that the demagnetising force is negligible: **H** will then be given by the intensity of the field in the interior of the coil, and **I** can be

determined from the magnetic moment of the magnetised wire and the dimensions of the wire; **B**, if required, can be got from the relation $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}$.

To realise these conditions practically, an iron wire whose length is four or five hundred times its diameter is placed in a long carefully wound coil of length about equal to that of the wire. The coil is then placed in a vertical position, with one end close to the needle of a mirror magnetometer, at a point behind the needle on a line drawn through its centre at right angles to its length. A very small current is then passed round the coil, and by means of an adjustable resistance in the circuit the strength of the current is gradually increased. As the current is increased the deflection of the spot on the magnetometer scale also increases—quickly at first, but afterwards more and more slowly, showing that the magnetic effect on the wire appears to tend to a maximum. The deflection of the magnetometer needle is evidently due to two things—the magnetic effect of the coil itself and the magnetic effect of the thin core of iron wire. These two effects can readily be separated; that due to the coil is proportional to the current flowing in the coil, so that if the deflection produced by the *coil alone* for one or two known values of the current be obtained by experiment, then the deflection corresponding to any given current can be determined: the effect due to the iron wire core for any given value of the current can then be readily obtained by subtracting the deflection due to the coil alone from that given by the coil and the wire. The effect of the coil can also be eliminated by means of a compensating coil (Fig. 166, c) placed in series with it and so adjusted in position that the effect of one coil on the magnetometer needle exactly balances the effect of the other. The deflection of the magnetometer due to the action of the wire core alone will, under the conditions of the experiment, be proportional to the intensity of magnetisation of the iron. The intensity of the magnetising field being given by $4\pi n\mathrm{C}$ is evidently proportional to C, and C can readily be determined by means of a current measurer placed in the circuit. Hence if we take for

abscissæ the values of C, and for ordinates the corresponding values of the magnetometer deflections, we obtain a curve similar to the dotted curve O A (Fig. 164), showing the relation between the magnetic moment of coil and wire and the intensity of the magnetie field in the coil. If we now similarly plot a curve O C, showing the deflections due to the coil alone for different values of C, then by subtracting the ordinates of the curve O C from the corresponding ordinates of O A we get the curve O B, showing the relation between the magnetic moment of the

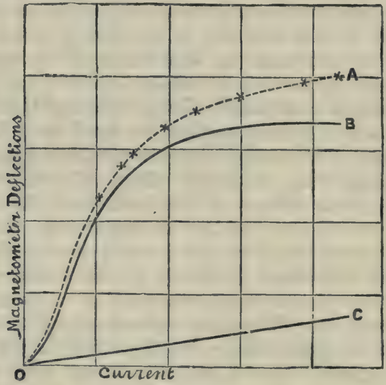


Fig. 164.

iron wire and the intensity of the magnetising field. An examination of this curve shows that the magnetic moment, and therefore the intensity of magnetisation, increases with increase in the value of H: the increase is at first somewhat slow, then more rapid, and finally slower and slower, until it becomes almost imperceptible. In the upper part of the curve, where a large increase in the intensity of the magnetising field produces a very small increase in the intensity of magnetisation, the iron has almost attained magnetic *saturation*; but experiment seems to show that absolute saturation cannot be attained—there is always

a slight increase in I when H is increased, and I only attains its maximum value when H is infinite.

Instead of plotting a curve from the values of the current for abscissæ, and the deflections as ordinates, the absolute values of I and H can be deduced from these and other data of the experiment, and a curve obtained by taking values of H as abscissæ, and the corresponding

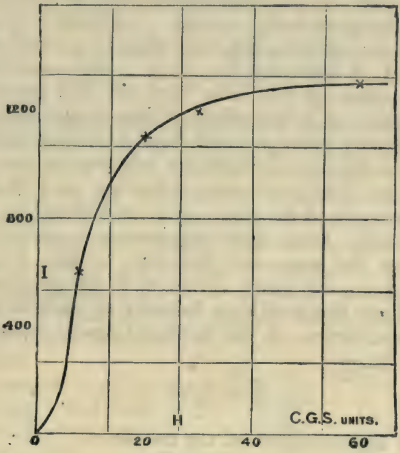


Fig. 165.

values of I as ordinates. The curve so obtained will be exactly similar in general form to that discussed above, but it has the additional advantage of showing the actual values of I and H; and since the magnetic susceptibility, κ, is given by the ratio I/H, the value of this constant for any point on the curve can be obtained by taking the ratio of the values of I and H at the given point. Similarly, if a curve be drawn showing the relation between B and H, then the *permeability*, B/H, for any point on the curve can be determined, or the value of this constant can be obtained from the I and H curve by means of the relation

$\mu = 1 + 4\pi\kappa$. Fig. 165 shows a curve for I and H in C.G.S. units. Looking at this curve, it is evident the values of κ and μ vary with the magnetic state of the iron, and calculation for a number of points on the curve will show that the values of these constants vary with the intensity of magnetisation of the iron.

The determination of I and H in absolute C.G.S. units form the data of an experiment such as that described above is comparatively simple. Omitting unimportant details and corrections, the following is a brief sketch of the method of reducing the data. In considering the magnetic action of the iron wire on the needle of the magnetometer, only the effect of the upper pole need be considered—the other pole will be at a considerable distance from the needle, and its position relative to the needle is such that its effect in causing deflection may be neglected. Let the strength of the upper pole be denoted by m; then if its distance from the magnetometer needle be d, the intensity of the magnetic field due to this pole at the needle is given by m/d^2. Then if H′ denote the uniform field in which the needle is initially at rest, it is evident that for equilibrium of the deflected needle we must have, as in Art. 86,—

$$\frac{m}{d^2} = \mathrm{H}' \tan \alpha,$$

where α is the deflection of the needle.

That is

$$m = d^2 \mathrm{H}' \tan \alpha.$$

Now I is given by m/a, where a denotes the area of cross section of the wire; therefore we have—

$$\mathrm{I} = \frac{m}{a} = \frac{d^2 \mathrm{H}' \tan \alpha}{a}.$$

To determine I it is therefore necessary to know d, a, $\tan \alpha$, and H′. The first two can be measured directly; the third, $\tan \alpha$, is readily given by the deflection on the magnetometer scale and the distance of the scale from the needle; for if s denote the former and L the latter distance, then, remembering that the angular deflection of the reflected

ray is twice that of the needle, we get $\tan a = \frac{s}{2L}$, provided the deflection is small enough for $\tan \frac{1}{2} a$ to be taken as $\frac{1}{2} \tan a$. Finally, the value of H′ can be determined by causing the needle of the magnetometer to oscillate first in the field which controls it during the experiment, and then by removing all control magnets, and taking its period of oscillation in the field of the earth only. By comparing the periods of oscillation as in Art. 87, the value of the control field can easily be obtained in terms of the horizontal component of the earth's field at the place of experiment.

The value of H, the intensity of the magnetising field, is more readily obtained; it is given at once by $4 \pi n C$, and is evidently known when n and C are known. The value of C, which must be expressed in absolute C.G.S. units, is given by the current measurer in the circuit, and n can be accurately obtained directly by counting the number of turns in a considerable length of the coil. When I is once obtained in this way, B can be easily calculated from the relation $B = H + 4 \pi I$; B can also be obtained by direct experiment as described later.

The apparatus used by Prof. Ewing for this method of experiment is shown in Fig. 166.

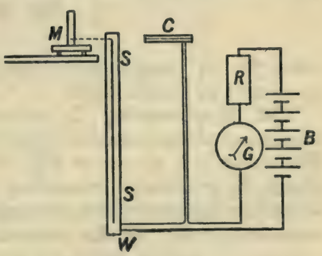


Fig. 166.

It should be noticed that in the method of experiment described above the iron wire is in a vertical position, and is therefore subject to the inductive action of V, the vertical component of the earth's field: for this reason the strength of the magnetising field is given, not by $4 \pi n C$, but by $4 \pi n C \pm V$. The value of V is comparatively small —about ·43 C.G.S. units—but its inductive effect is quite appreciable.

The behaviour of different magnetic substances during

magnetisation is indicated quantitatively in Fig. 167. The ordinates indicate magnetic induction, and abscissæ

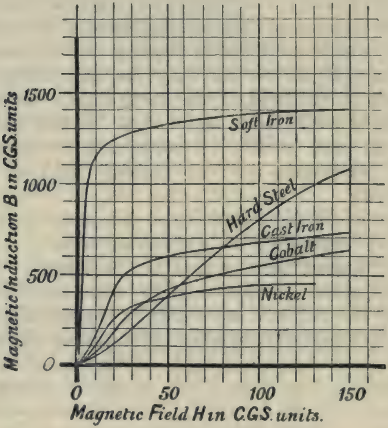


Fig. 167.

strength of magnetising field, both quantities being measured in C.G.S. units.

100. Hysteresis. In connection with this subject the important phenomenon of *hysteresis* must not be passed over. The curve O A shown in Fig. 168 is obtained, as explained above, by gradually increasing the current in the coil, and so subjecting the wire to the inductive action of a gradually increasing field. If this process is carried on until the wire practically attains magnetic saturation, and the current is then gradually *decreased*, it will be found that the stages in the magnetic condition of the wire during *increase* of the magnetising field are not repeated in the reverse order during the decrease of the field. The magnetic intensity for any given value of H during decrease is greater than during increase, so that when the magnetising field is zero the wire is not demagnetised, but possesses a considerable

amount of residual magnetism. Fig. 168 represents graphically what we have just described: O A is the curve, showing the increase of I with H as H is increased, A O′ shows how I decreases with H as H is decreased, and O O′ indicates the residual intensity of magnetisation when H is zero. The curve A O′ can, however, be continued. If the current after being reduced to zero is reversed and then gradually increased, the value of H changes sign, and under the influence of this reversed magnetic field the wire first loses its residual magnetism, and then becomes

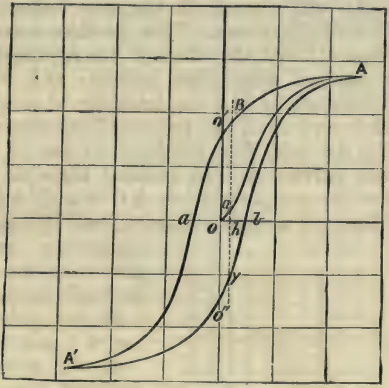
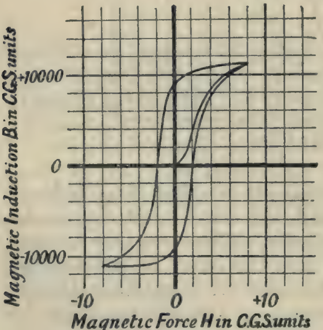


Fig. 168.

magnetised in the sense opposite to that first induced. This can be continued until the wire again approaches saturation with reversed polarity. The curve O′ *a* A′ represents this process: from O′ to *a* the effect of the reversed field is merely to reduce the residual magnetism to zero, and at *a* we have the wire in a neutral condition but the magnetising field is not zero; it has a negative value given by O *a*, and this value is sometimes called the *coercive force* for the material under experiment. From *a* to A′ the intensity of magnetisation, with sign reversed, gradually

increases and tends towards a maximum at A′, just as in the portion O A of the curve.

If the current be now decreased, the intensity of magnetisation during the decrease of H will not pass through its old values in the reverse order, but the values during decrease will be numerically greater than during increase. This is indicated in the figure by the portion A′ O″ of the curve corresponding to the portion A O′. At O″ the current has been reduced to zero; the residual intensity of magnetisation, however, is not zero, but represented by O O″. If the current be now reversed and gradually increased, the magnetic state will change in the way indicated by the portion O″ *b* A of the curve. This portion corresponds to O′ *a* A′. From O″ to *b* the effect of the reversed field is to reduce the residual magnetism to zero; at *b*, with a magnetic field indicated by O *b*, the intensity of magnetisation is zero; and from *b* to A the value of I increases with that of H. At A the wire has returned to one of its initial states on the curve O A. In passing through the states indicated by the complete curve A O′ *a* A′ O″ *b* A, the iron has passed through a cycle of states, and by repeating the operations involved in the cycle the iron may be made to pass through a similar cycle any number of times.



Soft Iron

Fig. 169.

It will be seen from a study of this curve that the magnetic state of the iron does not entirely depend on the intensity of the magnetising field. For example, take the value of H indicated by O *h* (Fig. 168). It is evident that there are *three* values of I corresponding to this value of H. These are indicated by $h\,\alpha$, $h\,\beta$, and $h\,\gamma$; and it is evident that the

difference in these values is connected with the previous history of the iron: thus, the value $h\alpha$ is for iron which was in a neutral condition before being subjected to the action of a gradually increasing field; $h\beta$ refers to iron which has been first magnetised up to saturation, and then subjected to the action of a gradually decreasing field; $h\gamma$ is for iron which, after being reduced to the neutral condition at a, is then magnetised in

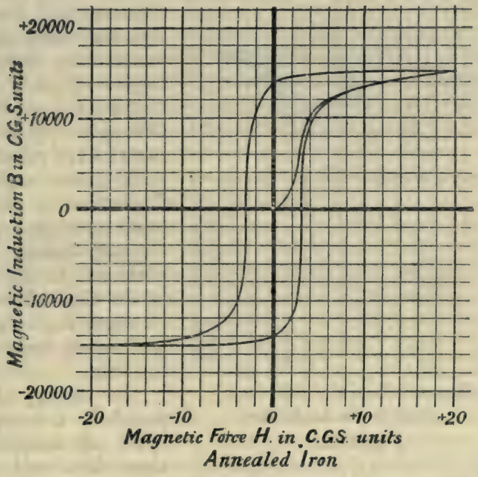


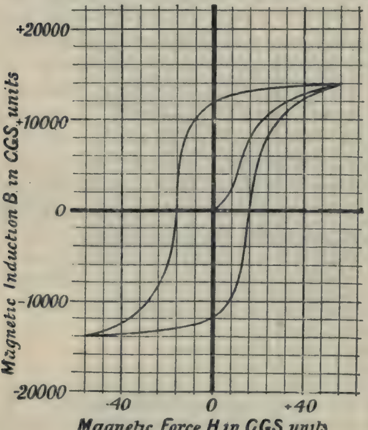
Fig. 170.

the reverse sense up to saturation, then subjected to a gradually decreasing field, and finally to a reversed field gradually increased from zero to the value indicated by O h.

It is evident from this that the magnetic state of the iron depends not only on the intensity of the magnetising field, but also on its magnetic history since last leaving the neutral condition. It will be noticed that the behaviour of the iron is such as to suggest that the original magnetic condition of the iron, whatever it may be, tends to persist

— that is, change in the magnetisation of the iron "lags" behind the changes in the magnetising force acting on it. This phenomenon has been called by Professor Ewing *Hysteresis*, from the Greek ὑστέρειν, to lag behind. Typical hysteresis curves for different materials are given in Figs. 169, 170, 171.

The existence of the *hysteresis* in iron is the cause of a considerable loss of energy when, as in the case of iron in dynamo and motor armatures, cores of transformers, etc., the iron has to go through cycles of magnetic changes with great rapidity. Each time a cycle is gone through a quantity of energy proportional to the area of the closed hysteresis curve is dissipated and causes heating of the iron.



Cycle for Annealed Steel

Fig. 171.

The energy dissipated in a complete cycle of magnetisation for soft iron varies from 40,000 ergs to 60,000 ergs per cubic centimetre. For steel the energy dissipated depends greatly on the nature of the steel and on whether it has been annealed or not. For piano wire steel, for example, the amount dissipated is about 90,000 ergs when annealed and 120,000 when not annealed. For steel with a small percentage of tungsten the energy dissipated in a cycle may be as high as 220,000 ergs.

The whole subject of magnetic induction is one of the greatest interest, and students wishing to gain further knowledge of the subject cannot do better than read

Professor Ewing's book on "Magnetic Induction in Iron and other Metals," where the whole subject is fully discussed.

101. The Ballistic Method. An important method adopted in studying the magnetisation of iron is known as the *Ballistic Method.* For this method the iron is best taken in the form of an anchor ring of diameter considerably larger than the diameter of its cross section. A quantity of iron wire coiled into a ring answers the purpose well. This ring is magnetised by passing a current round a coil wound evenly on the ring as a core. The current in the coil is produced by a battery of secondary cells, having the coil and an adjustable resistance connected up in series with it. The adjustable resistance is so constructed that by using it the current can be increased, step by step, from zero value up to the maximum value required, or decreased, step by step, from the maximum value to zero without breaking the circuit. A secondary induction coil consisting of a few turns of thin wire is wound, at any point, round the ring core of iron and connected to the terminals of a ballistic galvanometer. The first step in the experiment is to reduce the iron core to a perfectly neutral unmagnetised state. This may be done by rapidly reversing a steadily decreasing current in the magnetising coil. Then starting with a neutral core, the magnetising current is, by means of the adjustable resistance, increased step by step from zero up to the maximum value, and at each sudden increment of current the throw of the galvanometer needle is noted. This throw is proportional to the corresponding increment in the flow of induction through the induction coil wound on the iron core. If a standardising inductor is connected in the galvanometer circuit the absolute value of the *increase* in the flow of induction through the induction coil at each sudden increment of current can be determined, and the value of B, the flow of induction per unit area of cross section of the iron core, for each current value can readily be obtained. The strength of the magnetising current is given by a current measurer in the circuit of the magnetising coil, and the strength of the magnetising field can be calculated from the strength of the current. When

the diameter of the iron ring is large compared with its cross section, the magnetising field is approximately given by $H = 4\pi n C$, where C denotes the magnetising current in absolute units. When the diameter of the ring is not large compared with the cross section, the more exact relation should be used. The arrangement of apparatus necessary for this method is shown in Fig. 172, where a straight bar of iron, $b\,b$, fitted into, a yoke of soft iron, Y Y, takes the place of an anchor ring.

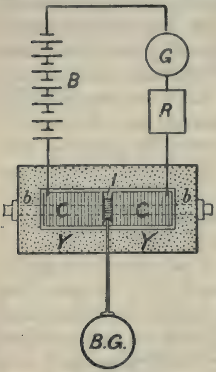


Fig. 172.

The complete hysteresis curve can be obtained by this method by noting the throw of the galvanometer needle at each increase or decrease of the magnetising current. It should be noticed that each throw of the needle indicates only the *change* in the flow of induction through the induction coil, and that the actual value of the induction flow at any stage in the experiment is determined by the appropriate summation of the changes up to that stage.

102. Variation of Permeability. The variation of permeability with the induction through the iron is shown by the diagram of Fig. 173. It will be seen that in weak fields μ increases rapidly to a maximum and then decreases as the strength of the field increases. In very intense fields permeability is found to decrease steadily for nearly all magnetic substances, as shown by the curves in Fig. 174. For

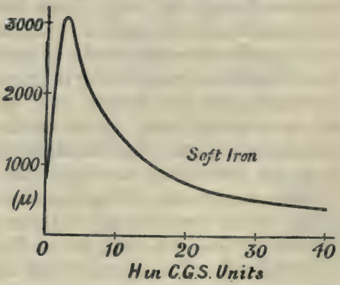


Fig. 173.

manganese steel, however, the permeability is practically constant for all values of the induction.

Permeability also varies considerably with temperature

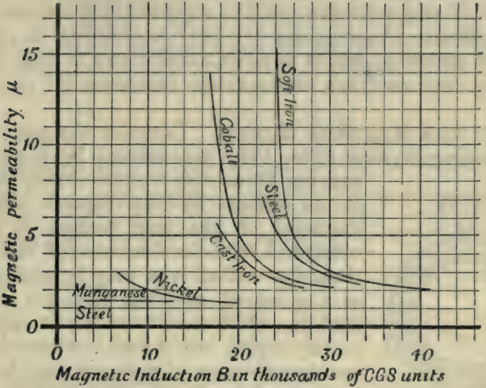


Fig. 174.

In the case of iron it increases with rise of temperature up to a critical temperature between 700° C. and 900° C. according to the character of the iron. The rate of increase is small at first but increases rapidly near the critical temperature. After passing the critical temperature the permeability falls very suddenly and at a few degrees above this temperature the iron ceases to be magnetic. The critical temperature is the temperature of recalescence, that is, the temperature at which a piece of iron gradually cooling from a red heat suddenly glows again to a bright red heat.

The most satisfactory method of determining the permeability of any specimen of iron is to obtain a B and H curve for the specimen. This, however, necessitates having a wire or anchor ring of the iron. If only a short rod is available it must be accurately fitted into a thick yoke of soft iron so that the rod and the soft iron of the yoke joining its ends practically form a continuous magnetic

circuit. Observations of its magnetisation are taken by the ballistic method, the magnetising and induction coils being wound round the rod or round a tube through which the rod passes. When the permeability for a particular intensity of magnetisation only is required it is necessary to take ballistic observations for only one variation of the current strength, but it is more satisfactory to take a few step by step readings over a range including the required value.

103. The Magnetic Circuit. Consider the case of an anchor ring of iron (Fig. 175) magnetised by a current passing in a wire wound closely round it, as shown in the figure. If the diameter of the ring section is small compared with the diameter of the ring, then the coil of wire may be considered as a solenoid, and the strength of the field inside it may be taken as approximately equal to $4\pi n C$, where n, the number of turns of wire per unit length, is obtained by dividing the total number of turns by the length of the *axis* of the iron ring.

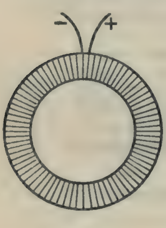


Fig. 175.

The induction across any section of the iron ring will therefore be given by $\mathbf{B} = \mu \cdot 4\pi n C$, where μ denotes the permeability of the iron. If a denote the area of cross section of the iron, l the length of the axis of the ring, and N the total number of turns of wire, we have

$$\mathbf{B} a = \frac{4\pi\mu N C a}{l}.$$

Here Ba denotes the total induction across any cross section of the iron ring, and if this be denoted by $\mathbf{F}$, we have

$$\mathbf{F} = \frac{4\pi\mu N C a}{l},$$

or

$$4\pi N C = \frac{1}{\mu} \cdot \frac{l}{a} \cdot \mathbf{F}.$$

Now $4\pi N C$, being equal to $4\pi n C \cdot l$, evidently denotes the work done in carrying unit pole round the axis of the

coil of wire against the magnetic force $4\pi n C$, that is, $4\pi N C$ gives the difference of magnetic potential for the circuit of the ring. Also **F** denotes the total flow of induction round the circuit of the iron ring. Comparing, then, this circuit with an electric current circuit, $4\pi NC$ evidently corresponds to the electromotive force, and **F** to the current or flow of electricity. Further, if the form of the relation

$$4\pi N C = \frac{1}{\mu} \cdot \frac{l}{a} \cdot \mathbf{F}$$

be compared with Ohm's law relation for an electric circuit in the form $E = CR$ or $E = C \cdot \frac{l}{a} S$ it will be seen that, taking $4\pi n C$ to correspond to E, and F to C, $\frac{1}{\mu}$ corresponds to the specific resistance, S, and $\frac{l}{a\mu}$ to R, the resistance of the electric circuit. The analogy here indicated has suggested the terms *magnetomotive force* for $4\pi NC$, and *magnetic resistance* or *reluctance* for $\frac{l}{a\mu}$, where l, a, and μ are as specified above. The term *reluctance* is used in preference to resistance; $\frac{l}{a\mu}$ is not the true analogue of electric resistance. Taking $\frac{1}{\mu}$ to correspond to specific resistance, μ should correspond to specific conductivity. But μ, the permeability of the medium, really corresponds to specific inductive capacity, and the real analogue of a piece of magnetised material is a polarised dielectric, the positive and negative charges at each end of a tube of induction in the dielectric corresponding to the positive and negative charges at each end of the tubes of induction passing through the magnetised material.

The idea of a magnetic circuit in which *magnetomotive force* ($4\pi N C$) and *reluctance* are related by a law corresponding to Ohm's law has, however, been found extremely useful in the design of dynamos, motors, and other electromagnetic machines. Thus in a magnetic circuit made up of soft iron, cast iron, and air gaps, if μ_1, μ_2, μ_3, denote

the permeability, l_1, l_2, l_3, the lengths, and a_1, a_2, a_3, the cross sections of these materials, we have for the circuit,

$$\text{Total reluctance} = \frac{l_1}{a_1 \mu_1} + \frac{l_2}{a_2 \mu_2} + \frac{l_3}{a_3 \mu_3}.$$

If the magnetising coil for the circuit has N turns and carries a current C, then the

$$\text{Magnetomotive force} = 4 \pi \text{N C},$$

and the flow of induction round the circuit is given by

$$\mathbf{F} = \frac{4 \pi \text{N C}}{\frac{l_1}{a_1 \mu_1} + \frac{l_2}{a_2 \mu_2} + \frac{l_3}{a_3 \mu_3}},$$

or generally

$$\mathbf{F} = \frac{4 \pi \text{N C}}{\Sigma \frac{l}{a \mu}}.$$

This result brings out very clearly how a very narrow air-gap in a magnetic circuit may, because of the comparatively small value of μ for air (unity), cause a large increase in the reluctance, and therefore seriously diminish the total flow of induction round the circuit. The case of an electromagnet with and without its soft iron keeper (for which μ is very large), is an important case in point.

104. Electromagnets. The theoretical lifting power of a magnet may be determined as follows. Let I denote the intensity of magnetisation of the iron. Then, assuming the magnetisation to be uniform in the magnet and its attached keeper, we may take the surface densities of the charges of magnetism on the opposing surfaces of the magnet and keeper as I and $-$I, and the force of attraction of one surface on the other per unit area of surface is, as in Art. 44, given by $2 \pi \text{I}^2$. Hence, if A denote the surface area of the poles, and the intensity of magnetisation at every point on the surface be the same, the lifting power of the magnet in dynes is given by $2 \pi \text{I}^2 \text{A}$, where I is expressed in C.G.S. units. Since B, the magnetic induction in the iron is equal to $4 \pi \text{I}$ (H being zero), we have $\text{I} = \text{B}/4 \pi$, and the lifting power $2 \pi \text{I}^2 \text{A}$, can be expressed in the form $\text{B}^2\text{A}/8 \pi$.

In designing an electromagnet the magnetising coil should be arranged so as to give the value of B corresponding to the maximum value of μ. The reluctance of the circuit is made as small as possible for the conditions under which the magnet will be required to work. In the case of magnets intended to give a magnetic field in the air gap between its pole pieces, it is evident that the character of the field will depend upon the terminal form of the pole pieces. Between two plane terminal surfaces of considerable area, compared with the width of the gap, the field will be approximately uniform except near the edges of pole pieces; between two conically pointed pole pieces, however, there will be a very intense field along the axis of the poles with a rapid decrease of intensity outwards from the axis along an equatorial line.

CHAPTER XVIII.

THE MAGNETIC FIELD.

105. Tubes of Induction. The conception of lines of force and tubes of force has already been explained. In a magnetic field the number of tubes of force per unit area, taken at right angles to the direction of the force at any point, is, by convention, numerically equal to the intensity of the field at that point. If we consider the induction in the field, that is, if we consider the magnetisation of the medium of the field, we have to deal with tubes of induction instead of tubes of force. If the medium were non-magnetisable then the tubes of induction and the tubes of force would exactly coincide, but if the medium is magnetised by the field existing in it, then, as explained in Art. 97, the number of tubes of force per unit area at any point is determined by the strength of the field in a narrow tubular space or tunnel, taken (Fig. 176, *a*) with its length parallel to the direction of magnetisation of the medium, and the number of tubes of induction is determined by the strength of the field in a very narrow air gap or crevasse, taken (Fig. 176, *b*) in the medium with its parallel faces at right angles to the direction of magnetisation. We have seen that, if H be the strength of the field in the tunnel and B in the gap, then $B = H + 4\pi I$, where I is the intensity of magnetisation induced in the medium. That is, the number of tubes of induction at any point exceeds the number of tubes of force by $4\pi I$.

Fig. 176.

When the magnetisation of the medium is induced by the field in it the direction of magnetisation at any point

is, except in certain crystalline media, the same as that of the force.

In the case of a bar magnet in air, assuming the magnet to be uniformly magnetised, the number of tubes of force starting from its north pole and curving round through the air to the south pole is, by Art. 97, $4\pi m$, where m is the strength of the pole. Also the number of tubes of induction crossing any cross section of the magnet is evidently, since there is no inducing field, equal to $4\pi \mathrm{I} a$, where I is the intensity of magnetisation and a the area of cross section. But $\mathrm{I} a = m$, so that the number of tubes of induction passing through the magnet from the south pole to the north is the same as the number of tubes of force curving round through the air from the north to the south pole. The two sets of tubes are, in fact, continuous, each tube of force in air being continued through the magnet as a tube of induction.

The difference between the conventions for the electric field and the magnetic field is indicated by the fact that the number of tubes of induction starting from a charged surface is numerically equal to σ, the surface density of the charge, while the number of tubes of induction crossing a surface taken at right angles to the direction of magnetisation is $4\pi \mathrm{I}$, where I is the intensity of magnetisation or surface density of magnetism. This corresponds to the fact that the number of tubes of force emanating from a pole of strength m, that is from a point charge m of north magnetism, is $4\pi m$, while the number of tubes of force emanating from a charge m of positive electricity is m.

When the magnetic field contains several media there are certain conditions which must obtain at the separating surfaces. These conditions correspond exactly with those specified for the electric field in Art. 47. In a magnetic field the conditions which must be satisfied may be stated as follows.

(*a*) The magnetic force tangential to the separating surface must be the same in the two media.

(*b*) The magnetic induction normal to the surface must be the same in the two media.

In comparing the second condition with the corresponding one in Art. 47 it must be remembered that magnetic induction and electric polarisation correspond. Also the magnetic induction at any point is μ times the magnetic force at the point just as the electric polarisation at any point is K times the electric force at that point, where μ and K denote respectively the permeability and specific inductive capacity of the medium.

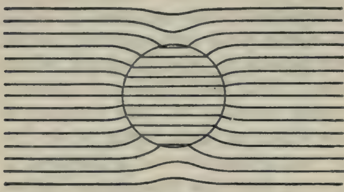


Fig. 177.

As a result of these conditions it can be shown as in Art. 47 that when tubes of induction pass from one medium to another they undergo refraction according to a law expressed by the relation

$$\mu_2 \tan \phi_1 = \mu_1 \tan \phi_2,$$

where μ_1 and μ_2 denote the permeabilities of the media, and ϕ_1 and ϕ_2 the angles which the lines of force make with the normal to the surface of separation.

It follows from this that when lines of force pass from one medium to another of smaller permeability the lines of force are bent towards the normal, and when they pass into one of greater permeability they are bent away from the normal. All paramagnetic substances have a greater permeability than air and all diamagnetic substances a lesser permeability. Hence it follows that lines of force passing from air into a paramagnetic substance such as iron are bent away from the normal, and when passing into a diamagnetic substance such as bismuth are bent towards the normal.

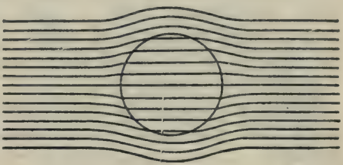


Fig. 178.

Figs. 177 and 178 show the lines of force in a field containing a sphere of paramagnetic substance and one of diamagnetic substance.

106. Energy in a Magnetic Field. In Art. 80 it has been shown that the magnetic potential at any point due to a small magnet is given by $\frac{M \cos \alpha}{d^2}$, where M, d, and α are as specified in Art. 80. This evidently means that the work done in bringing a unit north pole from an infinite distance up to the point P (Fig. 179), is measured by $\frac{M \cos \alpha}{d^2}$, or if a unit pole be placed at P the work that would be done in bringing the small magnet from an infinite distance up to its position in the field due to the unit pole is measured in the same way. That is, $\frac{M \cos \alpha}{d^2}$ is the mutual potential energy of the magnetic system, made up of the small magnet, N S, and the unit pole at P. This expression shows that the energy of the magnet varies with α, and is a maximum when $\alpha = 0$, that is, when the magnet is in a position of unstable equilibrium with its north pole, N, towards P. The minimum value obtains when $\alpha = \pi$, and the magnet is in its position of rest in the field with its south pole towards P. The potential energy of the magnet, as a part of this system, may also be expressed in terms of the strength of the field due to the pole at O. The strength of the field at

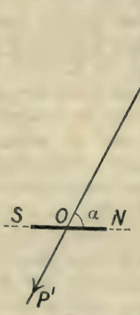


Fig. 179.

O, due to a unit north pole at P, is $\frac{1}{d^2}$ in the direction O P′, and the component of this field along the axis of the magnet is $-\frac{\cos \alpha}{d^2}$. Hence, the potential energy of the

magnet, being given by $M \cdot \frac{\cos a}{d^2}$, is equal to the magnetic moment of the magnet multiplied by the component of the strength of the field in a direction parallel to the axis of the magnet with the sign of the product changed. If I denote the intensity of magnetisation of the magnet, and H the strength of the field in the direction of its axis at the point where the magnet is placed, then the expression for the work done on the magnet in bringing it into the field is evidently $-I\,v\,H$, where v is the volume of the magnet, for $I\,v = M$ by definition. That is, the work done on the magnet per unit volume of its material in bringing it into the field is $-I\,H$. The magnetic field due to the pole is assumed to be undisturbed by the introduction of the magnet, so that there is by this assumption no change in the energy in the medium of the field.

In the case just considered the magnetic material of constant intensity of magnetisation, I, is brought from infinity to a point where the field is of strength H, and placed in the field with its direction of magnetisation parallel to the direction of the field. If, however, we consider a piece of *neutral* iron or other magnetic material placed in a field, the strength of which is supposed to increase from zero to any given value, the case is different. The iron is magnetised by the inductive action of the field, and if, when the strength of the field is H, the intensity of magnetisation of the iron is I, the work done on the iron per unit volume is evidently not given by $-I\,H$, for during the establishment of the field the values of the magnetisation and field intensities have increased from zero to the final values I and H. Following the analogy of Art. 25, for the energy of an electric charge, the work per unit volume would be given by $\frac{1}{2}I\,H$, if I were directly proportional to H, that is, if the magnetic susceptibility of the material were constant. This, however, is not the case. The relation between I and H is given by the curves of Art. 99. Let O A (Fig. 180) be the magnetisation curve for the material. Consider the step in the process of magnetisation indicated by the short length, P Q, of the

curve OA. This step is supposed to be so short that the field may be assumed constant during the step, and of magnitude indicated by O p or M P, which, for an infinitely short step, can be considered equal to O q or N Q. The intensity of magnetisation alters from I, denoted by p P or O M, to I + i, denoted by q Q or O N; so that M N represents i. The change of intensity i is equivalent to moving i units of N. magnetism for a unit distance; and, since the field is H, the work done *by* the field *on* the material in the small step considered is i H. But i H is represented by M N × M P, which ultimately is the area M P Q N in the figure. The total work done per unit volume of the material during the magnetisation, up to the condition represented by A, is the sum of such areas as M P Q N, which is equal to the area O P A B O. If, on reducing the magnetic field from H to zero again, the curve of magnetisation follows the path A O′, then in the same way the work done *by* the material is represented by the area O′ A B. Hence the total work done on unit volume for the paths O A and A O′ is equal to the difference of these areas O A O′.

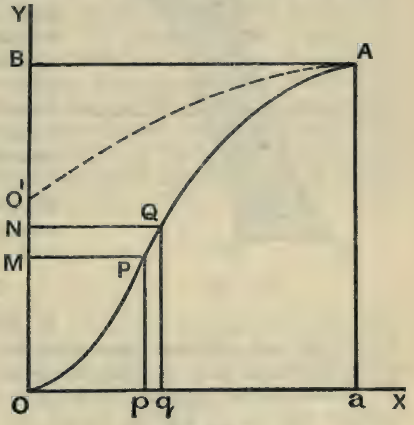


Fig. 180.

In Fig. 181, if the cycle be supposed to start at A and to follow the course indicated by A B C D A, the work done on the material during magnetisation, A to B, is the hori-

zontally shaded area A B M, and the work done by the material during demagnetisation, B to C, is the vertically shaded area C M B. Similarly for the areas to left to M N. Hence the total work done on the material per unit volume per cycle, that is, the excess of the work done on the material over the work done by it, is represented by the area of the loop A B C D A.

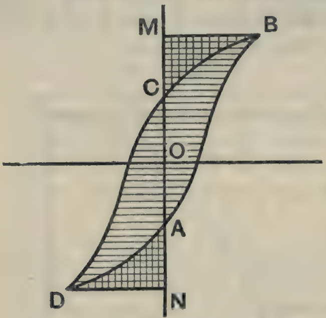


Fig. 181.

If in the hysteresis diagram the ordinates usually represent B and not I, then, since

$$B = 4\pi I + H,$$

it is evident that the length of each ordinate *intercepted by the loop* will be 4π times its length on a diagram where the ordinates represent I. Hence, if we suppose the loop to be divided into an infinite number of vertical strips, the *area* of each strip in the B, H diagram will be 4π times the area of the same strip in the I, H diagram. But we have shown that the area of the loop in the I, H diagram represents the energy dissipated as heat in unit volume of the material, therefore in a B, H diagram the area of the loop divided by 4π will represent this energy.

107. Energy in the Medium. If we consider a magnetic field due to a number of permanently magnetised magnets in a medium of which the permeability is constant it will be possible to specify the energy per unit volume of the field medium. As has been explained, the permeability of iron and similar substances is far from constant, but for weakly magnetic substances such as air the permeability seems to be practically a constant.

In such a medium I, the intensity of the magnetisation induced by the field due to the permanent magnets, would

be proportional to the field, and the number of tubes of induction per unit area at any point, where the strength of the field is F, would be μ F, where μ denotes the permeability of the medium. In this case the field is similar to an electric field due to a number of charges in the field, and the law of force in the two fields is the same. Hence, just as the energy in the electric field is given by $\frac{1}{2} \Sigma \mathrm{Q} \mathrm{V}$, so the energy in the magnetic field is given by $\frac{1}{2} \Sigma m \mathrm{P}$, where m denotes strength of pole, and P magnetic potential. Consider a very thin longitudinal element of any one of the permanent magnets in the field. Let a be the area of cross section of the element and I the intensity of magnetisation of its material. If P_n denote the magnetic potential of the north end point of the element, and P_s the potential of the south end point. Then the energy due to this element is equal to

$$\tfrac{1}{2} \Sigma m \mathrm{P},$$

that is, to

$$\tfrac{1}{2} \mathrm{I} a (\mathrm{P}_n - \mathrm{P}_s).$$

The number of tubes of induction emanating from the north end of this element and returning to the south end through the external medium is $4 \pi \mathrm{I} a$. The energy associated with each tube of induction is therefore

$$\tfrac{1}{2} \mathrm{I} a (\mathrm{P}_n - \mathrm{P}s)/4 \pi \mathrm{I} a,$$

or

$$(\Gamma_n - \mathrm{P}_s) / 8 \pi.$$

Also, if in any one of these tubes the magnetic force at any point is F, and l be a very short length of the tube at this point, then the difference of potential $\mathrm{P}_n - \mathrm{P}_s$ is equal to $\Sigma \mathrm{F} l$, the summation being taken over the whole length of the tube in the medium external to the magnet. This gives the energy for each tube equal to $\Sigma \mathrm{F} l / 8 \pi$, which indicates that at any point in the tube the energy *per unit length* of the tube is given by $\mathrm{F} / 8 \pi$, where F is the strength of the field at that point. At any point where the strength of the field is F, the number of tubes of induction per unit area is μ F, where μ is the permeability of the medium. Hence, the area of cross section of a tube at this point is $\frac{1}{\mu \mathrm{F}}$ and if we take a very short

length, l, of the tube at this point, the volume of this length is $\frac{l}{\mu F}$, and, since the energy in this volume is $\frac{F l}{8 \pi}$, the energy per unit volume of the medium is $\frac{F l}{8 \pi} \bigg/ \frac{l}{\mu F}$ or $\frac{\mu F^2}{8 \pi}$ where μ denotes the permeability of the medium (supposed constant) and F the intensity of the field at the point. Since $B = \mu F$ the energy per unit volume is also given by $B^2/8 \pi \mu$.

When a tube of magnetic induction passes from one medium to another of greater permeability the magnetic force along the tube decreases in the same ratio as the permeability increases. Hence, in field containing different media, the energy per unit length of a tube of induction is least in the medium of greatest permeability; and the tubes of induction, taking the path of minimum energy through the field will therefore tend to pass through the media of greatest permeability. If, therefore, a piece of paramagnetic material be placed in a magnetic field in air the tubes of induction will deviate from their initial path so as to pass through the paramagnetic material unless the increase in the length of the tube in air, caused by this deviation, involves a greater increase of energy than can be compensated for by the passage of the tube through the paramagnetic material. It follows from this that for every tube of induction which passes through the paramagnetic material there is a decrease in the total energy in the field, and that the greater the number of tubes that pass through the material the greater this decrease of energy will be. The energy in the field will therefore be a minimum when the material is in the strongest part of the field. Hence, if a piece of any material of greater permeability than that of the medium of the magnetic field be put into the field it tends to move towards the strongest part of the field, and the direction of the force acting on it is that in which the rate of decrease of the energy per unit volume is greatest. This question is exactly analogous to that discussed in Art. 33 for the electric field.

These considerations explain why a piece of paramagnetic substance in a magnetic field in air tends to move into the strongest part of the field while a piece of diamagnetic material tends to move into the weakest part of the field. They also help to explain why a short rod of paramagnetic material placed in the field between the poles of an electromagnet sets parallel to the field, while a rod of diamagnetic material sets at right angles to the field. The field is, in general, not uniform, and is usually strongest along the axial line of the magnet poles, and gets weaker as the distance from this axial line increases. Hence the paramagnetic rod sets axially in the strongest part of the field, while the diamagnetic rod sets equatorially, so that as much as possible of its material may be in the weaker parts of the field. In a perfectly uniform field* both rods would tend to set axially, but in the diamagnetic rod the controlling couple is extremely weak, and a very small decrease in the field intensity laterally causes the rod to take up an equatorial position. It will be seen from this that for Faraday's method of detecting diamagnetic substances the field between the poles should decrease in strength as rapidly as possible from the axis outwards along an equatorial line, and that dumb-bell shaped rods with a fairly long bar would be preferable to short cylindrical rods.

108. Rotation of the Plane of Polarisation of Light in a Magnetic Field. It is found that when any transparent substance is placed in a magnetic field it acquires the power of rotating the plane of polarisation of a beam of plane polarised light passed through it. The amount of rotation depends upon the material, and is also proportional to the component of the field intensity parallel to the direction of the beam. Hence, for a given substance the rotation is a maximum when the directions of the beam and the magnetic field are parallel and zero when they are at right angles. The direction of rotation is, however, not reversed by reversing the direction of the beam. Of the substances that have been tested, carbon bisulphide and

* Under the circumstances a perfectly uniform field could not be realised practically.

Faraday's heavy lead glass show this property most strongly.

In experimenting, the substance may be placed between the poles of an electromagnet or, more satisfactorily, in the interior of a long coil and a beam of plane polarised light passed through it parallel to the direction of the field. The amount of rotation produced can then be measured by a polarimeter in the usual way. The amount of rotation is proportional to the length of the substance traversed, and as the sense of the rotation is not reversed by reversal of path, it has been found convenient to increase the length of path by multiple reflexions, produced by silvering the ends of the piece of substance, as indicated in Fig. 182. The amount of rotation also depends upon the strength of the field, and results show that the general law of the phenomenon is that the rotation of the plane of polarisation along the path between any two points is directly proportional to the difference of magnetic potential between these two points. That is, if P denotes the *difference* of potential between any two points, and δ the observed rotation of the plane of polarisation due to the transmission of a beam of plane polarised light along the line joining the points, then

Fig. 182.

$$\delta = \mathrm{V}\,\mathrm{P},$$

where V is a constant depending on the material.

This law was enunciated by Verdet and is known as Verdet's Law, and the constant V is known as *Verdet's constant.* The value of V varies with the wave length of the light and is approximately inversely proportional to the square of the wave length.

For all diamagnetic substances the direction of rotation is the same as that of the current which would produce the magnetic field to which the rotation is due. For paramagnetic substances it is in the opposite direction.

For pure carbon bisulphide the value of Verdet's constant at t° C. is given by

$$V_t = V_o (1 - \cdot 00104\, t - \cdot 000014\, t^2),$$

V_o being equal to $0{\cdot}043'$.

This rotation of the plane of polarisation may be used in measuring a current. Let the current to be measured be passed round a uniformly wound solenoid having n turns per unit length. Then, if the rotation produced by a column of carbon bisulphide, of length l, placed in the interior of the coil with its length parallel to the axis of the coil, be denoted by δ, we have

$$\delta = \mathrm{V}\,\mathrm{P} \text{ or, since } \mathrm{P} = 4\,\pi\, n\, l\, \mathrm{C},$$

$$\delta = 4\,\pi\, n\, l\, \mathrm{C}\,\mathrm{V}.$$

That is

$$\mathrm{C} = \mathrm{V}\frac{4\,\pi\, n\, l}{\delta},$$

and C is determined if V is known and n, l, and δ are observed.

The plane of polarisation of light is also found to be rotated by reflexion at the polished surface of a magnet. This was discovered by Dr. Kerr in 1877.

PART III.

CURRENT ELECTRICITY.

CHAPTER XIX.

INTRODUCTORY.

inf. **109. Voltaic Cells and their Action.** From Part I., the student has already learnt that electricity flows along conductors—thus, if a positively charged body be connected to the earth by means of a long copper wire, the positive electricity at once flows along the wire, and the body is discharged. In this case the flow lasts for a mere instant of time, but if it were possible to supply electricity to the body as quickly as it loses it, then we should have a continuous *current* of electricity along the wire. It is found possible by chemical means to maintain for some time a constant supply of electricity, and the arrangement employed for this purpose is known as a *voltaic cell.* It was discovered by Volta (hence the name *voltaic*) that if a strip of copper and a strip of zinc be placed in a vessel containing dilute sulphuric acid, and connected by a wire, then a current of electricity is maintained through the wire. This arrangement (Fig. 183) is the simplest form of voltaic cell, and we have now to consider the explanation of its action.

As to the two metals, zinc and copper, the essential chemical difference between them is that zinc is more easily oxidised than copper, that is, zinc has a greater attraction for oxygen than copper has. Again, consider the

liquid in which the metals are placed,—it is practically acidulated water,—and, therefore, its molecules are principally water molecules containing two atoms of hydrogen and one of oxygen. Now, it is supposed that the atoms of any one molecule are not always the same, but are continually changing, that is, there is a constant interchange of atoms among the molecules, and thus there are instants at which a given oxygen atom in passing from one molecule to another is free or *dissociated* from the molecules of the liquid. Hence, considering the immense number of molecules in the liquid, it is evident that there must *always* be a large number of dissociated atoms present.

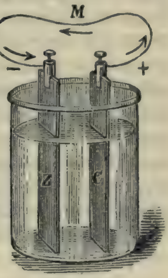


Fig. 183.

Now both zinc and copper attract oxygen atoms, and when placed in the liquid these metals attract all dissociated oxygen atoms coming within range* of their attraction. In this way both the copper and the zinc attract the oxygen atoms in the liquid, but the attraction of the zinc is greater than that of the copper. Now there is a good deal of evidence in support of the hypothesis that all atoms are charged, some positively and some negatively; oxygen is supposed to be *electro-negative*, that is, its atoms are supposed to be negatively charged, and hydrogen is the type of the *electro-positive* elements.

Hence, as the metals combine with the attracted oxygen atoms, they become negatively charged, and soon repel the free oxygen atoms electrically, as strongly as they attract them chemically. Equilibrium is thus quickly attained, and both metals are negatively charged; but the attraction of the zinc for oxygen being much greater than that of the copper, the charge on the zinc is greater than that on the copper.

So far, then, the effect of immersing the metals in the acid is to charge both negatively, but the zinc to a greater

* The range of this molecular attraction is very small—perhaps about the ten-millionth part of a millimetre.

degree than the copper, which is thus at a higher potential than the zinc. Hence, if the two metals be now joined by a piece of wire, positive electricity at once flows from the copper to the zinc, or negative electricity flows from the zinc to the copper, and the equilibrium in the cell is at once destroyed. The zinc plate becomes *less* strongly charged with negative electricity, and therefore again attracts adjacent oxygen atoms, while the copper plate becomes *more* strongly charged with negative electricity, and therefore repels instead of attracting the free oxygen atoms near it. In this way a continuous current of *negative* electricity is maintained from the zinc through the wire to the copper, and back to the zinc through the liquid, as a sort of convection current of negatively charged oxygen atoms. Similarly, by considering zinc to *repel* hydrogen *more* than copper does, we may consider the current, as a flow of *positive* electricity from the copper through the wire to the zinc and back to the copper through the liquid, as a convection current of *positively* charged hydrogen atoms. Since the flow of oxygen atoms through the liquid in one direction necessarily implies a flow of hydrogen atoms in the opposite direction, this explanation of the action of a voltaic cell points to the fact that a current possibly consists in a transfer of positive and negative electricity in opposite directions. It is, however, usual to consider only the "positive current," and the current is said to travel in the direction in which positive electricity is supposed to be carried. Thus, in the simple cell shown above, the current, starting from the copper plate, passes round its *circuit* through the wire to the zinc plate, and then through the liquid back to the copper plate. The copper plate is called the *positive pole* of the cell, and the zinc plate the *negative pole*; in the wire or *external circuit* of the cell, the current passes from the copper to the zinc, or from the positive to the negative pole, while in the liquid or *internal circuit* it passes from the zinc to the copper, or from the negative to the positive pole.

The metals here used as the elements of the cell are copper and zinc, but any two substances which attract oxygen to markedly different extents may be employed—

thus zinc and platinum, zinc and carbon, copper and carbon, zinc and silver, iron and platinum, and many other pairs of substances, act more or less efficiently as the plates of a voltaic cell. In fact, if the metals be arranged in order of their chemical affinity for oxygen, and any two of the series be chosen as the elements of a cell, the efficiency of the cell will be greater the further apart these metals are in the series. Thus, the following is a list indicating decreasing affinity for oxygen, and from it we see that a cell having plates of zinc and copper is less efficient than one having, say, zinc and carbon as its elements—further, if a cell be made up with any two substances on the list, the higher on the list having the stronger affinity for oxygen must always be the negative pole of the cell.

1. Magnesium.	6. Copper.
2. Zinc.	7. Silver.
3. Lead.	8. Gold.
4. Tin.	9. Platinum.
5. Iron.	10. Carbon.

The chemical action of the cell should here be considered. The zinc on combination with the oxygen becomes zinc oxide, and as this is formed it is acted on by the sulphuric acid present, and zinc sulphate is formed. In this way the zinc and the acid are used up, zinc sulphate being formed and hydrogen liberated, and the energy of the electric current produced is the equivalent of the chemical work done in the cell. The hydrogen is liberated at the copper plate, and copper is not chemically acted on to any extent.

imp. **110. Polarisation.** In theory this simple cell is quite satisfactory and should work well, but in practice it is found that after working for a few minutes its action is greatly weakened and may ultimately stop.

Experimental investigation into the cause of this defect has shown that it is due to the fact that a layer of hydrogen bubbles collects on the copper plate. This layer of hydrogen spoils the action of the cell in two ways: first, it interposes a thin insulating layer in the path of the current, and thus an additional resistance is opposed to the passage of the current round its circuit; second,

the hydrogen bubbles attached to the copper plate attract the oxygen atoms in the liquid even more strongly than the zinc plate does, and thus the layer of gas, acting in combination with the zinc plate, opposes the action of the zinc and copper plates. In fact the action of the cell is now of a double nature—the zinc and copper plates, owing to their different attractions for oxygen, cause a current to travel in the internal circuit from the zinc to the copper; in a precisely similar way, the hydrogen layer and the zinc plate, by virtue of their different attractions for oxygen, tend to produce a current in the internal circuit from the hydrogen to the zinc, that is, in the opposite direction to that due to the zinc and copper plates. In this way the total current produced by the cell is greatly diminished, and may be stopped altogether if the hydrogen be allowed to accumulate to such an extent as to interpose a completely insulating layer in the circuit.

This defect in the action of the cell is known as **polarisation,** and the various modifications of Volta's simple cell have been devised with the intention of removing this defect as completely as possible.

111. Various Forms of Cells. The most obvious method of preventing polarisation is to remove the layer of hydrogen bubbles mechanically. Thus, if the copper plate be repeatedly brushed, the cell continues to act satisfactorily, and does not become polarised. This, however, is a troublesome method, and Smee was the first to suggest an improvement. He constructed a cell having a platinised silver plate instead of a copper plate. The rough surface formed by the finely divided platinum which was deposited on the plate allowed the hydrogen bubbles to escape freely from its numerous sharp points, and thus polarisation was in a great measure prevented without the trouble of brushing the plate.

These mechanical methods of preventing polarisation were, however, not very satisfactory, and it was next proposed to adopt a chemical method of preventing the accumulation of the hydrogen on the positive plate.

Poggendorff's bichromate cell is constructed on this

principle. It is made up with zinc and carbon elements, but the fluid—a solution of potassium bichromate mixed with strong sulphuric acid—is one which, on account of its powerful oxidising properties, rapidly oxidises the hydrogen, and thus prevents polarisation. Fig. 184 shows the usual form of the cell—the two outer plates shown are of carbon (gas coke), and *together* form the positive plate of the cell. The middle part is of zinc, and can be raised out of the liquid, by means of a sliding rod to which it is attached, when the cell is not in use. The plates are connected to the *binding screws*, *a* and *b*; the screw *a* is connected with both carbon plates, forming the positive pole of the cell, and *b*, connected with the zinc through the brass sliding rod, forms the negative pole. Hence, when a piece of wire connects *a* and *b*, a current at once passes from *a* to *b*, but no current passes unless this connection be made. The defect of this cell is that, owing to the action of the hydrogen and zinc on the liquid, the latter quickly changes in character and gradually becomes less and less efficient, so that for a short time the cell gives a good current, which quickly falls off, though it recovers considerably if the cell be put out of action for some hours.

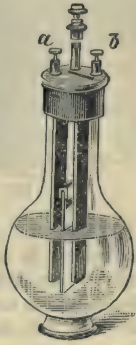


Fig. 184.

A more effectual method of preventing polarisation is afforded by the use of *two-fluid* cells. The cells described above are all made up with one fluid, but in the two-fluid cells two fluids are used, one to act on the zinc, the other to deal with the hydrogen tending to accumulate on the positive plate. The two fluids are separated by a porous partition, which allows diffusion to go on, but prevents direct mixture of the liquids. *Bunsen's cell* is a good example of this class. The plates are zinc and carbon arranged in two cells—the zinc in an outer one containing dilute sulphuric acid, and the carbon in an inner *porous* cell containing *strong nitric acid.* Fig. 185 shows the arrangement and general appearance of the cell; but as it

is absolutely essential for the student to see, handle, and work with these cells, we shall not waste space on a detailed description. The chemical action in the cell is readily understood. The zinc acted on by the *dilute* acid gives zinc sulphate and sets hydrogen free; thus—

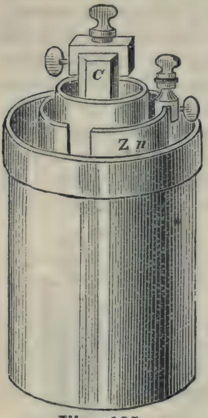


Fig. 185.

$$\underset{\text{(Zinc)}}{Zn} + \underset{\text{(Sulphuric Acid)}}{H_2SO_4} = \underset{\text{(Zinc Sulphate)}}{ZnSO_4} + \underset{\text{(Hydrogen)}}{H_2}.$$

Possibly, as explained above, this reaction takes place in two steps—the first involving the oxidation of the zinc and the liberation of hydrogen, and the second the formation of zinc sulphate; thus—

$$\mathbf{Zn} + \underset{\text{(Water)}}{H_2O} = \underset{\text{(Zinc Oxide)}}{ZnO} + \mathbf{H_2}. \quad (1)$$

$$ZnO + \mathbf{H_2SO_4} = \mathbf{ZnSO_4} + H_2O. \quad (2)$$

Whatever be the exact nature of the reaction, free hydrogen is certainly liberated, and travels by alternate combination and dissociation towards the positive pole. In this cell, however, before reaching the carbon plate, it passes through the wall of the porous cell and meets the nitric acid. Now nitric acid being a strongly oxidising agent, the hydrogen is rapidly oxidised, causing reduction of the acid. The lower oxides of nitrogen resulting from this reduction are mostly dissolved in the acid, and thus prevented from escaping into the air.

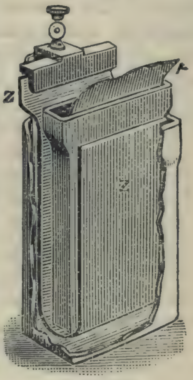


Fig. 186.

Grove's cell (Fig. 186) is essentially the same as Bunsen's, the only difference being that a sheet of platinum foil is used instead of the plate of carbon.

Daniell's cell is one of the most familiar of two-fluid

cells, and differs from the Bunsen and Grove cells in the fact that the hydrogen is not got rid of by oxidation, but by a substitution reaction. It is made up with copper and zinc plates, and the two fluids employed are dilute sulphuric acid (or a semi-saturated solution of zinc sulphate) and a saturated solution of copper sulphate. In the form that the cell usually takes (Fig. 187) the outer cell is of copper and forms the positive plate of the cell. This cell contains the copper sulphate, and is usually fitted with a ledge on which a quantity of crystals are placed, which serve to keep the solution saturated. The zinc plate or rod is fitted by means of a wooden stopper into the inner porous cell, which contains the acid or solution of zinc sulphate. The action of the cell is as follows: first in the inner cell the zinc and dilute acid act on one another so as to give zinc sulphate and free hydrogen—

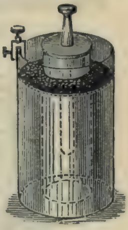


Fig. 187.

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

This free hydrogen passes from molecule to molecule through the acid towards the copper, until it meets the porous cell wall. Here it soon encounters a molecule of copper sulphate ($CuSO_4$) and at once displaces the copper; thus—

$$H_2 + \underset{\text{(Copper Sulphate)}}{CuSO_4} = H_2SO_4 + \underset{\text{(Copper)}}{Cu.}$$

The displaced copper atom then travels from molecule to molecule, until it reaches the copper plate on which it is deposited. Hence, instead of a layer of hydrogen forming on the plate, a thin film of pure copper is deposited on it, and as a consequence no polarisation results, for the copper film is an excellent conductor, and being of the same material as the plate on which it is deposited, there is no tendency to set up an opposing current.

In the form of Daniell's cell, known as the *gravity* Daniell's cell, there is no porous pot, the liquids being separated merely by the action of gravity. The denser copper sulphate solution fills the bottom of the cell and the

lighter zinc sulphate rests on it, the surface of separation of the two liquids remaining quite distinct so long as the cell is not disturbed.

The *Callaud* gravity cell is shown in Fig. 188. The

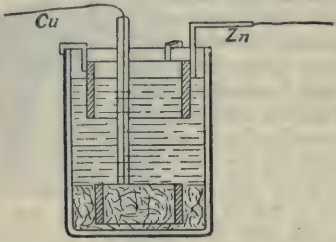


Fig. 188.

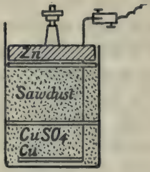


Fig. 189.

gravity cell has the advantage of having a low resistance but it is not portable.

The *Minotto cell* is a high resistance portable form of Daniell's cell. It is shown in Fig. 189. The copper plate rests on the bottom of the cell. Over it is a layer of copper sulphate crystals, then a layer of sand or sawdust moistened with zinc sulphate solution, and at the top the zinc plate.

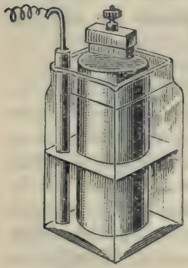


Fig. 190.

The *Leclanché cell* (Fig. 190) is a convenient cell now greatly used; its elements are zinc and carbon. The zinc, in the form of a rod, is placed in the outer cell in a solution of ammonium chloride, and the carbon in the inner porous cell is packed round with black oxide of manganese and pieces of carbon. The solution of ammonium chloride acts on the zinc, producing a double chloride of zinc and ammonium, and liberating ammonia gas and hydrogen. The hydrogen, on penetrating into the inner cell, is slowly oxidised by the manganese dioxide. This oxidising action goes on very slowly, so

that if the cell is worked for some time it becomes partially polarised, but gradually recovers if left to itself.

omit In the *Leclanché agglomerate cell*, shown in Fig. 191, the porous pot and its contents are replaced by two blocks of porous agglomerate, applied by elastic bands to the surface of the carbon plate. The agglomerate blocks used for this purpose are made of a mixture of 45 parts of manganese dioxide, 50 parts of carbon, 3 parts of shellac, 2 parts of potassium sulphate, and a trace of sulphur. This mixture is heated to 100° C., carefully mixed, and then pressed into moulds.

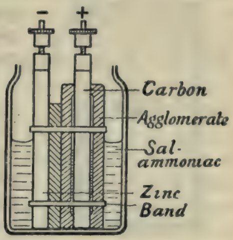


Fig. 191.

In the cell known as the *Aylmer-Leclanché cell*, Fig. 192, the agglomerate blocks take the form shown in the figure, and the zinc rod is replaced by a zinc cylinder similar to that used in some forms of Bunsen cells. This type of Leclanché cell seems to be the most satisfactory modification of the original form.

The various forms of dry cells are in most cases modified Leclanché cells. The Burnley or E.C.C. cell, shown in section in Fig. 193, is a typical form. It consists of a carbon rod, C, surrounded by a black paste, B, made of manganese dioxide and carbon powder mixed with a solution of ammonium chloride and zinc chloride. Outside this is a thinner layer of white paste, W, made of plaster of Paris and flour, mixed also with a solution of ammonium chloride and zinc chloride. Next to this paste is the zinc case, Z, forming the negative plate, and the whole cell is cased with tough card board as an

Fig. 192.

insulating and protective casing. The top of the cell is closed by pouring on a layer of melted pitch, which when it solidifies effectually seals the cell. The tube passing through the pitch into the paste, B, allows for the escape of gas from the cell.

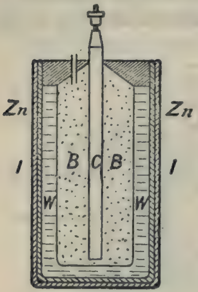


Fig. 193.

The *Edison Lalande cell* is a one-fluid cell with a positive plate of compressed copper oxide, mounted in a frame of copper, and a negative plate of zinc. The fluid used in the cell is a solution of caustic potash made by dissolving one part by weight of potash in three parts of water.

The chemical action in the cell is probably given by the equation—

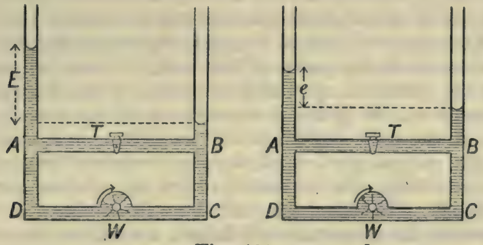
$$CuO + 2\,KHO + Zn = Cu + H_2O + K_2ZnO_2.$$

The E.M.F. of this cell is about ·75 volt, but the resistance is low. For a cell about one foot high and six inches in diameter, the resistance may be as low as ·02 ohm, and this cell could give a fairly constant current of from ten to fifteen amperes for twenty-four hours. There is practically no local action in the cell, so that it can stand unused without deterioration. It has, however, the disadvantage of being comparatively costly.

112. Electromotive Force. In the description given above it has been stated that on joining the copper and zinc of Volta's simple cell by a conducting wire, a current of positive electricity flows from the copper to the zinc in the external circuit, because the copper is at a higher potential than the zinc. That is, the flow is due to the *difference of potential* of the plates of the cell, and this difference of potential, to which the current is due, is the measure of the **electromotive force** of the cell. Generally defined, electromotive force* is whatever causes

* The term *force* is used from analogy to mechanical force, which causes motion of *matter*, but it does not imply that electromotive force is a force in the dynamical sense of the term.

motion of electricity. Difference of potential is an electromotive force, but it is not the only one. Just as the difference of potential energy, due to difference of level in a liquid, produces flow of the liquid, so difference of potential causes flow of electricity; but a flow of liquid can be produced by other means than by the aid of difference of level, for example, by the use of a force pump; so with electricity a current may be produced without being primarily due to difference of potential. Now, a voltaic cell of any kind is practically an electricity pump, the seat of its electromotive force lying in the unexplained attraction of zinc (or other electropositive metal) for oxygen; and when the circuit of the cell is complete this electromotive force drives a current of electricity round it, but if the circuit is incomplete, then the cell works until a difference of potential is set up at its poles which is equivalent to the electromotive force to which it is due. Hence, when the circuit of a cell is *open*, that is, when its poles are not connected by a conductor, the difference of potential between the poles measures the electromotive force of the cell; but if the



a Fig. 194. *b*

circuit is closed, then the electromotive force is spent in driving the current round the circuit and cannot therefore, set up a difference of potential equal to itself between the poles of the cell. The action of the cell will probably be more easily understood by considering the following hydrostatic analogy. Let A B C D (Fig. 194) represent a circuit of pipes filled with water, and having a stop-cock

at T, and a small paddle-wheel at W. By rotating the paddle-wheel the water can, when T is open, be driven round the circuit in the direction C D A B; but if T is closed, then no current can be produced, but the water is driven into the tube D A, until the pressure due to the difference of level in the tubes D A and C B is sufficient to balance the force exerted by the driving wheel at W. The left-hand drawing in Fig. 194 indicates this case, and the difference of level, E, is a measure of the driving force of the paddle wheel. When, however, the cock T is opened and a current established in the circuit, the levels in the tubes D A and C B change in the way shown in the right-hand drawing of the figure, and *the difference of level* in the vertical tubes is now considerably reduced. The work done by the paddle wheel is now spent in driving the current of water round the circuit, and the difference of the levels at A and B (e) is a measure of that portion of the driving force of the wheel which is spent in driving the current through the tube A B. Now this hydrostatic arrangement roughly illustrates the action of a voltaic cell—the pressure exerted by the paddle wheel at W corresponds to the electromotive force of the cell; and if A B represent the external portion of the circuit, the difference of levels at A and B represents the difference of potential at the poles of the cell. Hence, we see that when the circuit is open and no current flows, the potential difference at the poles is a measure of the electromotive force of the cell; but when the circuit is closed, it is only a measure of that portion of the electromotive force which is spent in driving the current through the external portion of the circuit. Further, the work done in driving the paddle represents the chemical work done in the cell, and the energy of the current of water which is equivalent to the work done on the wheel corresponds to the energy of the electrical current resulting from the chemical work done in the cell. If the tubes A B C D be filled with small shot, the analogy to an electric circuit will be more complete, for the shot may be taken to represent the molecules of the conductors composing the circuit, and the resistance which they offer to the current of water corresponds to the **electrical resistance** which even the best conductors offer to the passage of

electricity through their substance. Moreover, just as in one case the energy of the stream of water is spent in doing work against the friction of the shot, so, in the case of an electric circuit, the energy of the current is expended in doing work against molecular forces, and in both cases transformation of energy takes place and the energy of the current is dissipated as heat in the circuit.

Considering the question of electrical resistance introduced above, it is evident that from its assumed nature the resistance of a given conductor will vary with the substance of the conductor, and also with its dimensions. Thus, similar conductors of different material will have different resistances, and for a given material the resistance will vary with the length of substance which the current has to traverse, and with the area of cross section of the path along which it travels. The greater the length traversed the greater the resistance offered to the current, but the greater the cross section of the conductor the smaller the resistance, for if the hydrostatic analogy holds, the resistance will be less for a wide than for a narrow path. Now, an essential part of every circuit is the cell itself, and from what has been said it is evident that the resistance of the cell is that offered by the layer of liquid or liquids between the two plates, and must, therefore, depend upon these plates and their distance apart—the larger the plates and the nearer they are together the smaller will be the resistance. The arrangement of the plates in Figs. 184, 186 will now be better understood—the doubling of the carbon plate in Fig. 184 and of the zinc plates in Fig. 186 not only adds to the compactness of the cell, but also by increasing the size of the plates diminishes the resistance in it without unduly increasing its size.

113. Batteries of Cells. In practice cells are seldom used singly, but usually in *batteries* of two or three, or more, suitably connected. Fig. 195 shows a battery of four Bunsen cells arranged *in series*; that is, commencing with the end cell on the left hand, the negative pole is connected to the positive pole of the next cell, and the negative pole of that cell is in turn connected to the positive of the third, and so on.

The *poles of the battery* are the free poles of the *end* cells—Fig. 196 shows a similar arrangement of Leclanché cells with its circuit closed by a wire joining its poles.

In this series arrangement the electromotive force and

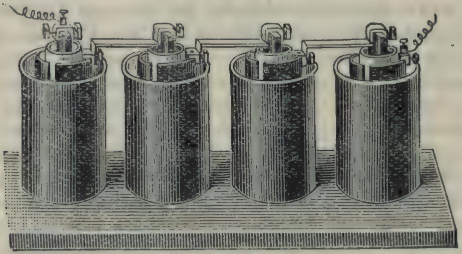


Fig. 195.

the resistance *of the battery* are proportional to the number of cells employed. For example, if the battery be made up of five similar cells, then the electromotive force of the battery is five times that of one cell, and its resistance is

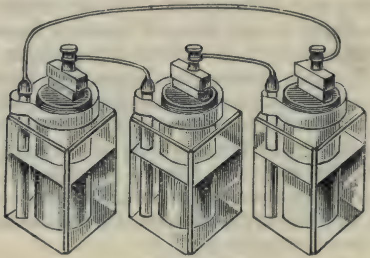


Fig. 196.

also five times the resistance of a single cell. If the cells are not similar, then the electromotive force of the arrangement is the sum of the electromotive forces of the constituent cells, and, similarly, the resistance is the sum of the individual resistance of the cells.

Another arrangement frequently adopted is that shown in Fig. 197. The four cells shown are said to be arranged "*in parallel*" or "*for quantity*"—all the positive poles are connected together and all the negative poles separately, and the four cells are thus practically converted into one cell four times the size of a single cell. Hence the electromotive force of the combined cells is the same as that of one, but the resistance of the arrangement is only one-fourth that of a single cell. Since in this arrangement all similar poles are connected together and practically made one, *any* two unlike poles may be taken as the poles of the battery.

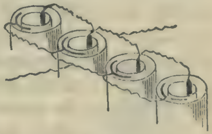


Fig. 197.

114. Local Action. There is one point of some practical importance in the use of cells which should here be noticed. It is well known that ordinary commercial zinc is readily dissolved by dilute sulphuric acid with formation of zinc sulphate, whereas *pure* zinc is only very slightly attacked. Now the zinc plates used in cells are always made of ordinary commercial zinc, and would, therefore, be quickly used up if some precaution were not adopted to prevent their dissolving in the acid when the cell is not giving a current. It is impracticable to use pure zinc plates, but it is found that by amalgamating * the ordinary plates with mercury, they are protected from the action of the acid, except when the cell is giving a current. The chemical action that goes on in a battery with unamalgamated zincs when the circuit is open and no current flows is called *local action*, and its energy is spent in maintaining a multitude of small currents. These circulate between portions of the impure zinc plates, which differ from each other, either chemically or physically, to such an extent as to act as the plates of minute cells. The energy resulting from the solution of the zinc in the acid is thus wasted

* Zinc plates are readily amalgamated by rubbing them alternately with dilute sulphuric acid and mercury. A pad of wash-leather mounted on a handle may be conveniently adopted for this purpose. Very little mercury shoul l be used.

in local work, and it is to prevent this waste of energy that we resort to amalgamation.

115. Voltaic Pile and Crown of Cups. In investigating the action of voltaic cells, Volta constructed an interesting apparatus known as the *Voltaic pile* or the *dry pile.* It usually takes some such form as that shown in Fig. 198, and is made by arranging alternately in a pile zinc and copper discs, separated by discs of felt or flannel soaked in brine; each couple acts as a small cell, and if arranged so that the zinc of one couple is in contact with the copper of the next, the pile acts as a battery made up of a large number of separate cells. Such a pile may have a considerable electromotive force, equal to the sum of the individual electromotive forces of the component cells, but its resistance will also be large, so that it cannot give a strong current. Another arrangement devised by Volta was his *Crown of Cups,*—this was merely a battery made up of a large number of the simple zinc and copper cells described above.

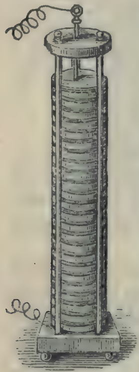


Fig. 198.

116. Essentials of a Good Cell. Having now described the voltaic cell in some detail, it may be instructive to consider what are the essentials of a good cell, and to notice how far the cells described fulfil the necessary conditions. A good voltaic cell should meet the following requirements:—

(1) Its electromotive force should be high and constant.

(2) Its resistance should be small.

(3) It should be free from polarisation.

(4) It should give a constant current for a considerable time, and should therefore have a good supply of suitable working materials, which should not be rapidly exhaustible.

(5) No chemical action should go on in it except when the current is passing.

(6) It should be convenient and economical in use.

None of the cells described above fulfil all these conditions,

but each of them fulfils some of them; and in order to choose the proper cell for use under given conditions, it is necessary to know their several characteristics. Bunsen and Grove cells fulfil conditions (1) and (2), and consequently give a strong current, but unless they are of large size the current will not remain constant for more than a few hours, They emit a small quantity of corrosive and suffocating fumes, and should, therefore, not be used where the fumes may give rise to annoyance, or attack the metal work of instruments or fittings. Daniell's cell has a constant, but not very high electromotive force (rather more than half that of a Bunsen or Grove), and its resistance is rather high, but when in good order it will give a constant current of medium strength for a very long time. It is convenient to use, and emits no fumes whatever. The electromotive force of a Leclanché cell is somewhat higher than that of a Daniell, and its resistance is also higher, but its current is by no means constant. As already explained, it becomes polarised when in continued use, but as it recovers gradually and, unlike the other cells, may always be kept made up ready for use, it is one of the most convenient cells when a fairly good current is wanted for only a few minutes at a time, for example, in ringing an electric bell.

Volta's simple cell is of theoretical interest only, and Poggendorff's bichromate cell is not greatly used, because, although its electromotive force is high and its resistance low, its current, though strong at first, quickly diminishes almost to zero. Care also has to be taken always to remove the zinc out of the liquid when the cell is not in use, but Fuller's two-fluid form of the cell has been much used in the Post Office.

imp. **117. Volta's Theory. Ayrton and Perry's Experiment.** Volta believed that the main cause of the working of a cell was the contact of the metals. When zinc touches copper there is a flow of positive electricity from the copper to the zinc which causes the zinc to be of higher potential. Experiments were designed by Volta and others to test this, but most of these earlier experiments are capable of other explanations and are therefore inconclusive. A more recent experiment by Ayrton and

Perry is comparatively free from objection. In Fig. 199, G represents diagrammatically a quadrant electrometer, whose two pairs of quadrants are connected to flat horizontal brass plates A and B. L and M are two other plates parallel to and at the same distance from A and

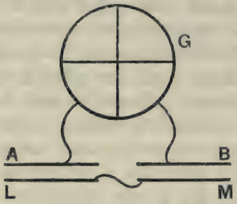


Fig. 199.

B, and connected to one another by a wire. It was argued that if L and M be at different potentials, A and B will have a proportional difference of potential of the same kind. The experiment clearly showed a P.D., when zinc and copper were used, of about ·75 volt, the zinc being positive.

imp. **118. Contact Theory. Explanation.** This result appears to indicate that zinc and copper have a contact P.D. of nearly a volt. Modern investigation, however, goes far to show that this theory must be abandoned: contact of dissimilar metals or other substances does give rise to electrical separation, but not to *the* separation we have to deal with in a voltaic cell. For metals the difference of potential established by contact may be exceedingly minute compared with the electromotive force of a cell, and has been held to have little or nothing to do with voltaic currents, in the ordinary meaning of the term. The most probable explanation of the action of a voltaic cell is given in Article 109, and an explanation of the experiment just described can be given in an exactly similar way. Consider a piece of zinc (Fig. 200) surrounded by air —here, just as when immersed in dilute acid, it attracts the neighbouring oxygen atoms, but in the liquid these atoms *while dissociated,* are free to obey this attraction, whereas

in air, *where there is no dissociation*, they are not free to do so.

The molecules of free oxygen are uncharged, but each molecule may be supposed to be resolvable into two oppositely charged atoms, one atom charged positively and the other negatively. Under the influence of the attraction of the zinc the layer of oxygen molecules adjacent to it may become polarised, forming an inner layer of negatively charged atoms in contact with the zinc and an outer layer of positively charged atoms surrounding this inner layer.

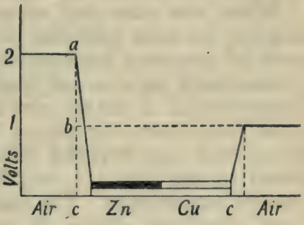


Fig. 200.

Equilibrium will be attained when the attraction between the zinc and the adjacent layer of negatively charged oxygen atoms equals the attraction between the two layers of atoms, and the work done in effecting the polarisation of the oxygen molecules is equal to the energy of the charged condenser constituted by the two layers of oppositely charged atoms. Whether the inner layer of oxygen atoms actually combines with the zinc and so transfers its charge to the metal or simply remains in close contact with it the zinc is practically negatively charged, and a difference of potential will be set up between it, or the layer in contact with it, and the adjacent layer of positively charged atoms. The magnitude of this difference of potential will depend upon the force of attraction between the zinc and the oxygen atoms. The relation between these two quantities may perhaps be expressed by means of the formula for the attracted disc electrometer plates. If v be the difference of potential set up, δ the distance between the two layers of atoms, and f the force of attraction per unit area of the zinc on the inner oxygen layer, then we have $v = \delta\sqrt{8\pi f}$. In the case of the copper the details would be the same as for the zinc, but as the chemical attraction between copper and oxygen is less than between zinc and

oxygen the difference of potential set up between the two layers of oxygen atoms would be less in the case of the copper. In the case of zinc this difference is about 1·8 volts and in the case of copper about ·8 volt.

When the zinc and copper are put in contact they acquire the same potential but there will now be a difference of potential between the outer layer of oxygen in contact with the zinc and that in contact with the copper. The potential of the air just above the zinc will be higher than that above the copper by a quantity of the order of one volt; and it is this difference which is actually measured by the plates A and B (Fig. 199) in Ayrton and Perry's experiment. The distribution of potential difference, described above, is indicated graphically in the diagram of Fig. 200. In this diagram the metals are in contact, and the difference of potential between the films of oxygen on the two metals is represented by $a\,b$ and is here shown to be about one volt.

The short distances at c, c in the figure represent the distance between the two layers of oxygen atoms which surround the metals. As their distance is of molecular dimensions it cannot be represented to scale, but is shown diagrammatically on a much exaggerated scale in the figure. In the case of a voltaic cell the intervening medium is a conductor and not an insulator; and therefore instead of a difference of potential being produced in it a current is set up through it.

This experiment of Volta's is thus merely an illustration of the action of a voltaic cell in which the medium which surrounds the plates is an insulator, and not an *electrolyte*, or liquid which conducts by virtue of the dissociation which goes on in it.

The distribution of potential in a simple cell such as Volta's may be shown diagrammatically by a diagram similar to that given in Fig. 200. The diagram of Fig. 201 gives the distribution for the case when the two plates of a simple Volta cell are placed in the dilute acid.

In this case there is no difference of potential between the zinc and copper in the air. The differences of potential

between the air and the metal, and between the acid and the metal, are shown to be the same for each metal, because in each case the magnitude of the differences depends upon the same chemical affinity. This explains

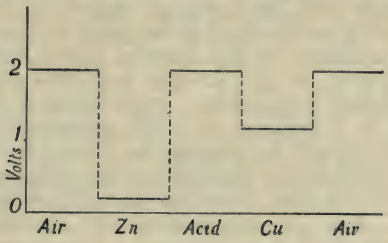


Fig. 201.

why there is no slope of potential in the intervening air. If, however, a copper wire is attached to each plate of the cell, the distribution changes as shown in Fig. 202, and a difference of potential is set up between the copper wires

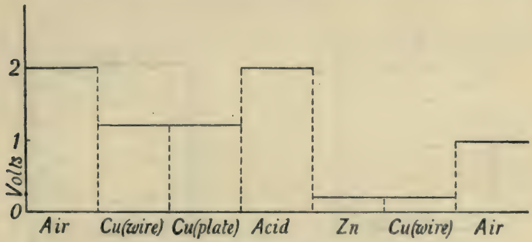


Fig. 202.

attached to the two plates of the cell. This figure is really a combination of Fig. 200 and Fig. 201.

The essential point of difference between the two theories of the voltaic cell—Volta's contact theory and the chemical action theory—is the explanation of the source of the

electromotive force of the cell. The contact theory states that the electromotive force results from metallic contact between the two metallic elements of the cell. The chemical theory states that it results in the way detailed above from the chemical action going on in the cell. The evidence of Volta's experiment gave strong support to the contact theory, until it was shown that the result of the experiment could be explained by the chemical action theory. The chemical theory is also strongly supported by the fact that it supplies a satisfactory explanation of the source of energy in a working cell. On the contact theory it would be necessary to suppose the energy to be derived from the heat of the elements of the cell, and not, as is obviously the case, from the chemical reactions going on in the cell. In thermo-electricity, considered in a later Chapter, we have a true case of electromotive force resulting from contact between dissimilar metals, but the electromotive forces developed are extremely small compared with those due to chemical action in a cell.

119. Path of Energy in the Circuit of a Voltaic Cell. In speaking of the circuit of a voltaic cell and of the current in this circuit, it is convenient to speak as if the

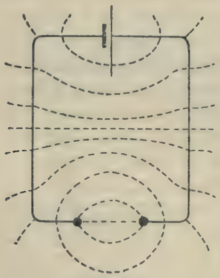


Fig. 203.

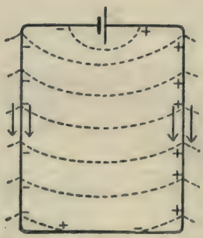


Fig. 204.

electrical actions going on were confined to the conducting circuit. It will, however, be understood from what has been said in discussing the energy in an electrical field,

that the ether medium surrounding the circuit is really the vehicle of the electrical energy liberated by chemical action in the cell. If the circuit of the cell is not closed the terminals of the cell are oppositely charged, and tubes of force pass, as shown in Fig. 203, from one terminal to the other, and each tube of force is, as explained in Art. 33, the seat of a definite quantity of energy. When the circuit is closed these tubes of force travel outwards from the cell as indicated in Fig. 204. As the ends of each tube approach each other along the conducting circuit, the energy in the tube decreases by transformation into heat in the circuit. The energy from the cell thus passes to any point in the circuit through the ether, and the conducting circuit through which the current is usually said to pass determines the direction of transmission of the energy, and is the seat of the transformation of this energy into heat. The resistance of the circuit must from this point of view be associated with the rate of dissipation of energy in the circuit, and may be defined, as will be seen later, as the ratio of this rate of dissipation to the square of the current strength.

CHAPTER XX.

GENERAL EFFECTS OF CURRENTS.

120. Introductory. In the preceding articles we have spoken of a current of electricity flowing round the circuit of a voltaic cell, but we have not stated how this current manifests itself, nor how its existence can be detected. In the cell itself, it is true, chemical action indicates the passage of a current; but as it is necessary to be able to detect and even to measure the current in any conductor, we must consider whether the passage of a current of electricity through that conductor has any effects of which we may take advantage for this purpose.

There are two broad classes of conductors to be considered. The first consists of those conductors which conduct electricity like they conduct heat, with little or no permanent effect on their physical or chemical state. The second comprises *electrolytic conductors* or *electrolytes*, in which the passage of current is accompanied by chemical action.

Of the first class, metallic conductors afford typical examples, and, as we shall have to deal mostly with conductors of this kind, we shall here consider a few experiments illustrating the effects attendant on the passage of a current through metal wires.

121. Heating Effect. Let the poles of a Bunsen cell be joined by a short piece of iron wire about half a millimetre in diameter. When the connection is made good, and the current flows through the wire, it will be found, on holding it between the fingers, that it gradually becomes hot, and before long may become too hot to touch. This proves that one of the effects of passing the current through the wire is to heat it; and by varying the experiment a little we may learn something more of this **heating effect**

of the currents. Let the piece of wire joining the poles of the cell be replaced by a much thinner piece of the same length and composed of the same material. It will now be found that the heating effect is much more evident, for the wire quickly becomes red hot. This seems to point to the fact that the heating effect varies with the resistance which the conductor offers to the passage of the current, for the only difference between the two wires is one of diameter; and as the thin wire offers a much narrower path to the current than the thick one, its electrical resistance, as explained above, must necessarily be greater. Again, if thin wires of the same dimensions but of different material be employed, it will be found that the heating effect is different for each. For example, let short thin wires of copper, iron, brass, and platinum be tried: it will be found that, with the exception of the copper wire, all the wires quickly become white hot. The copper wire may slowly attain a dull red heat, and of the other three the platinum heats most quickly, the iron next, and the brass at nearly the same rate. In each of these experiments approximately the same current is used, but the same results may be exhibited in a more satisfactory way as follows. Take short pieces of thick copper wire, thin copper wire, platinum wire, iron wire, and brass wire, and fasten them end to end, so as to form one compound conductor. This is most conveniently done by the help of small binding screws of the form shown in Fig. 205. If the poles of a battery of three or four Bunsen cells be joined by this conductor, then precisely the same current passes through each wire composing it, but the heating effect in each will be very different. The platinum, iron, and brass wires will, at slightly different rates, become white hot, the thin copper wire will be nearly red hot, but the thick copper wire will be only slightly heated, and the thick brass binding screws may not be perceptibly heated at all. From these experiments we may deduce the following general results:—

Fig. 205.

(*a*) The passage of a current through a conductor heats it.

(*b*) For conductors of the same material the amount of heat developed in a conductor by a given current varies with the resistance of the conductor. The greater the resistance, the greater the amount of heat developed, but further experiment is necessary to determine the quantitative nature of the relation.

(*c*) The amounts of heat developed by a given current in similar conductors of different material are different. This points to the fact that electrical resistance varies with the material of the conductor, and that the amount of heat developed in it will be small or great, according as it is composed of good or bad conducting material. Looked at in this way, this result is practically the same as the preceding one—here the difference of resistance is due to difference of material, in (*b*) it is due to difference in dimensions. As a corollary to this result, it may be noticed that copper appears to be a good conducting material, while brass, iron, and platinum, though readily conducting the current, offer considerable resistance to its passage.

(*d*) When a conductor is made up of several different parts of different conducting powers, the heat developed in any portion of it depends upon the resistance offered *by that portion* to the passage of the current.

If the experiments described above be repeated with, say, Daniell or Leclanché cells to supply the current, the same general results will be obtained, but the heating effects noticed will be very slight. In these cases the current used will be much weaker than in the first case, and we may therefore infer that—

(*e*) The heat developed in a given conductor varies with the current used. The stronger the current, the greater the amount of heat developed, but the quantitative nature of this relation has yet to be found.

It is probable that the passage of a current through a conductor affects most of the physical properties of its material, but the heating effect is the only one we need consider at present. Its importance will be realised from the following considerations. Imagine a Bunsen cell working, with its poles joined by a piece of wire. So long as the cell is working, chemical energy is being used up in

the cell, and a question arises as to what becomes of this energy. The answer is, that it produces an electrical current which develops heat in the circuit, the amount of heat energy evolved being equivalent to the chemical energy used up in the cell, and thus furnishing a measure of the energy of the current. In this case, the circuit is made up of the piece of wire and the cell itself: part of the heat is developed in the wire and the remainder in the cell, for every conductor which offers resistance to the passage of the current must become heated when the current passes through it, the amount of heat developed in it being equivalent to the work done by the cell in driving the current through it.

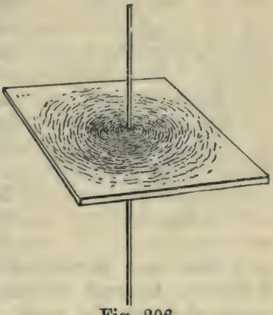


Fig. 206.

122. Magnetic Effects. The passage of a current through a conductor is attended by another effect which is even more noteworthy than the development of heat. It is found by experiment that a conductor which is carrying a current has a magnetic field surrounding it, and consequently currents are capable of influencing magnets, and may be employed for the magnetisation of magnetic substances.

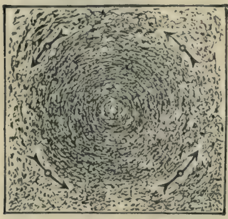


Fig. 207.

In order to exhibit the magnetic field of a current, let a hole be bored in a piece of glass or cardboard, and let a wire carrying a strong current be fitted through it, as in Fig. 206, with its length at right angles to the horizontal surface of the glass. Then, on sprinkling iron filings over the glass and gently tapping, the filings at once arrange themselves in curves of force, taking the form of concentric circles round the wire as their common centre. This

arrangement is shown in Fig. 207, where the current is supposed to come up through the centre of the field, in a direction at right angles to the plane of the paper. The direction of the magnetic force at any point in the field due to the current is thus seen to be at right angles to the plane determined by the direction of the wire and the point considered.

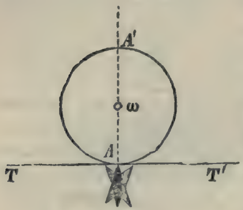


Fig. 208.

Thus, if w (Fig. 208) denote the section of a wire running vertically through the paper, the direction of the magnetic force at A due to a current down through the wire is along T T′, the tangent to the curve of force A A′ which passes through the point A. That is, the direction of the force is at right angles to the vertical plane represented by the trace A w A′ passing through the wire and the point A. If the current in the wire flows down into the paper, the experiment shows that the magnetic force at A is along A T, but if the current flows upwards, out of

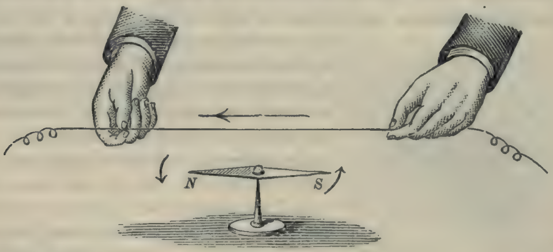


Fig. 209.

the paper, it is along A T′. Hence, if a magnetic needle be placed, as in the figure, with north pole at A, this pole is in the first case directed to the left and in the second to the right.

This effect is more strikingly shown by an experiment due to Œrstedt. A wire carrying a current is held, as in Fig. 209, over a magnetic needle with its length parallel to the axis of the needle. The poles of the needle at once experience the magnetic force due to the current, and the needle, acted on by a horizontal couple, is deflected out of the magnetic meridian until it reaches a position of equilibrium, where the moment of the couple due to the magnetic field of the current is balanced by the moment of the couple due to the magnetic field of the earth. If the direction of the current be from north to south, then the north pole of the needle is urged eastwards, but if the current flow from south to north, the north pole is urged towards the west. It the current pass below the needle, then a consideration of the conditions shows that these effects are reversed. As it is somewhat puzzling to deduce or remember these results, the following rule due to Ampère, and commonly called *Ampère's rule*, will be found very useful: *To determine the direction in which the* **north pole** *of a needle will be urged by the magnetic action of a current, imagine a man swimming* **with** *the current with his face turned so as to look at the north pole considered, then that pole will in all cases be deflected towards his* **left** *hand.*

Another convenient form of this rule is this. If one look along a conductor, *in the direction in which the current flows*, then the direction in which a *north* pole would be urged round the circular lines of force in the field is the same as that of the hands of a watch.

To consider the magnetising action of a current, let a wire insulated by a gutta percha covering be coiled round a bar of iron or steel in the way shown in Fig. 210, and let a current be passed through the wire. Immediately the current is established the bar becomes a magnet while the current passes. If made of steel the bar remains permanently magnetised after the current is stopped, but if made of iron the magnetisation is more or less temporary according to the quality of the iron, so that with a soft iron core the magnetisation comes and goes with the current.

This method is the most satisfactory for the magnetisation of steel bars as permanent magnets, and with a strong current much more powerful magnets can be obtained by this method than by any of the methods described in Art. 73.

With a given current a much more powerful magnet can be obtained while the current passes by using a soft iron core instead of a steel core; hence, when very strong magnets are required, soft iron bars wound with coils of wire suitable for carrying strong currents are used. These

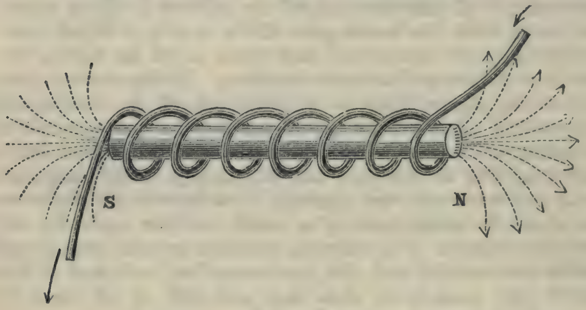


Fig. 210.

magnets are called *electromagnets*, and are greatly used in practical electrical work. Small electromagnets are also much used in automatic appliances, where rapid magnetisation and demagnetisation are produced by making and breaking the circuit round the coils.

The polarity of the bar in Fig. 210 is readily determined by the application of Ampère's rule. The molecules of the bar, on experiencing the magnetic field due to the current, tend to set themselves parallel to the lines of force inside the coil, and their north poles will all lie to the left of a person swimming round the coil and looking inwards at the bar, that is, the north pole of the bar will be to his left and the south pole to his right. This result is more easily remembered in the following equivalent form. When

one looks, end on, at the south pole of the bar, the current in the coil will be found to be flowing round the bar in the same direction as the hands of a watch.

In what has been said above we have spoken of the direction of "the lines of force inside the coil," and it may be well to consider here the general nature of the magnetic field due to a coil of wire carrying a current. At any point in the wire the lines of force run round it in concentric circles, but from the arrangement of the coil it is evident that in the interior the general direction of the resultant magnetic force is parallel to the axis of the coil; similarly, in the region outside the coil the line of action of the resultant force is approximately parallel to the axis, but in a direction opposed to that in the interior. In the spaces between the turns of the coil, however, the magnetic field due to one turn is more or less neutralised by the field due to the adjacent turns on either side. Thus, in Fig. 211, which represents a section of a few turns of the coil, the general direction of the resultant force in the interior is along *ab*, and in the region outside the coil along *c d*, but at *e* and *f*, between the turns, the field due to one turn is neutralised by that due to the adjacent turn. Hence, if a wire be wound in a coil on a sheet of cardboard in the way shown in Fig. 212, the magnetic field produced when a current is passed

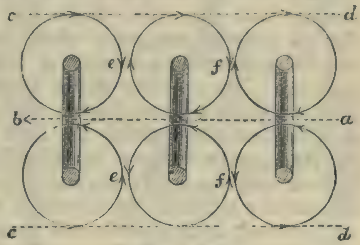


Fig. 211.

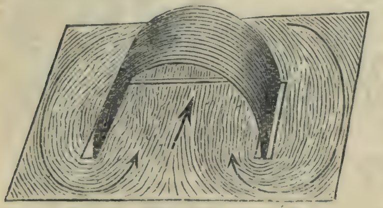


Fig. 212.

through the coil may be exhibited by sprinkling iron filings on the card, and in accordance with what has been said above the lines of force will have the distribution shown in the figure. If the magnetic field shown here be compared with that of Fig. 122, it will be found that the two fields are similar, and that, as far as external magnetic effect is concerned, a conducting helix carrying a current is practically equivalent to a magnet. The polarity of the helix may be determined by Ampère's rule; remembering that the north-seeking pole will be that from which the lines of force* diverge, it is only necessary to know the direction of the current in the coil in order to determine the poles. Thus in Fig. 210, if the arrows indicate the direction of the current, the lines of force will by Ampère's rule have the direction indicated, and the polarity of the needle will be as shown in the figure. From this it will be seen that the rule for determining the polarity of a bar magnetised by a current flowing round it in a coil applies also to the polarity of the coil itself.

These results are strikingly illustrated by an arrangement known as the *mounted solenoid.* This is a coil of wire arranged as shown in Fig. 213 so as to be capable of free suspension in a horizontal plane by pivoting the terminals of the coil in the small cups *m m* (Fig. 213). These cups can be filled with mercury, and a current passed through the coil by connecting these cups by means of the wires, *w w*, to the poles of a battery. Immediately the current passes, the coil acquires magnetic properties, and being free to move, at once sets itself in the magnetic meridian. Its poles are attracted and repelled by the poles of an ordinary

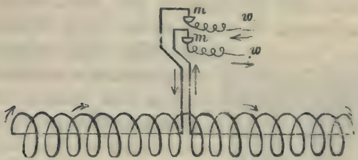


Fig. 213.

* The direction of a line of magnetic force being that in which a north pole would be urged.

magnet, and in every respect the coil behaves as a freely suspended bar magnet would. The floating battery of Fig. 214 also illustrates these effects in a simple and striking manner. It consists of zinc and copper plates fixed through a piece of cork and carrying a short coil of copper wire. On floating the arrangement in dilute acid, a current passes through the coil and endows it with magnetic properties. If the poles of a magnet be presented to one of the poles of the coil, the resulting attraction or repulsion is at once made evident by the motion of the coil.

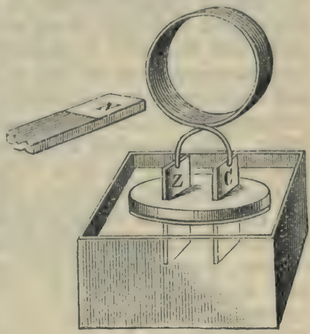


Fig. 214.

imp. **123. Chemical Effects: Electrolysis.** Another important effect of the passage of a current through a conductor has still to be noticed. This is the *chemical effect* which is produced when a current is passed through an electrolyte. As already noticed in the theory of the voltaic cell, the electrical current is the result of chemical action going on in the cell; conversely, if an electrical current be passed through a conductor capable of electrolytic conduction, then the passage of this current is attended with chemical action. Thus, if the poles of a Bunsen's cell be connected to two pieces of platinum foil immersed in water acidulated with sulphuric acid, the passage of the current through the liquid decomposes it into oxygen and hydrogen. These gases are liberated separately at the surfaces of the pieces of platinum foil—the oxygen coming off from that connected with the positive pole of the cell, and the hydrogen from the other. Remembering that the dissociated atoms of oxygen and hydrogen in the liquid are supposed to be charged respectively with negative and positive electricity, this result is just what should be expected; for the foil connected with the positive pole of

the cell, being charged positively relative to that connected to the negative pole, at once attracts up to its surface the dissociated oxygen atoms in its neighbourhood, and the dissociated hydrogen atoms seek the other piece. Thus the gases are liberated separately at the two platinum terminals—oxygen at the one and hydrogen at the other—and the current is maintained through the liquid by the stream of dissociated atoms passing from one piece to the other. The chemical action in this case is very similar to that which goes on in a voltaic cell, but it is essential to distinguish clearly between the two cases. In the voltaic cell, the stream of dissociated atoms is set up by a difference between the chemical attractions of the two cell plates for oxygen; but in the case just considered the platinum plates used are exactly similar and have no chemical attraction for oxygen at all, so that it is only on electrifying them to different extents, by putting them in contact with the poles of a cell, that the necessary difference of attraction for the dissociated atoms is established, and a current thereby set up. Further, since in the voltaic cell the motion of the dissociated oxygen atoms is due to the greater attraction of one of the plates, these atoms on reaching that plate combine with it, and the chemical action which maintains the current goes on. In the case here considered, however, neither the oxygen nor hydrogen atoms combine with the plates to which they are attracted, but their charges, which they give up before liberation, neutralise the charges on the plates at the same rate as these charges are supplied by the cell to which the plates are connected. In this way a current is maintained round the circuit, and the chemical decomposition produced is the result, and not the cause, of its existence.

The chemical decomposition resulting from the passage of a current through an electrolyte is called *electrolysis*. The terminals at which the current enters and leaves the liquid are known as *electrodes*—the positive electrode, or *anode*, being that connected with the positive pole of the cell, and by which the current enters the liquid; and the negative electrode, or *cathode*, that connected to the negative pole of the cell, and by which the current leaves the liquid. The

products of electrolysis which appear at these electrodes are sometimes called *ions*. An apparatus in which the electrolysis of a given electrolyte may be suitably effected, and by which one or both of the products of electrolysis may be retained and measured, is called a *voltameter*. Fig. 215 shows one form of voltameter for the electrolysis of water.

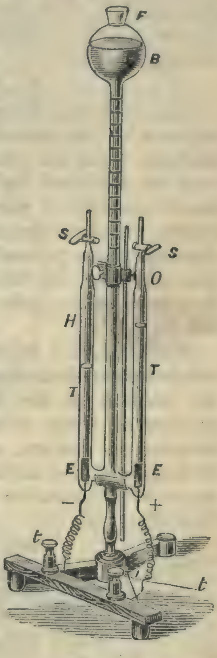


Fig. 215.

The taps at S S being open, acidulated water is poured into the apparatus by the funnel F until the vertical tubes T T are filled. The taps are then closed, and on joining the terminals *t t* to the poles of a battery, a current flows between the platinum-foil electrodes E E, which are attached to platinum wires fused into the bottom of the vertical tubes T T. The gas liberated at each electrode rises in the tube in which the electrode is fixed, and collects in the upper part of the tube, displacing the liquid up into the bulb B. By this means oxygen is collected in one tube, and hydrogen in the other, and it will then be seen that the volume of the hydrogen is approximately twice that of the oxygen. The identity of the gases may be verified by opening the taps S S, and testing them in the ordinary way by means of a lighted taper and a glowing splinter of wood—the hydrogen burns with a pale blue flame, and the oxygen ignites the glowing splinter.

Many other substances are decomposed by the passage of a current through them. For example, hydrochloric acid is at once decomposed into hydrogen and chlorine, the former being liberated at the negative pole and the latter at the positive pole. A solution of common salt, or

sodium chloride ($NaCl$), is first decomposed into sodium (Na) and chlorine (Cl), thus:—

$$NaCl = Na + Cl.$$

The chlorine is liberated at the anode, or positive electrode; but the sodium does not appear at the cathode, for immediately it is set free it reacts on the water present in the solution, forming sodium hydrate, and liberating hydrogen, thus:—

$$Na_2 + 2H_2O = 2NaOH + H_2.$$

The hydrogen liberated by this *secondary* reaction then appears at the cathode as one of the products of the electrolysis, while the presence of the alkaline sodium hydrate may be exhibited by means of litmus solution. For, if a small quantity of reddened litmus be added to the solution before the electrolysis begins, it will be found that, as the decomposition proceeds, the colour of the solution gradually becomes blue, the blue colour first appearing in the neighbourhood of the cathode, but quickly extending throughout the liquid.

Similarly, if a solution of sodium sulphate (Na_2SO_4) be subjected to electrolysis, the first products of decomposition are sodium at the cathode and SO_4 at the anode; but neither of these substances are allowed to appear—the sodium, as before, attacking the water and liberating hydrogen at the cathode, while the SO_4 also attacks the water with formation of sulphuric acid (H_2SO_4) and liberation of oxygen:—

$$SO_4 + H_2O = H_2SO_4 + O.$$

Thus the final results of electrolysis will be hydrogen and sodium hydrate at the cathode, and oxygen and sulphuric acid at the anode. The gases are liberated, and the hydrate and acid slowly diffuse from the electrodes through the liquid. If the electrolysis be performed in the U-tube (Fig. 216), the solution coloured with litmus, the liquid in the cathode limb of the tube gradually becomes blue, owing to the formation of the sodium hydrate, while the liquid in the other limb slowly becomes red, owing to the presence of the free sulphuric acid.

In the same way a solution of caustic soda or caustic

potash may be decomposed by electrolysis, yielding oxygen at one electrode and hydrogen at the other. Potassium iodide is also readily decomposed, yielding free iodine at the anode and hydrogen at the cathode, and this reaction furnishes a very delicate test for the existence of weak currents. The free iodine liberated at the anode turns starch-paste blue, and if the terminals of a circuit be placed on starch iodide paper, the point at which the positive terminal rests is at once marked by a blue dot.

Solutions of copper sulphate and silver nitrate are electrolytes of some practical importance. Let two pieces of

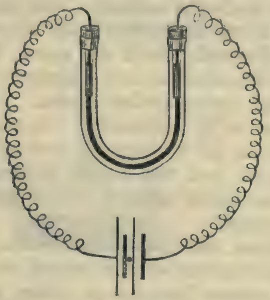


Fig. 216.

platinum foil, dipping into a beaker containing a solution of copper sulphate ($CuSO_4$), be connected to the poles of a battery. When the current passes the liquid is decomposed into Cu and SO_4. The copper, like all other metals, appears at the cathode *on which it is deposited as a thin film*, but the SO_4, as described above, reacts with the water, producing sulphuric acid and oxygen. If copper plates be used instead of platinum foil, copper is deposited on one plate and the SO_4 attacks the other plate, forming $CuSO_4$; and thus copper is, as it were, carried through the liquid with the current from one plate to the other, and the average concentration of the solution remains unchanged. Similarly, if a solution of silver nitrate ($AgNO_3$) be electrolysed by

a current passing between two silver plates, silver, being a metal, is deposited on the cathode, and the NO_3 set free at the anode dissolves it with re-formation of $AgNO_3$, and thus, while the quantity of silver in the solution remains unchanged, silver is carried with the current from one plate to the other.

These experiments illustrate how electro-plating is effected. For example, suppose it is required to coat a brass or copper spoon with silver, the result might be effected in the following simple way. Take a silver coin or a strip of silver and the spoon, and arrange them as electrodes for the electrolysis of a solution of silver nitrate, the spoon being made the cathode. On passing the current a thin layer of silver is deposited on the spoon, and when this layer is sufficiently thick the spoon may be removed, and its surface burnished so as to present the usual polished appearance of an electro-plated spoon. In practice, however, the process is not so simple; special arrangements have to be made to get a coherent layer of silver that will take a good polish, and the liquid generally subjected to electrolysis is not silver nitrate, but a double cyanide of silver and potassium.

Most electrolytes, like those described above, are liquids, and this is quite what we should expect, for electrolysis can only take place where dissociation exists, and it is difficult to imagine dissociation occurring in anything but a fluid wherein molecular motion is possible. It is not, however, always necessary to make a solution of a salt in order to effect its electrolysis, for many salts, especially chlorides, when fused conduct electrolytically. Thus, when fused, silver, magnesium, and aluminium chlorides are readily decomposed by a current, the pure metal appearing at the cathode. This, in fact, is one important method of obtaining a number of metals in a pure state, and the metal potassium was first discovered by Sir Humphry Davy by subjecting the solid hydrate to electrolysis. The hydrate was allowed to deliquesce slightly, so as to become like a paste; then, on passing a strong current through it, it quickly liquefied, and small globules of potassium appeared round the wire where the current left the hydrate. The

metal here appeared in a free state, because there was not sufficient water present to combine with all of it as it formed.

omit **124. Less Important Effects.** The effect of electric currents in stimulating the nerves of a living body belong rather to Physiology than to Physics.

The *Capillary Electrometer* is an application of the effect of electric currents on the surface tension, or surface energy, of the boundary between mercury and an acid liquid. As where air meets zinc (§ 118), so here a bounding film is formed whose positively charged face is in the acid and negative face on the metal. A certain potential

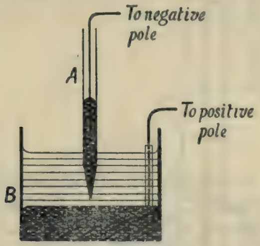


Fig. 217.

difference is produced, say V, and the film can be compared to the simplest kind of condenser (§ 37) and has energy per unit surface proportional to V^2. Now the energy per unit surface is the same as the surface tension T. If T be partly independent of V and partly electrical, as explained above, there must be an equation of the form $T = L + M V^2$.

If a current be made to flow from the acid to the mercury, this obviously increases V by a quantity v, say; hence now $T = L + M (V + v)^2$. So the surface tension increases.

In the Capillary Electrometer the connections indicated (Fig. 217) cause a small current to flow from the mercury in the tube A to the layer of mercury at the bottom of the vessel B through the intervening layer of dilute sulphuric acid. The tube A is drawn out to a fine point

and, under given conditions, the position of the lower end of the column of mercury in the tube depends upon the surface tension at the separating surface between the mercury and the acid. When the mercury in A is connected to the mercury in B and both are therefore at the

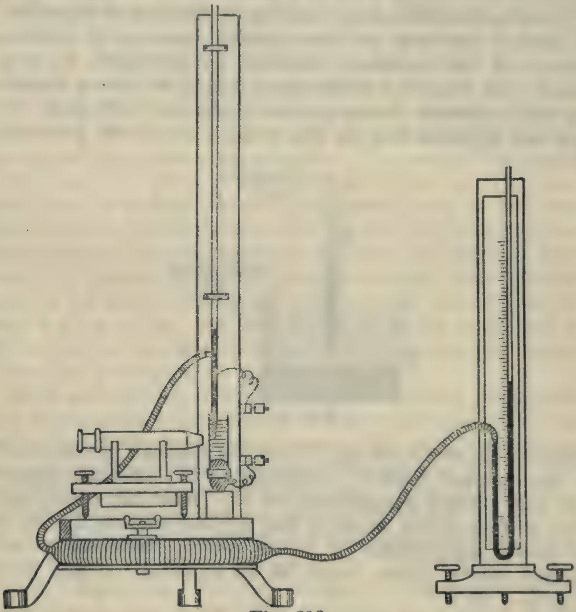


Fig. 218.

same potential, the mercury surface in the lower part of the tube A will have a definite position, which may be marked. If, however, the potential difference indicated in the figure be produced, so that a current passes from B to A, the surface tension at the lower end of the mercury in A increases, and the surface rises in the tube. The extent of this change of position, or the change of pressure

necessary to bring the surface back to the same position, may be taken to indicate the difference of potential between the mercury in A and that in B.

Fig. 218 shows a form of instrument constructed to measure the increase of the pressure on the upper surface of the mercury in the tube necessary to adjust the lower sur-

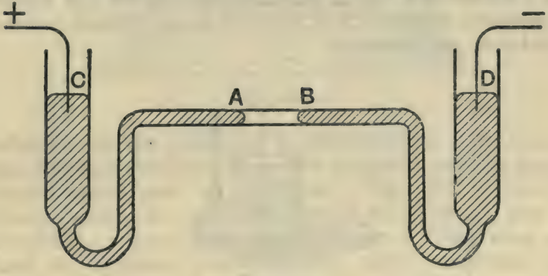


Fig. 219.

face to the position indicating no difference of potential between the two masses of mercury. The connections are so arranged that the lower mass of mercury is at a higher potential than the upper. This causes the lower level of the mercury in the tube to rise and the pressure in the upper part of the tube has to be increased to bring it back to the zero position. The increase of pressure is proportional to the difference of potential, but the instrument should be calibrated for use and cannot be used for a difference of potential greater than about ·9 volt. This form of electrometer is due to Lippmann.

A simple electrometer in which the potential difference is indicated by motion is shown in the figure. A B is a bubble of dilute sulphuric acid, in a capillary tube, between two mercury surfaces. If C be connected to a positive electrode and D to a negative, a small current flows which increases the surface tension at B and diminishes it at A. The bubble therefore moves in direction A B till brought to rest by the difference of mercury level produced at C and D.

125. Galvanoscopes. We have seen above that when a current passes near a magnet it tends to deflect it out of the magnetic meridian, and that, under known conditions, the sense of the deflection of the magnet indicates the direction of the current. Any arrangement in which advantage is taken of this effect to indicate the existence of a current is called a galvanoscope. Thus the simple arrangement shown in Fig. 220 acts as an efficient current indicator or galvanoscope.

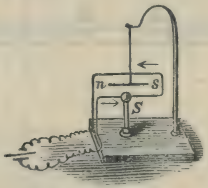


Fig. 220.

The magnet *n s* is freely suspended inside the rectangular loop of wire supported by the stand S, and indicates by its deflection the existence and direction of a current in the wire. It will be noticed that the current passing *under* the needle in one direction and back *over* it in the *opposite* direction tends, in both positions, to deflect the needle in the same direction. The arrangement may be made more effective by coiling the wire a large number of times round the needle, and thus increasing the force with which the current tends to deflect the needle.

With a given current a thin coil of twenty turns is twenty times as effective as a coil of one turn, for there are then practically twenty currents flowing round the needle, and each one has an equal effect in producing deflection. It must however be carefully noticed that this result is true only when we compare the action of a given current in one coil with *an equal* current in the other coil. Fig. 221 shows a galvanoscope arranged with a coil of several turns.

As explained above, the magnetic field due to the earth

opposes the deflection of a magnet by the magnetic action of a current; hence it is evident that if the current be very weak it may not be able to produce any appreciable

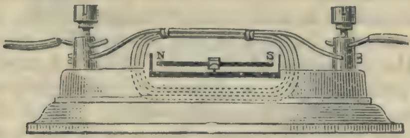


Fig. 221.

deflection of the needle of a galvanoscope. To remedy this defect, a combination of two magnetic needles known as an *astatic pair* is employed. Fig. 222 shows the arrangement: two magnets of equal strength and size are attached to a short piece of aluminium or brass wire, and fixed *in opposed positions* with their magnetic axes parallel. If the magnets are exactly equal and accurately fixed, the combination, when freely suspended, will set in any position, for the couple tending to set one magnet into the magnetic meridian is exactly counterbalanced by the opposite couple acting on the other. In practice, however,

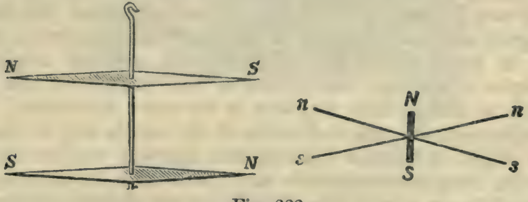


Fig. 222.

it is impossible to secure the exact equilibrium of an astatic pair; for it is difficult to obtain magnets of exactly equal strength, and it is very difficult to fix them accurately parallel to one another. The ordinary astatic needle is therefore imperfectly astatic and when freely suspended in a magnetic field sets approximately at right

angles to the field. A diagrammatic plan of a very imperfectly astatic pair is shown in Fig. 222. The two north poles n, n, act as one compound north pole, and the two south poles as one compound south pole, and the pair sets in the field as if it were a short magnet N S. The more perfectly astatic the pair is the shorter N S becomes, and the less the influence of the field on it. A carefully constructed astatic needle is, however, but slightly subject to controlling action of the earth's field; and, if the coil be arranged so that the current passing through it tends to deflect both needles in the same direction, a very sensitive

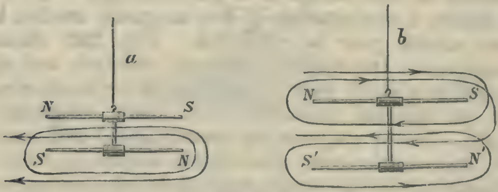


Fig. 223.

galvanoscope may be constructed. Fig. 223 shows how the coil may be arranged—it may be wound round the lower magnet only, as shown at a, or round both magnets, as at b; in both cases the student will see, by applying Ampère's law, that the action of the current in the coil tends to deflect *both* magnets in the same direction. The couple opposing the deflection of the needle will be due to the slight difference between the opposed couples acting on the component magnets, and the torsion on the suspension fibre or wire. Fig. 224 shows an elaborate form of galvanoscope, with an astatic needle, known as *Nobili's Astatic Galvanometer*, though it is in principle a current indicator, and not a current measurer as its name implies. The coil is wound round the lower needle, and the upper needle moves over a circular scale graduated in degrees. When in use, the torsion head should be turned until the needle sets with its length at right angles to the magnetic

meridian; the frame of the coil should be set accurately parallel to the needle, and the pointer should then indicate the zero of the scale.

A galvanoscope of this kind, in which the deflection produced may be read off on a scale, may be *calibrated* for

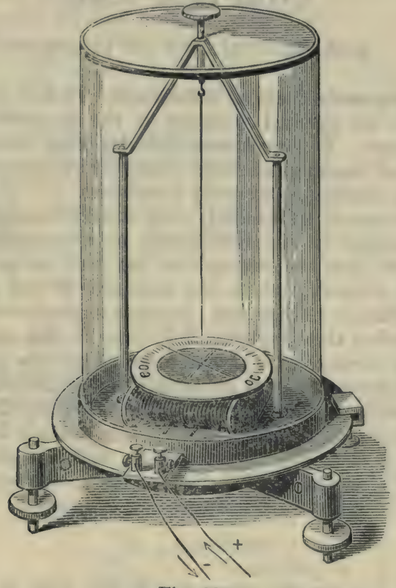


Fig. 224.

use as a galvanometer. That is, by comparing its indications with those of a measuring instrument, a table of current strength corresponding to the divisions of the scale may be drawn up, and the current strength for any given deflection may then be determined by reference to this table or by means of curve constructed from the table.

CHAPTER XXI.

MAGNETIC EFFECT OF CURRENTS.

126. Magnetic Field of a Current. In Art. 122 we have discussed the general character of the magnetic field due to a current. We must now consider what determines the *intensity* of the magnetic field at any point in the neighbourhood of a conductor carrying a current.

In the case of an infinitely long straight conductor carrying a current it was established experimentally by Biot and Savart that the strength of the field at any point is inversely proportional to the distance of the point from the conductor. Biot and Savart's experiment consisted in comparing the magnetic field due to the current in a very long vertical wire at a number of points at different distances from the wire by noting the oscillations of a very small needle at these points, and eliminating the influence of the horizontal component of the earth's field by the method of Art. 87. The magnetic field was also found to be directly proportional to the current in the wire. Hence, as a result of these experiments we may write

$$I \propto \frac{C}{r} \text{ or } I = A\frac{C}{r}$$

where I denotes the strength of the field at a distance r from a very long straight conductor carrying a current, C, and A is a constant which involves constants depending upon the unit selected for the measurement of current, and also upon the permeability of the medium.

In discussing the law established by Biot and Savart, Laplace showed that the result may be deduced mathematically from the following assumptions:—

Let A B (Fig. 225) represent a wire carrying a current; then, considering the field due to a very small element, *a b*,

it is assumed that the intensity of the field at c due to the current in that element is

1. *Directly* proportional to the strength of the current in A B.

2. *Directly* proportional to the distance, $a\,b'$, that is, to the apparent length of the element $a\,b$ as seen from c. (The line $a\,b'$ is drawn at right angles to the line $c\,o$, which joins c to o, the middle point of $a\,b$.)

3. *Inversely* proportional to the square of the distance of the element $a\,b$ from c, that is to $c\,o^2$.

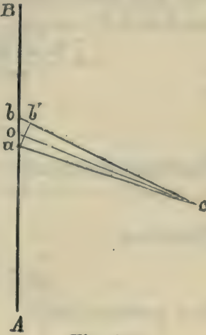


Fig. 225.

Hence, if i denote the intensity of the field at c due to the element $a\,b$, and C the strength of the current in A B, we have

$$i \propto \frac{C\,.\,a\,b'}{(o\,c)^2} \quad \text{or} \quad i \propto \frac{C\,.\,a\,b\sin\alpha}{(o\,c)^2}$$

where α denotes the angle $a\,o\,c$.

The direction of i is perpendicular to the plane through c and A B. To determine, by application of this result, the intensity, at any point of the field, due to a current in a conductor of given form and position, it is evidently necessary to sum up the effects due to each element of the conductor considered separately — the result of the summation giving the total intensity, at the point considered, of the field due to the current in the conductor taken as a whole.

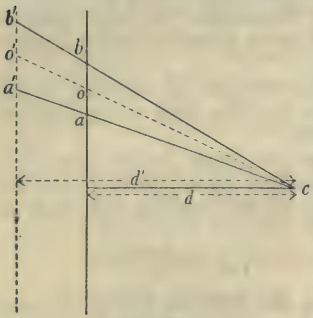


Fig. 226.

In the case of an infinitely long straight conductor, by

applying Laplace's assumptions to the two cases shown in Fig. 226, where the distances of the point c from the wire are d and d', we get

$$\iota \propto \frac{C \,.\, a\,b \sin a}{o\,c^2} \quad \text{and} \quad \iota' \propto \frac{C \,.\, a'\,b' \sin a}{o'\,c^2},$$

or

$$\frac{\iota}{\iota'} = \frac{a\ b}{a'\,b'} \cdot \left(\frac{o'\,c}{o\,c}\right)^2.$$

But from the figure

$$\frac{o'\,c}{o\,c} = \frac{a'\,b'}{a\,b}.$$

Therefore

$$\frac{\iota}{\iota'} = \frac{a\,b}{a'\,b'} \cdot \left(\frac{a'\,b'}{a\,b}\right)^2 = \frac{a'\,b'}{a\,b}.$$

And geometrically

$$\frac{a\,b}{a'\,b'} = \frac{d}{d'};$$

hence,

$$\frac{\iota}{\iota'} = \frac{d'}{d}.$$

This indicates plainly that as the distance of the wire from the point c increases, the effect of each element into which the wire may be divided decreases inversely as the distance increases, and therefore by summation the total effect of the whole wire at any point varies inversely as the distance of the point from the wire.

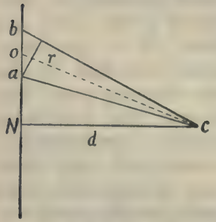


Fig. 227.

The strength of the field at any point near the wire is calculated as follows.

Let the angle NCO (Fig. 227) be denoted by a. Then the field at C due to the element $a\,b$, is given by

$$\iota = \frac{C \,.\, ab \cos a}{r^2}.$$

where the distance CO is denoted by r and C is measured in the units defined in Art. 129.

But the element $a\,b$ corresponds to an infinitely small increment $d\,a$ in the angle a, and

$a\,b \cos a = r\,.\,d\,a.$ Also

$$\cos a = \frac{d}{r} \text{ or } r = \frac{d}{\cos a},$$

and therefore

$$i = \frac{\text{C}\,.\,r\,d\,a}{r^2} = \frac{\text{C}\,.\,d\,a}{r} = \frac{\text{C}\,.\cos a\,.\,d\,a}{d}.$$

Now the field at C due to the infinitely long conductor in which $a\,b$ is an element is evidently the sum or integral of

$$\frac{\text{C}\,.\cos a\,.\,d\,a}{d}$$

between the limits for which

$$a = -\frac{\pi}{2} \text{ to } a = \frac{\pi}{2},$$

that is $$\text{F} = \frac{\text{C}}{d}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos a\,.\,d\,a = \frac{\text{C}}{d}\,.\,2 \sin\frac{\pi}{2} = \frac{2\,\text{C}}{d}.$$

A simple case of great importance must here be considered. Consider the conductor A B to be looped into a circle as shown in Fig. 228, and let us determine the intensity of the magnetic field at c, the centre of the circle, due to the current in the circular conductor A B.

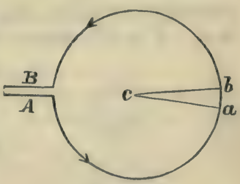


Fig. 228.

Considering the action of the element $a\,b$, the intensity of the field due to this element is evidently proportional to $\frac{\text{C} \times ab}{r^2}$, where C denotes the strength of the current in A B, and r the radius of the circle. Now, since each element of the conductor is similarly placed relatively to c, the intensity of the field at c due to the current in the conductor A C B taken as a whole is proportional to $\frac{\text{C} \times \text{A B}}{r^2}$, that is, since

$AB = 2\pi r$, to $\dfrac{C \times 2\pi r}{r^2}$, or to $\dfrac{2\pi C}{r}$ Hence, if I denote the total intensity of the field at C, then

$$I \propto \frac{2\pi C}{r}.$$

The line of action of the force will be at right angles to the plane of the coil, and the direction along this line may be determined by Ampère's rule when the direction of the current is known.

Thus, for the case of Fig. 228, the direction of the force, that is, the direction in which a north pole would be urged, is upwards, towards the reader, along the normal to the plane of the paper at *c*.

If the circular coil A B consist of n turns (sufficiently thin to be regarded as coincident) instead of one, then, since the current passes n times round the coil, the intensity of the field will be n times as great as for one turn, that is

$$I \propto \frac{2\pi n C}{r}.$$

where r is, as before, the radius of the coil, and C the current in it. Fig. 229 shows roughly the lines of force for the magnetic field inside a coil of wire carrying a current, and it should be noticed that, within a small space near the centre of the coil, the field is practically uniform (Art. 75), that is, the lines of force are approximately parallel equidistant straight lines.

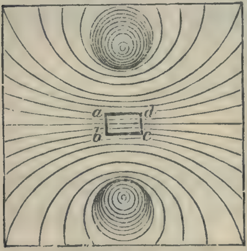


Fig. 229.

These results may be readily verified experimentally by looping a long thin wire carrying a constant current successively into a number of coils varying in size and number of turns, and comparing the magnetic fields produced at the centre of each coil. This may be done by noting in

each case the number of oscillations performed by a small needle placed at the centre of the coil. Care must be taken to arrange the plane of the coils at right angles to the magnetic meridian, so that the field due to the current may be parallel to that due to earth; then, for example, if the needle make 20 vibrations per minute when vibrating in the earth's field *only*, and 25, 30, and 40 respectively when vibrating at the centre of three different coils, then the intensities of the three fields *due to the current in the coils* are as $(25^2-20^2) : (30^2-20^2) : (40^2-20^2)$, that is, as $225 : 500 : 1200$, or as $9 : 20 : 48$. Results obtained in this way will be found to verify the statement that $I \propto 2\pi n C/r$, and it should here be noticed that by proving the field for n turns of a current C to be n times the field for one turn, we practically prove that $I \propto C$, for n turns of a current C are practically equivalent to one turn of a current nC; and if by increasing the current n-fold the magnetic field is also increased n-fold, then $I \propto C$.

A slight extension of the results obtained above shows that the intensity of the field at any point *on the axis* of a thin coil of n turns, at a distance x from the centre of the coil, is proportional to

$$2\pi n C \frac{r^2}{(r^2+x^2)^{\frac{3}{2}}},$$

where C denotes the current in the coil.

For, by the law given above, the intensity of the magnetic field at any point on the axis of the coil, due to a small element of the coil, is proportional to

$$\frac{lC}{(r^2+x^2)},$$

where l is the length of the small element, r the radius of the coil, and x the distance of the point on the axis from the centre of the coil. The direction of this force is at right angles to the plane passing through the given point on the axis and the element of the coil, and its component along the axis of the coil is therefore proportional to

$$\frac{r\,l\,C}{(r^2+x^2)^{\frac{3}{2}}},$$

The total resultant force along the axis is got by summing up the effects of all the elements of the coil, that is, by substituting for l the total length of the coil; hence the resultant intensity along the axis is proportional to

$$2\pi n C \frac{r^2}{(r^2+x^2)^{\frac{3}{2}}}.$$

Evidently, when $x=0$ this expression reduces to $2\pi n C/r$, the result obtained above for the intensity at the centre of the coil.

A further extension of this result shows that the intensity of the magnetic field in the interior of a coil similar to that shown in Fig. 210, but closely wound, is approximately proportional to

$$4\pi n C,$$

where n denotes the number of turns *per unit length* of the coil, and C the current flowing in it. The magnetic field in the interior of such a coil is practically uniform except near the ends.

127. Force exerted on a Conductor carrying a Current in a Magnetic Field. It has been shown in Article 126 that the intensity of the magnetic field at c due to the element $a\,b$ is given by

$$i=\frac{C \times a\,b'}{o\,c^2}$$

the direction of the field at c being at right angles to the plane $a\,b\,c$.

If we denote $a\,b$ by l, $o\,c$ by r, and the angle $a\,o\,c$ by a, then

$$i=\frac{C\,l \sin a}{r^2}.$$

This expression for the intensity of the magnetic field at c gives the force which a unit north pole would experience if placed at that point. It therefore gives the force which a unit north pole at c would exert on the element $a\,b$. But the strength of the magnetic field at o, due to a unit north pole at c, is $1/r^2$. Hence the force exerted on the element $a\,b$ in a magnetic field of intensity $1/r^2$ is

$$C\,.\,l \sin a\,.\,1/r^2$$

That is, in a field of intensity H the force exerted on the element $a\,b$ of the conductor A B carrying a current C is $C\,l \sin a \,.\, H$ or $C\,H\,l \sin a$, that is, $C\,H \sin a$ per unit length and the direction of this force is at right angles to the plane determined by $a\,b$ and the direction of the magnetic field at o.

The result given above may evidently be put in the following simple form. The product $H\,l \sin a$ is the area of a parallelogram whose adjacent sides, inclined at an angle a, represent H and l respectively in magnitude and direction. Hence, if we construct a parallelogram whose adjacent sides represent in magnitude and direction the intensity of the magnetic field and the length of an element of the conductor carrying a current C, and if A denote the area of this parallelogram, then the force exerted on this element is given by C A, and is at right angles to the plane of the parallelogram. If the conductor is straight and the field uniform then this result may be applied to a conductor of finite length.

The direction of this force is readily deduced by Ampère's rule. Thus, in Fig. 225, if the current flow from A to B the force on a north pole at c would be downwards at right angles to the plane of the figure, that is, the force on the element $a\,b$ would be upwards through o at right angles to the plane of the figure. But the field due to a north pole at o is along $c\,o$, hence, if the observer at o looks along the direction of the field *and* the direction of the current, the force on the element will be to his *left* in a direction at right angles to the plane determined by these two directions.

128. Work done in Displacing, in a Magnetic Field, a Circuit carrying a Current. Let $a\,b$ (Fig. 230) represent an element of a conductor carrying a current in the direction of the arrow, and let $a\,\mathrm{I}$ denote the direction and strength of the magnetic field acting on the element. Also let $a\,\mathrm{F}$, drawn in a direction at right angles to the plane of $a\,b$ and $a\,\mathrm{I}$, represent the direction of the force F acting on the element $a\,b$. If now the element remaining parallel to its first position suffer a very small displacement to the

position $a'\,b'$, the work done in effecting the displacement by the force acting on the element is measured by $F \,.\, a\,a''$, where $a\,a''$ is the projection of $a\,a'$ on the direction of F. But if H denote the intensity of the field, then by Art. 127, we have $F = C\,H\,a\,b \sin \alpha$, where α denotes the angle $b\,a\,I$ and C the current in the element, that is, the work done in the displacement is equal to $C\,H\,a\,b \sin \alpha \,.\, a\,a''$. But $a\,b \sin \alpha \,.\, a\,a''$ is $a\,b'' \,.\, a\,a''$, the area of $a\,a'''\,b'''\,b''$ which is the projection of the area $a\,a'\,b'\,b$ on a plane through a at right angles to the direction of the magnetic field. The quantity $H \,.\, a\,b \sin \alpha \,.\, a\,a''$ is, therefore, the flow of force through $a\,a'\,b'\,b$ or *the number of lines of force cut by the element* $a\,b$ *during its displacement* into the portion $a'\,b'$. The work done by the force along $a\,F$ is positive or negative according as the projection $a\,a''$ of $a\,a'$ is in a direction the same as, or opposite to, that of $a\,F$. That is, if the displacement is to the left for an observer, swimming with the current and looking along the line of force, the work done by the electromagnetic force is positive; if to the right the work is negative.

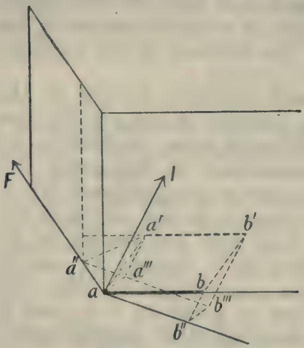


Fig. 230.

If a closed circuit be displaced in any way in a field it is evident that the work done may be determined by imagining the circuit divided up into very small elements and summing up the work done in the displacement of each element. Thus, if C denote the current in the circuit and n_1, n_2, n_3, . . . the number of lines of force cut by the elements of the circuit, then the total work for the given displacement is $\Sigma\, C\, n$ or $C\, \Sigma\, n$. In determining $\Sigma\, n$, however, the *sign* of n for each element has to be considered. By applying the rule given above it will be found that,

assuming the lines of force at any point to pass through the circuit from the negative side to the positive side (that is, in the direction which the lines of force due to current in the circuit would pass), then n is positive if the lines are so cut as to pass from without to within the area of the circuit, and negative if the lines are so cut as to pass from within to without this area. It follows from this that $\Sigma\, n$ is really equivalent to the increase of the flow of force through the circuit in the positive direction. If this *increase* be denoted by N then the work done *by* the electromagnetic force acting on the circuit in the magnetic field is measured by N C, where C is the current in the circuit. If the displacement is a very small linear displacement l, then N C/l is the work done *by* the electromagnetic forces per unit length, that is N C/l is the force causing the displacement. Similarly, if the circuit be rotated round an axis through a very small angle θ, then N C/θ gives the moment of the couple causing the displacement.

If the circuit be taken from an infinite distance up to any point in the field then the work done against electromagnetic forces in bringing the circuit to that point is given by $-$**F** C, where **F** denotes the total flow of force through the circuit in the position it occupies in the field. The quantity $-$**F** C therefore measures the potential of the circuit in the field. The potential here defined evidently varies with the position of the circuit in the field and may be zero if **F** is zero. Its maximum value obtains when **F** is negative and has the greatest arithmetical value possible, and for its minimum value **F** is positive and as great as possible. The two positions of the circuit corresponding to the maximum and minimum values evidently make an angle of 180° with each other. For a circuit free to move in the field the position of rest is that corresponding to the minimum potential energy, that is, to the position in which the lines of force pass through it in the greatest number and in the same direction as the lines of force due to the current in the circuit. A plane circuit, for example, freely suspended in a uniform field tends to set itself at right angles to the field in such a position that the positive direction of its axis is the same as that of the field. It is

evident also that if the circuit is flexible or made up of movable parts it will, when a current is passing through it, tend to make the area enclosed by the circuit a maximum. A flexible circuit therefore tends to become circular in form and in a circuit with a movable part, the movable part moves in such a way as to increase the area of the circuit.

129. Electromagnetic Unit of Current Strength. It is evident from the preceding article that we may *compare* the strengths of current by comparing the intensities of the fields which they produce at the centre of a given coil. For if the same coil be used for different currents, then, in the relation $I \propto 2\pi n C/r$, n and r are constants and C is the only variable, and we have $I \propto C$ or $C \propto I$.

In order to measure a given current absolutely, however, it is necessary to decide upon a unit of current strength. This is most conveniently done by adopting a unit such that the relation $I \propto 2\pi n C/r$ may be written as an equality, thus

$$I = \frac{2\pi n C}{r}.$$

That is, *unit current is that current which, flowing in a single coil of unit radius, produces a magnetic field of* 2π *units of intensity at the centre of the coil.* For, from the relation given above

$$C = \frac{I\, r}{2\pi n},$$

and if $I = 2\pi$ units, r be of unit length, and $n = 1$, then

$$C = 1,$$

that is, the current is of unit strength.

This definition is perhaps more neatly expressed by saying that *unit current is that current which, flowing in an arc of unit length and unit radius, produces a magnetic field of unit intensity at the centre of the arc.* If it be remembered that a single complete coil of unit radius contains 2π arcs of unit length, it will be seen that the two definitions are identical. If the unit of length be the centimetre, and the unit of force the dyne, then the unit of current defined above is known as the *absolute electromagnetic unit* of current strength, and is equal to 10 ampères.

130. Fixed Coil Galvanometers. The general principle of the action of all fixed coil galvanometers depends upon the effect of the current in a coil of wire on a magnetic needle freely suspended at the centre of the coil. When no current passes the needle sets in the magnetic meridian, and the plane of the coil should also be in the magnetic meridian, that is, the needle should set with its length in the plane of the coil. Then, on passing a current through the coil, the direction of the magnetic field, due to the current, being at right angles to the plane of the coil, tends to deflect the needle out of the magnetic meridian, and sets it in such a position that the moment of the couple due to the field of the current is balanced by the moment of the opposing couple due to the field of the earth. The magnitude of this deflection is directly connected with the strength of the current, and by noting its value it is possible with a properly constructed instrument to obtain a measure of the current.

From this it is evident that the essential parts of any galvanometer are (*a*) a coil of wire, (*b*) a magnetic needle freely suspended at the centre of the coil, and (*c*) a scale on which the deflection of the needle may be read. The forms of galvanometer of this type most generally used are the *Tangent* and *Sine galvanometers* and *Thomson's mirror or reflecting galvanometer.*

Speaking generally, with reference to their use in practical work, galvanometers may be divided into two classes.

1. Instruments adapted for exact measurement of current. Of this class the *tangent* and *sine galvanometers* are the best types.

2. Instruments constructed so as to secure extreme sensitiveness. Galvanometers of this class are not suitable for the measurement of current, but they are the most useful instruments in the general work of electrical measurement, where it is often more essential to have a sensitive instrument capable of indicating, and if necessary measuring, the feeblest current, rather than one capable of exact measurement of current generally. Reflecting galvanometers are instruments of this type.

131. The Tangent Galvanometer. A simple form of

tangent galvanometer is shown in Fig. 231. The needle, which may be suspended or pivoted at the exact centre of the coil, is very short, and carries a light wire pointer moving over a circular scale. The needle and scale are enclosed in the shallow cylindrical box B by a glass cover, and the arrangements for accurately reading the deflection of the needle are exactly those described in Art. 85 in connection with the apparatus of Fig. 141, which may be taken to represent the needle and scale of a tangent galvanometer.

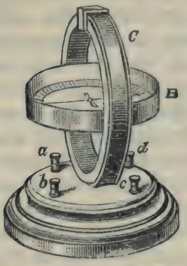


Fig. 231.

The coil of carefully insulated copper wire is wound round the frame C, and the ends of the wire are brought out under the base of the instrument and connected to screw terminals fixed in the base-board. Most instruments, however, are usually fitted with several coils of different lengths wound on the same frame, but the ends of each coil are connected to the terminal screws in such a way that any one coil may be used separately, or any two or more combined into one by choosing the proper screws as terminals. Thus, in Fig. 231, if the screws *a* and *b* are taken as terminals the current passes only through the coil joining them; but if the screws *a* and *c* are taken, the current passes through the coil joining *a* and *b*, and also through that joining *b* and *c*. Similarly, if *a* and *d* be taken, the current passes through the coils *a* to *b*, *b* to *c*, *c* to *d*. Three coils are sufficient for ordinary purposes—one consisting of a single turn of thick copper wire, a second of a number of turns of wire of medium thickness, and a third composed of a large number of turns of thin wire. To adjust the galvanometer for use it is carefully levelled, and the plane of the coil adjusted in the magnetic meridian. The needle then lies in the plane of the coil, and the ends of the pointer should point to the zeros of the scale. To eliminate errors due to inaccurate adjustment of the needle and pointer, both ends of the pointer should be read, and the deflection taken first on one side

of the zero, and then, by reversing the current in the coil, on the other side. The mean of the four readings thus obtained gives the true value of the deflection. The coil to be used in any given case must be chosen in accordance with principles to be discussed in the next chapter, but it will be readily understood that to prevent injury to the instrument all very strong currents should be sent through the single coil of stout wire.

We have now to consider the theory of the instrument in order establish a relation between the strength of a current and the magnitude of the deflection produced by it.

Let Fig. 232 represent a horizontal section of coil in the plane of the needle. The plane of the coil is supposed to lie in the magnet meridian N S, at right angles to the plane of the paper, the sections at C C representing the cut ends of the coil, and $a\,b$ the needle in its deflected position. For the sake of clearness the length of the needle is here shown much longer than it should be relative to the diameter of the coil: it should be comparatively short, so that in all positions it may lie within the small space in which the magnetic field due to the current may be considered of uniform intensity, equal to the intensity at the centre of the coil. That is, referring to Fig. 229, the needle should be small enough to lie within the small space $a\,b\,c\,d$ at the centre of the coil where the lines of force are approximately parallel and equidistant.

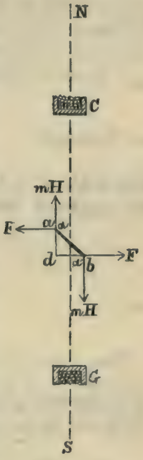


Fig. 232.

The intensity of the magnetic field within this space, due to a current C in the coil, is given approximately by

$$I = \frac{2\pi n C}{r},$$

where r denotes the radius of the coil, and n the number of turns in the coil.

Hence, if m denote the strength of either pole of the needle, the forces constituting the couple tending to deflect it out of the magnetic meridian are each given by

$$F = m\,I = \frac{2\,\pi\,m\,n\,C}{r},$$

and act at right angles to the plane of the coil.

When the needle is in its position of equilibrium, the moment of the couple due to the earth's field balances the moment due to this couple. Hence, for equilibrium we have

$$F\,.\,a\,d = m\,H\,.\,b\,d,$$

that is,

$$\frac{2\,\pi\,m\,n\,C}{r}\,.\,a\,d = m\,H\,.\,b\,d,$$

or

$$\frac{2\,\pi\,n\,C}{r}\,.\,a\,d = H\,.\,b\,d,$$

and therefore

$$C = \frac{r}{2\,\pi\,n}\,.\,H\,.\,\frac{b\,d}{a\,d}.$$

But $b\,d/a\,d = \tan b\,a\,d$, and as the angle $b\,a\,d$ is equal to α, the angle of deflection of the needle, we have

$$\frac{b\,d}{a\,d} = \tan \alpha,$$

and

$$C = \frac{r}{2\,\pi\,n}\,.\,H\,.\,\tan \alpha.$$

The quantity $\dfrac{r\,H}{2\,\pi\,n}$ is known as the *working constant* or *reduction factor* of the galvanometer,* and denoting it by k, we have

$$C = k \tan \alpha,$$

where k is a constant. That is, *the strength of the current varies as the tangent of the angle of deflection.* For example, if one current, C_1, produce a deflection of 30°, and another, C_2, a deflection of 60°, then

$$\frac{C_1}{C_2} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}.$$

* The constant of the galvanometer *coil* is $\dfrac{2\,\pi\,n}{r}$.

that is, C_2 is three times a great as C_1. We are thus able to *compare* currents by simply noting the deflections of the needle and taking the value of the tangents of the angles; but if we wish to measure a current absolutely, then the value of the constant k must be determined, either by calculation from the relation $\frac{r\,H}{2\,\pi\,n}$ or by experiment, as explained later.

Direct application of the method of Art. 86 gives a more concise statement of the theory of the tangent galvanometer than is given above. The galvanometer is adjusted for use with its plane in the magnetic meridian. The field at the centre of the coil, due to the current in the coil, and the horizontal component of the earth's field are then at right angles to each other. Hence, for the equilibrium of the deflected needle we have

$$I = H \tan \alpha.$$

But
$$I = \frac{2\,\pi\,n\,C}{r}$$

and therefore
$$\frac{2\,\pi\,n\,C}{r} = H \tan \alpha$$

or
$$C = \frac{r\,H}{2\,\pi\,n} \tan \alpha.$$

The constant $2\,\pi\,n/r$ is the field at the centre of the coil due to *unit current* in the coil. This constant is usually called the constant of the coil and denoted by G. With this notation, therefore, we have

$$C = \frac{H}{G} \tan \alpha$$

for any galvanometer, if G denote the magnetic field at the point where the needle is placed, and if the direction of this field is at right angles to the field H which controls the needle.

In Art. 126 it has been shown that the magnetic field at

a point on the axis of a circular coil of radius r, at a distance x from the centre of the coil, is given by

$$I = 2\pi n C \frac{r^2}{(r^2+x^2)^{\frac{3}{2}}},$$

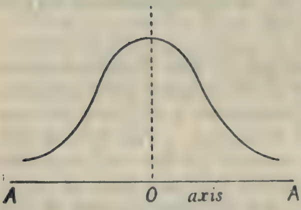


Fig. 233.

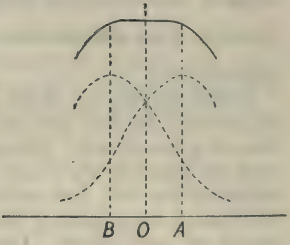


Fig. 234.

that is, G, the field for unit current is given by

$$G = \frac{2\pi n r^2}{(r^2+x^2)^{\frac{3}{2}}}.$$

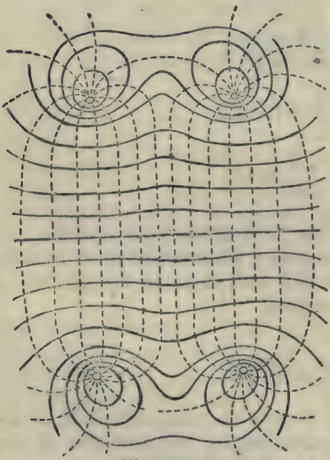


Fig. 235.

The variation of the field along the axis, in accordance with this relation is indicated by the curve drawn in Fig. 233. The abscissæ represent distances along the axis, measured from O, the centre of the coil. The curve is at first concave towards O, but the curvature becomes less and less, and quickly changes sign, the curves becoming convex towards C. The point of inflection or change of curvature is at the point where $x = \frac{1}{2}r$ and, at this point, the curve is,

for a short length, practically a straight line. If, therefore, we have two equal circular coils placed with their axes coincident and at a distance apart equal to the radius of either, then, for the same direction along the common axis, the rate of increase of the field due to one coil at a point midway between the two coils is equal to rate of decrease of the field due to the other coil at the same point, and the field for a short distance on each side of this point, will be practically uniform. This is shown graphically in Fig. 234. The dotted curves show the fields for the separate coils and the full line curve, the resultant of the two dotted curves, represents the resultant field between the two coils. This field evidently varies very little in the central part of the space between the coils. Fig. 235 shows the field of force between the coils. This result has been applied in the construction of a tangent galvanometer. The Helmholtz tangent galvanometer consists of two equal similar coils arranged as described above at a distance apart equal to the radius of the coils. The needle of the galvanometer is suspended so as to hang at the point on the axis midway between the coils and, as the variation of the field is less than one-tenth per cent. over a distance equal to one-eighth the radius on each side of this point, there is no appreciable error due to the length of the needle or to the inaccuracy of its adjustment at the central point. Fig. 236 shows a good form of the Helmholtz galvanometer.

Fig. 236.

When $x = \frac{1}{2} r$ the value of

$$G = \frac{2 \pi n r^2}{(r^2 + x^2)^{\frac{3}{2}}},$$

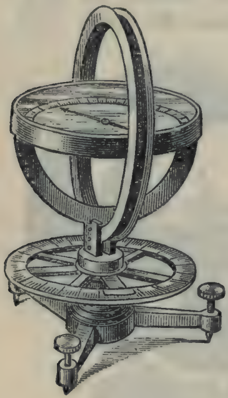
for the double coil is very approximately given by

$$G = 4{\cdot}5 \frac{n}{r}.$$

And as $$C = \frac{H}{G} \tan \alpha,$$

we have in this case $C = \frac{Hr}{4{\cdot}5\, n} \tan \alpha$; or $C = \frac{2\,Hr}{9\, n} \tan \alpha.$

132. The Sine Galvanometer, one form of which is shown in Fig. 237, is an instrument exactly similar in



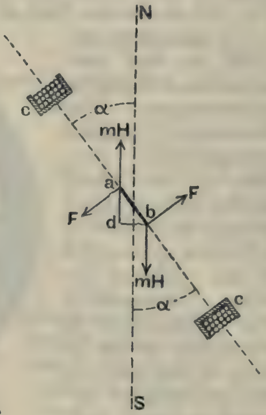


Fig. 237.

principle and construction to the tangent galvanometer It differs from it only in the fact that the coil and needle box can be rotated round a central vertical axis, and a horizontal circular scale is provided on which the amount of this rotation can be accurately read.

For use the instrument is adjusted in the same way as the tangent galvanometer, but when the needle is deflected

the coil is rotated after it until the needle is overtaken by it and *in its deflected position* lies in the plane of the coil. The diagram of Fig. 237 shows the conditions of equilibrium of the needle, and as before we must have

$$F \,.\, a\,b = m\mathrm{H} \,.\, b\,d,$$

that is,
$$\frac{2\,\pi\,m\,n\,\mathrm{C}}{r} \,.\, a\,b = m\,\mathrm{H} \,.\, b\,d,$$

or
$$\frac{2\,\pi\,n\,\mathrm{C}}{r} \,.\, a\,b = \mathrm{H} \,.\, b\,d,$$

and therefore
$$\mathrm{C} = \frac{r}{2\,\pi\,n} \,.\, \mathrm{H} \,.\, \frac{b\,d}{a\,b},$$

that is, since
$$\frac{b\,d}{a\,b} = \sin b\,a\,d = \sin a,$$

where a denotes the deflection of the needle or the rotation of the coil, we have

$$\mathrm{C} = \frac{r}{2\,\pi\,n} \,.\, \mathrm{H} \,.\, \sin a,$$

or
$$\mathrm{C} = k \,.\, \sin a,$$

where k denotes the reduction factor of the instrument. That is, the current is proportional to the *sine* of the angle of deflection of the needle; for example, if one current, C_1, produce a deflection of 30°, and another, C_2, a deflection of 60°, then

$$\frac{C_1}{C_2} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

It should be noticed that with the sine galvanometer the needle, when in its position of equilibrium, lies in the plane of the coil; and therefore the forces of the couple F, F, always act at right angles to the length of the needle, and therefore the arm of the couple is always given by the length of the needle $a\,b$.

This is true whatever be the length of the needle, and hence, for a needle of given length, the moment of the couple varies only with the strength of the current. For *comparative measurements,* therefore, the instrument may have a long needle, but for absolute measurements the

needle must be short, in order that the intensity of the field in which it moves may be uniform and easily determined.

133. The Mirror Galvanometer. A good form of Thomson's *mirror galvanometer* is shown in Fig. 238. The coil is wound on a comparatively small circular reel, R, enclosed in B, a cylindrical brass box with a glass front. The needle of the instrument is a short, carefully magnetised strip of steel, attached by shellac or cement to the back of a small concave mirror, *m*, suspended by a single silk fibre at the centre of the coil. The deflection of the magnet is measured by the mirror and scale method explained in Art. 36, and Fig. 239 shows the galvanometer and scale arranged for use. The large magnet, N S, supported above the coil, has several important uses.

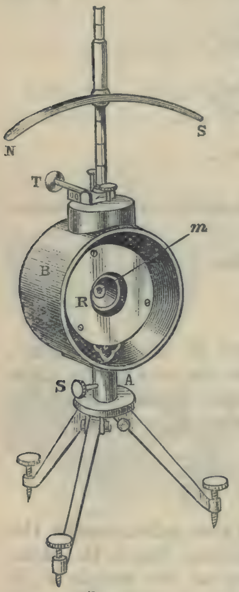


Fig. 238.

If this magnet be removed, the needle of the galvanometer sets in the magnetic meridian, and the magnetic field in which it lies is that due to the horizontal component of the earth's field. If now the magnet be replaced with its length in the magnetic meridian, and its *south* pole pointing northwards, the magnetic field which it produces at the centre of the coil will be added to that due to the earth. The needle will now be more difficult to deflect, and, consequently, the galvanometer will be less sensitive. The nearer the magnet is to the coil, the greater will this effect be; hence, by lowering the magnet on its support, the sensitiveness may be very considerably diminished. If, however, the magnet be replaced with its *north* pole pointing northwards, the field it produces at the centre of the

coil tends to neutralise that due to the earth; and hence, by lowering the magnet, the field in the coil may be decreased until, at a certain point, the field of the magnet exactly balances that of the earth. For this position of the magnet the needle is unstable, and remains at rest in any position; and if the magnet be lowered still more, the direction of the field is reversed, and the needle tends to turn round through 180°. Hence, in this case, the galvanometer cannot be used with the controlling magnet lower than the position of instability of the needle, but with the

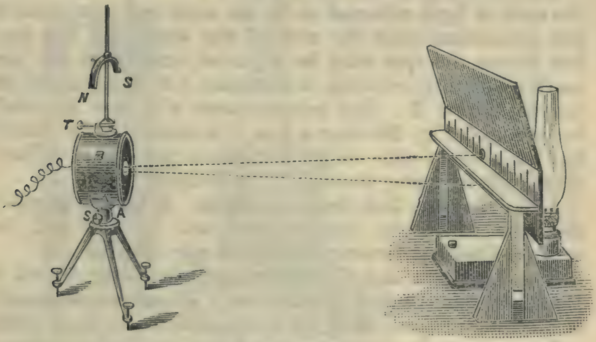


Fig. 239.

magnet slightly above this position the instrument is extremely sensitive. Its sensitiveness may be slightly decreased by raising the magnet, or, in a greater degree, by removing it, or, more effectually, as explained above, by reversing it and lowering it on its support.

So far we have considered the galvanometer placed with the plane of its coil in the magnetic meridian, but this is not an essential adjustment—by turning the controlling magnet on its support the needle may be made to set in almost any required plane, and the coils are then set parallel to it by rotating the body of the instrument round a central axis at A.

In adjusting the galvanometer for use it is first levelled so that the mirror *m* hangs truly at the centre of the coil. The position of the magnet N S is then adjusted so as to secure the required sensitiveness, and to set the needle in a plane convenient for work. The coil is then set parallel to the plane of the needle, and fixed in position by the screw at S. The scale must now be arranged with its length parallel to the plain of the coil, and the line joining the centre of the mirror to the zero of the scale should be approximately at right angles to this.

On adjusting the lamp and scale to the proper height, the spot of light reflected on to the scale will now come to rest somewhere near the zero of the scale, and the dark line across the spot may be adjusted exactly to the zero mark by slightly rotating the magnet N S by means of the tangent screw T. The galvanometer will now be ready for use, and Fig. 239 shows how it is connected in circuit, but the student can realise the entire process of adjustment only by going through it for himself.

134. Moving Coil Galvanometer. A type of galvanometer in which the coil is small and movable and the magnet fixed has recently come into very general use.

The Despretz and D'Arsonval galvanometer, shown in Fig. 240, was the first of this type. It consists essentially of a strong magnet, M M, of horse-shoe form, fixed vertically as shown in the figure, and a coil suspended between the poles of the magnet so as to be capable of deflection round the suspension wire as an axis. The coil is a small rectangular coil suspended by a single fine wire, *w w*, so as to hang in the field between the poles of the magnet. The upper part of the suspension wire is connected to one end of the coil, and the lower part to the other end, so that the upper and lower parts of the wire serve as leads for the current to the coil.

The lower end of the suspension wire is attached to the end of a spring which is directly connected to one of the terminals of the instrument, the other terminal being connected to the upper end of the wire through the torsion head. In this way the suspension wire is kept tightly stretched, and the coil, while free to be deflected, is kept

in position between the poles. In order to intensify the field in the space in which the vertical branches of the coil move, a cylinder of soft iron, C, of diameter nearly equal to the internal width of the coil, is fixed inside the coil with its axis in the line of the suspension wire. This arrangement gives a very intense field in the narrow space between the magnet poles and the surface of the soft iron cylinder. The coil is adjusted in position, so that with no torsion on the wire it hangs with its plane parallel to the

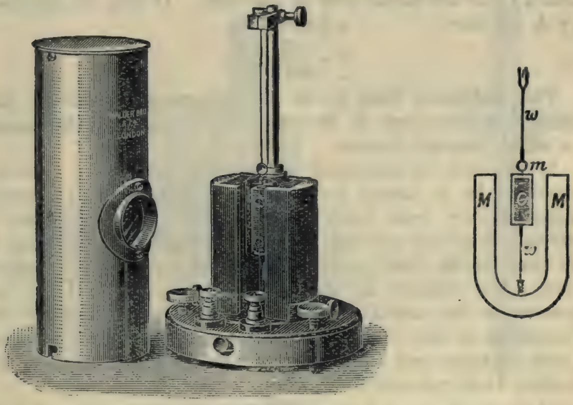


Fig. 240.

direction of the field. When the current passes through the coil it tends, by Art. 128, to set with its plane at right angles to the field. The coil is therefore deflected by the current until the angle of deflection is such that the moment of the deflecting couple is balanced by the moment of the couple due to the torsion of the wire. The deflection of the coil is noted and measured by the usual scale and mirror method, the small mirror being attached to the coil as indicated at m in the figure.

Let 2 d denote the mean width of the coil, l its mean length, n the number of turns, and H the strength of the

field in which the vertical branches of the coil hang. Let a, supposed small, be the twist of the wire. Then* for a small additional angular deflection θ the number of lines of force cut by each branch is approximately $l d \theta \mathrm{H}$, and the work done is, as in Art. 128, given by $2 n \mathrm{C} . l d \theta \mathrm{H}$. This gives the moment of the couple causing deflection as equal to $2 n \mathrm{C} . l d \mathrm{H}$. If T denote the moment of the torsion couple for unit angular twist of the wire we therefore get

$$2 n \mathrm{C} l d \mathrm{H} = \mathrm{T} a.$$

But $2 l d$ is the mean area of the coil. If this be denoted by A we have

$$n \mathrm{C} \mathrm{A} \mathrm{H} = \mathrm{T} a$$

or

$$\mathrm{C} = \frac{\mathrm{T}}{n \mathrm{A} \mathrm{H}} a.$$

This indicates that C is directly proportional to the deflection, provided H is constant throughout the space in which the coil moves.

If the coil be mounted on a metal frame, or (better) enclosed in a metal box, the instrument is practically a-periodic or dead beat in its action. That is to say a current does not set the coil in oscillation but swings it at once to its final position as given by the above equation. The reason is that the currents induced by the motion in the metal box or frame are large, because the resistance is very small; and the magnetic action between these currents and the permanent field always opposes the motion that induced them (Lenz's Law, § 166, (5)).

If the instrument is intended for ballistic work (see § 135) it is important that the oscillations should be unchecked, and therefore a non-conducting box or frame is used. The adjustment and setting up of this form of galvanometer is also simpler, and more convenient, than in the case of the ordinary fixed coil instruments, for it may be set up facing in any direction without the usual troublesome adjustment of the control magnet necessary with the latter instrument.

The pattern shown in Fig. 240 has been greatly improved in the forms made in recent years. Figs. 241 and 242 show some of the patterns now in general use.

* Method of Virtual Work. See *Tutorial Statics*, p. 281.

135. The Ballistic Galvanometer. A ballistic galvanometer is an ordinary reflecting galvanometer with a needle of large moment of inertia. It is used for measuring the *quantity* of electricity passed through it not as a continuous current but as a sudden discharge, and the needle has a large moment of inertia, so that it may be slow in beginning to move under the impulse of the sudden

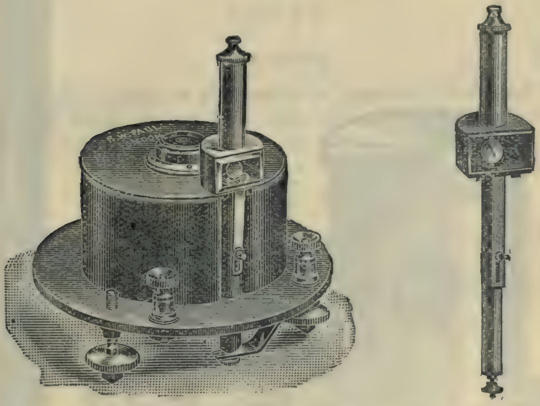


Fig. 241.

discharge, and will therefore not have moved appreciably from its position of rest during the time the discharge takes to pass through the galvanometer.

Let G denote the constant of the galvanometer coil, H the intensity of the control field, I the moment of inertia of the needle and M its magnetic moment. If the current at any instant during the discharge has the value C, then C G is the strength of the deflecting field, and, if the needle is supposed to be inappreciably deflected from its position of rest during the discharge, C G M is the

moment of the couple tending to deflect the needle. The angular acceleration due to this couple is $\frac{C\,G\,M}{I}$ and, there-

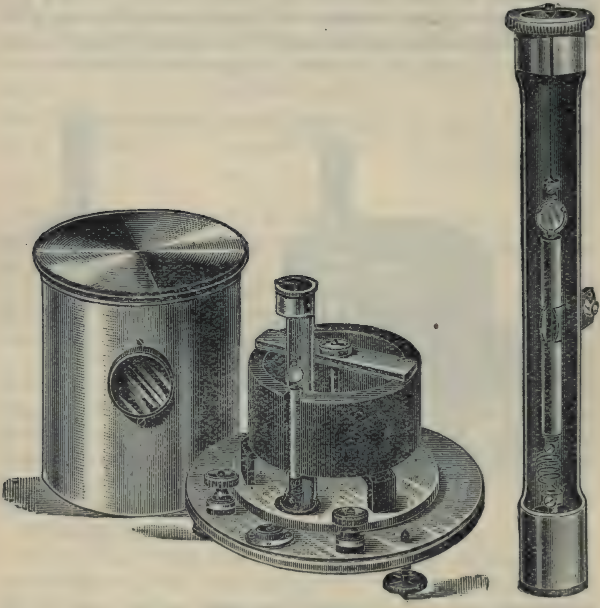


Fig. 242.

fore, during the very short time, δt, for which the current has the value C, the gain of angular velocity is $\frac{C\,G\,M}{I}\,\delta t$. During the whole discharge, therefore, the angular velocity imparted to the needle is given by $\Sigma \frac{C\,G\,M}{I}\,\delta t$, the

summation being for the whole time of the discharge. But

$$\Sigma \frac{C G M}{I} \delta t = \frac{G M}{I} \Sigma C . \delta t$$

and

$$\Sigma C . \delta t = Q,$$

where Q denotes the total quantity of electricity discharged through the galvanometer. Hence ω, the final angular velocity of the needle is given by

$$\omega = \frac{M G Q}{I}$$

or

$$I \omega = M G Q. \tag{1}$$

The kinetic energy of the needle is given by $\frac{1}{2} I \omega^2$, and this must be equal to the work done against the couple due to the control field H during the deflection throw of the needle. Let a be the angular deflection of the needle. Then

$$\tfrac{1}{2} I \omega^2 = M H (1 - \cos a)$$

or

$$I \omega^2 = 4 M H \sin^2 \frac{a}{2} \tag{2}$$

for $M H (1 - \cos a)$ is the work done in deflecting the small needle through the angle a in the field H.

Now if t denote the time of the swing of the needle

$$t = 2 \pi \sqrt{\frac{I}{M H}}$$

and therefore

$$I = \frac{M H t^2}{4 \pi^2}. \tag{3}$$

Combining the relations (2) and (3) we get

$$(I \omega)^2 = \frac{M^2 H^2 t^2 \sin^2 \frac{1}{2} a}{\pi^2}$$

or

$$I \omega = \frac{M H t}{\pi} \sin \frac{a}{2}.$$

Substituting the value of $I \omega$ given by (1) in this we get

$$M G Q = \frac{M H t}{\pi} \sin \frac{a}{2}.$$

or

$$Q = \frac{H t}{\pi G} \sin \frac{a}{2}.$$

This determines Q in terms of the galvanometer and needle constants, and shows that, under the conditions assumed, the quantity of electricity discharged through the galvanometer is proportional to the sine of half the angle of the first swing of the needle In determining a correction must be made for the damping, which takes place during the throw of the needle. This is most simply done by noting the amplitude of the next swing of the needle in the same direction. Let a_1 be the first swing and a_2 the next swing of the needle in the same direction. Then the difference, $a_1 - a_2$, is due to the damping during *four* successive half swings and, therefore, $\frac{a_1 - a_2}{4}$ is approximately the correction for damping during one half swing. Hence the corrected value of the first swing is

$$a = a_1 + \frac{a_1 - a_2}{4}.$$

If the deflections of the spot on the scale are observed, then d_1 and d_2 correspond to a_1 and a_2 above, we have

$$d = d_1 + \frac{d_1 - d_2}{4}$$

and

$$Q = \frac{\mathrm{H}\,t}{\pi \mathrm{G}} \cdot \frac{d}{4\,\mathrm{D}}$$

where D is the distance between the mirror and scale expressed in the same units as those in which d is given.

136. Mutual Action of Currents. If we place two flat spirals of wire a short distance apart with their axes in the same straight line and pass currents round them, then (provided they have suitable freedom of movement) they are found to approach or recede from one another, thus indicating respectively attraction or repulsion, according as the currents circulate round them in the same or in opposite directions. Bearing in mind that the current renders each spiral practically a magnet, and also the fundamental law of electromagnetic polarity, it is easy to see that these actions are in accordance with the laws of attraction and repulsion between two magnets.

But they may also be referred directly to the currents, in fact, seeing that a curcuit of whatever shape carrying a current is equivalent to a magnet, we should expect a pair of such in general to attract or repel each other according

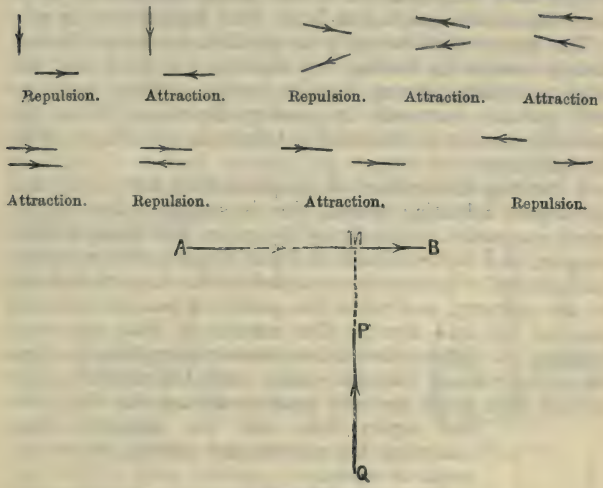


Fig. 243.

to circumstances. That this is so was first shown experimentally by Ampère, who demonstrated in the case of straight circuits the following laws:—

(1) *Two* PARALLEL *currents attract or repel one another according as they flow in the same or in opposite directions.*

(2) *Two* NON-PARALLEL *currents attract one another if both approach or both recede from the point of meeting of their directions while they repel one another if one approaches and the other recedes from that point*.*

* If the currents be not in the same plane the words "point of meeting of" should be replaced by "common perpendicular to."

These laws are illustrated in Fig. 243, which the student should carefully study. In the right-hand figure the vertical current attracts the portion A M of the horizontal one and repels the portion M B, but since A M is longer than M B there is, on the whole, attraction; if M were the middle point of A B the total force would be *nil.*

The experiments whereby Ampère's laws are established merely consist in having two circuits, one fixed and the other moveable, placing them in various relative positions and passing currents in sundry directions, when the movable circuit is observed to behave in accordance with the said laws.

A striking illustration of Ampère's first law is afforded by *Roget's vibrating spiral;* this is simply a fine copper spiral suspended vertically by its upper end from a fixed support, and with its lower end just dipping into mercury; one pole of a battery is connected to the support and one to the mercury, and when the current is passed the spiral vibrates up and down. The reason is that the coils carrying the current first attract one another; this shortens the spiral, causing its lower end to leave the mercury. The current then stops, the coils cease to attract, and the lower end again falls into the mercury, thus renewing the current and causing the movement to repeat.

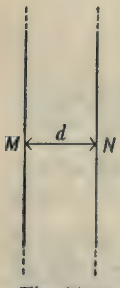


Fig. 244.

The force of attraction or repulsion between two infinitely long parallel conductors is readily determined.

Let C and C′ be the currents in the two conductors M and N (Fig. 244). The magnetic field at any point in N due to the current in M is equal to $2\,\mathrm{C}/d$, where d is the distance between the conductors and the direction of the field is at right angles to the plane of the conductors. The force per unit length exerted on N in this uniform field of intensity $2\,\mathrm{C}/d$ is, by Art. 127, $2\,\mathrm{C\,C'}/d$ and its direction is in the plane of the conductors at right angles to N. Similarly, for the force on M due to the current in N, and by applying Ampère's rules it will be found that the

forces give attraction or repulsion according as the two currents are in the same or in opposite directions.

137. Electromagnetic Rotations. The forces acting between two currents or between a magnet and a current can be applied to produce rotation of a magnet or of a circuit carrying a current. If, for example, a bar magnet be suitably weighted so that it floats vertically in mercury it will rotate round an insulated vertical conductor dipping into the mercury when a current is passed through the conductor. The pole of the magnet will move in the direction of the lines of force due to the current in the conductor. Similarly, if a fixed vertical bar magnet be surrounded by a ring trough full of mercury up to about the level of the mid-point of the magnet and a circuit pivoted on the end of the magnet be arranged to carry a current, as shown in Fig. 245, the forces acting on the two arms of the conductor will evidently tend to keep the circuit in rotation. If a coil of wire carrying a strong current be substituted for the magnet the same effect takes place as the result of the forces acting between the two circuits.

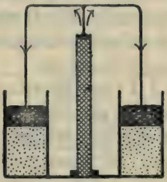


Fig. 245.

In any of these cases of rotation, the work done during rotation is measured (Art. 128) by the product of the current strength and the number of lines of force cut by the circuit. In the case of Fig. 245, for example, if m be the strength of the magnet pole, the number of lines of force emanating from it is $4\pi m$. In one revolution *each* branch of the conductor cuts all these lines, and, therefore, the total work done during a revolution is $8\pi m\mathrm{C}$ units. The moment of the couple causing rotation is therefore given by $8\pi m\mathrm{C}/2\pi$ or $4m\mathrm{C}$ and depends only on m and C. The energy necessary to maintain continuous rotation is of course derived from the chemical action in the battery supplying the current.

138. Ampere's Theorem. It has been shown in Art. 83 that the potential of a magnetic shell in a magnetic field is $\Phi\mathbf{F}$, where Φ is the strength of the shell and $\mathbf{F}$ the flow of

force through the contour of the shell from its negative side to its positive side. It has also been shown in Art. 128 that the potential of a closed circuit carrying a current C in a magnetic field is $-\mathrm{C}\,\mathbf{F}$, where C is the strength of the current and **F** the flow of force through the circuit from its negative to its positive side. It is evident that if $\Phi = \mathrm{C}$ these two quantities become equal. Ampère's theorem is based on this, and states that a closed circuit gives rise to the same field, and is subject to the same forces in a magnetic field, as a magnetic shell of the same contour as the circuit, if the strength of the shell is equal to the strength of the current. This shell is sometimes called the *equivalent magnetic shell* for the circuit. The electromagnetic unit of current (Art. 129) is chosen so that this provision is true, and it may be defined in this connection as that unit which makes the measure of the current strength the same as the measure of the strength of the equivalent magnetic shell.

All the theorems relating to magnetic shells can therefore be applied to a closed circuit by supposing it replaced by the equivalent magnetic shell. Thus, for a magnetic shell of strength Φ the potential at any point (Art. 83) is given by $\Phi\,\omega$, where ω is the angle which the contour of the shell subtends at the point. Hence, the potential due to a closed circuit at any point is $\mathrm{C}\,\omega$, where C is the electromagnetic measure of the current and ω the angle which the circuit subtends at the point. It should be noted, however, that this is the potential at the point, assuming that unit pole is brought from an infinite distance up to that point *without passing through the circuit.* If the pole passes through the circuit and back to the given point by a path outside the circuit, then the angle subtended by the circuit from points along the path of the pole changes from ω to $4\pi + \omega$, and the potential changes from $\mathrm{C}\,\omega$ to $\mathrm{C}\,(4\pi + \omega)$. If the path of the pole threads the circuit n times then the potential at the point becomes $\mathrm{C}\,(4 n \pi + \omega)$. This indicates that the work done on a unit pole in threading the circuit from any point back to the same point is $4\pi\,\mathrm{C}$.

As in the case of the shell it is also evident that the difference of potential between the two faces of the circuit

is $4\pi C$, the change being from $2\pi C$ to $-2\pi C$, and therefore the work done in carrying a unit north pole through the circuit from its negative to its positive side is $4\pi C$. Similarly, in a case of a long closely-coiled solenoid, the work done in moving a unit north pole for a distance l along the axis is $4\pi NC$, where N is the total number of coils passed through in the distance l. But if F denote the magnetic force along the axis, we have, since F is uniform

$$Fl = 4\pi NC,$$

or

$$F = 4\pi C\frac{N}{l} = 4\pi nC,$$

where n is the number of turns per unit length of the coil.

139. The Hall Effect. If a current passes between two points, P and Q, in a thin metal plate the lines of flow diverging from P converge to Q as shown in the diagram (Fig. 246), and the equipotential lines, everywhere at right angles to the lines of flow, run as indicated by the dotted lines in the figure. If this thin plate carrying a current be placed in a magnetic field with its plane at right angles to the field it is found that the system of lines of flow and therefore the equipotential lines, suffer distortion. The points P and Q remaining fixed, the lines appear as if rotated round the direction of the magnetic field as an axis. Looking along the lines of force the system of lines is deformed as if twisted to the right in iron, cobalt, zinc, antimony, and to the left in bismuth, nickel, and gold. In platinum and lead the effect is not perceptible. This effect is discovered experimentally by noticing the displacement of an equipotential line. If the point p be connected to one terminal of a sensitive galvanometer it is easy to find another point q at the same potential as p by adjusting the position of the point of contact with the other terminal until no current passes through the galvanometer. Let this adjustment be

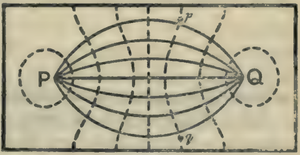


Fig. 246.

made before placing the plate in the magnetic field. It will then be found that when the plate is placed in the field a current at once passes through the galvanometer, indicating that p and q are no longer at the same potential. If the equipotential line initially passing through p and q be supposed to be rotated to the right by the action of the field, the point p evidently passes into a region of higher potential and q into one of lower potential than at first, and the current in the galvanometer will be from p to q. If, however, the equipotential line be rotated to the left the current will be from q to p. Hence, the direction of the current in the galvanometer indicates the sense of the distortion of the lines of flow and the magnitude of the current indicates the extent of the distortion. If e denote the difference of potential thus produced between two points equidistant from P and Q by a field of strength H, in the case of a plate of thickness t carrying a current C, it is found experimentally that

$$e = \mathrm{A}\frac{\mathrm{C}\,\mathrm{H}}{t}$$

where A is a constant under given conditions.

The Hall effect is very marked in bismuth, and it is associated with the fact that, under the conditions of the experiment, the specific resistance of bismuth increases to such an extent that its variation has been applied to the measurement of a magnetic field.

CHAPTER XXII.

OHM'S LAW.

140. Electromotive Force. Electromotive force has been defined generally as that which causes motion of electricity. The absolute electromagnetic unit of electromotive force is defined in Chapter XXVI., and is approximately equivalent to $1/3 \times 10^{10}$ of an electrostatic unit of potential. The unit of electromotive force in practical use is, however, equal to one hundred million (10^8) absolute electromagnetic units, and is called a **volt.** The electromotive force, or, as it is usually written, the E. M. F. of a Daniell's cell is about 1·07 volts, and may therefore be taken as a rough realisation of the practical unit. The electromotive forces of different cells may be *compared* by comparing the differences of potential set up at their poles when no current is allowed to pass. Thus, if one pole of a cell be connected to one pair of quadrants of a quadrant electrometer, and the other pole and the other pair of quadrants earthed, then the two pairs of quadrants being unconnected no current passes, but a difference of potential equal to the E. M. F. of the cell is set up between them, and the needle of the electrometer is deflected through an angle proportional to it. By noting in this way the several deflections produced by the cells to be compared, we may by comparing the deflections determine the comparative electromotive forces of the cells; and if the E. M. F. of any one cell be known absolutely, this method may be applied to determine the absolute E. M. F. of any given cell. For example, if the E. M. F. of a Daniell's cell be taken as 1 volt, and it produces a deflection of 100 divisions on the electrometer scale, then the E. M. F. of a cell giving 190 divisions deflection is evidently 1·9 volts.

The following table gives the approximate E. M. F. of the cells described in Art. 111.

Cell.	E. M. F. in volts.
Volta's simple cell	1 volt (about)
Smee's cell	0·7 ,, (,,)
Poggendorff bichromate cell ..	1·8 to 2·3 volts
Daniell's cell	1·1 volt (about)
Grove's cell	1·9 ,, (,,)
Bunsen's cell	1·9 ,, (,,)
Leclanché cell	1·4 to 1·6 volts
Latimer Clark cell (standard) ..	1·434 volt at 15° C.
Edison's Lalande cell	0·75 volt

141. The Latimer Clark Cell. The Latimer Clark cell is a standard cell of very constant E. M. F. Mercury and zinc are the elements; the mercury is at the bottom of the cell, and on it rests a paste made by mixing mercurous sulphate with zinc sulphate. The zinc element takes the form of a rod or strip of zinc immersed in the paste, but not going deep enough to touch the mercury. Contact is made with the mercury by means of a platinum wire passing through the bottom of the cell and forming the positive pole of the cell. Fig. 247 shows a convenient form of the cell; it is used only as a standard of E. M. F., and is soon spoiled if allowed to give a current.

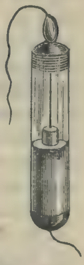


Fig. 247.

The following specification on the preparation of the Clark cell, issued by the Board of Trade in 1894, describes the cell fully:—

On the preparation of the Clark Cell.

"DEFINITION OF THE CELL."

"The cell consists of zinc, or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess.

"PREPARATION OF THE MATERIALS.

"1. *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled *in vacuo.*

"2. *The Zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass-paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

"3. *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water and repeat the process at least twice. After the last washing drain off as much of the water as possible.

"4. *The Zinc Sulphate Solution.*—Prepare a *neutral* saturated solution of pure ('pure re-crystallised') zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate and adding zinc oxide in the proportion of about two per cent by weight of the zinc sulphate crystals to neutralise any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30° C. Mercurous sulphate treated as described in 3, should be added in the proportion of about 12 per cent by weight of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

"5. *The Mercurous Sulphate and Zinc Sulphate Paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to ensure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30° C. Keep the paste for an hour at this temperature, agitating

it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case add more crystals from the stock bottle, and repeat the whole process.

"This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water.

"To set up the Cell.

"The cell may conveniently be set up in a small test-tube of about two centimetres' diameter, and four or five centimetres deep. Place the mercury in the bottom of this tube, filling it to a depth of, say, ·5 centimetre. Cut a cork about ·5 centimetre thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water some hours before use. Pass the zinc rod about one centimetre through the cork.

"Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

"Shake up the paste and introduce it without contact with the upper part of the walls of the test tube, filling the tube above the mercury to a depth of rather more than one centimetre.

"Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least

twenty-four hours before sealing, which should be done as follows:—

"Melt some marine glue until it is liquid enough to pour by its own weight and pour it into the test tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

"The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

"The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in the air.

"In using the cell sudden variations of temperature should as far as possible be avoided.

"The form of the vessel containing the cell may be varied. In the H form the zinc is replaced by an amalgam of ten parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one eg of the cell, and with the mercury in the other, by means of platinum wire sealed through the glass."

The Board of Trade form of the cell constructed in accordance with this specification, is shown full size in Fig. 248. The different materials employed and the space allotted to each are shown to scale.

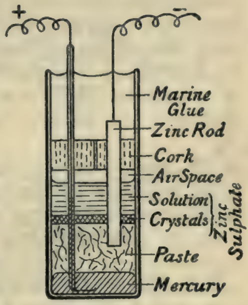


Fig. 248.

When a number of similar cells are arranged *in series* so as to form a battery, the E. M. F. of the battery is found by summing up the E. M. Forces of the component cells. Thus, the E. M. F. of a battery of n similar cells is n times the E. M. F. of one cell. If, however, the

cells are arranged *in parallel*, then they practically constitute one large cell, and the E. M. F. of the arrangement is the same as that of one cell. Similarly, if $m\,n$ cells be arranged in n rows, each row containing m cells arranged in parallel and the rows connected in series, the E. M. F. of the arrangement is n times the E. M. F. of one cell, for we have practically n large cells in series, each cell being m times as large as a single cell.

142. Current. The absolute electromagnetic unit of current has already been defined in Art. 129, but the practical unit, called the **ampere,** is only one-tenth of this absolute unit. The quantity of electricity which a current of one ampere conveys in one second past any section of the conductor is called the **coulomb,** and is equivalent to 3×10^9 * electrostatic units of quantity as defined in Art. 6.

The quantity of electricity conveyed by a current C in time t is given by the relation $Q = C\,t$. If C is expressed in amperes and t in seconds, then Q is given in coulombs.

The beginner should realise that when a current travels round a continuous undivided circuit the strength of the current is the same at all points in the circuit. But in the case of a *divided circuit*, such as that shown in Fig. 249, when the current reaches the point of division, a, it splits up along the several branches meeting at a, and, as explained in the next article, the relative strengths of these branch currents depend upon the relative resistances of the branches.

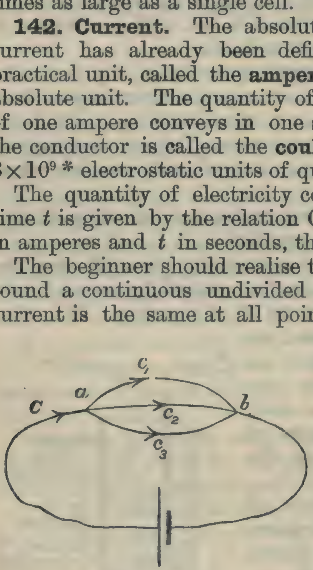


Fig. 249.

If C denotes the *total* current flowing in the circuit, and c_1, c_2, and c_3 the branch currents, then we evidently must have

$$C = c_1 + c_2 + c_3;$$

* The electromagnetic unit of quantity is 3×10^{10} electrostatic units, and the coulomb, being $\frac{1}{10}$th the electromagnetic unit, is 3×10^9 electrostatic units.

that is, $C - c_1 - c_2 - c_3 = 0$, or the *algebraic* sum of the currents meeting at points a and b is zero. This result is an example of *Kirchhoff's first law*, which states that in any network of wires carrying currents the algebraic sum of the currents meeting at any point is zero.

143. Resistance. The resistance of a conductor may be defined as the ratio of the electromotive force producing a current in the conductor to the strength of the current so produced.

Thus, if an electromotive force, E, applied to a given conductor, produce in it a current, C, then the resistance of that conductor is given by the ratio E/C. That is, if R denote the resistance of the conductor, we have

$$R = \frac{E}{C}.$$

If E and C be expressed in absolute electromagnetic units then R is also expressed in electromagnetic units, the absolute electromagnetic unit of resistance being the resistance of a conductor in which unit E. M. F. produces unit current. The practical unit of resistance is that resistance in which an E. M. F. of one volt produces a current of one ampere. It is called the **ohm,** and is equal to one thousand million (10^9) absolute units. It may conveniently be taken as the resistance of a column of mercury 106·3 cm. long, and having a uniform section of one square millimetre, the temperature of the mercury being 0° C.

In preparing practical standards of resistance, it is extremely difficult to arrange columns of mercury in a form convenient for general use; hence the resistances used in practical measurements are generally sets of coils of insulated wire wound on bobbins, and so arranged that they may be conveniently used in any desired combination. Such an arrangement of coils is called a *resistance box,* the bobbins carrying the coils being arranged inside a box. The ends of each coil are attached to thick pieces of brass fitted in rows on the lid of the box. These brass pieces are insulated from each other, but may be joined together by brass plugs, fitting into the spaces between them.

Fig. 250 shows diagrammatically how the coils of a resistance box are arranged, and how they may be combined so as to place any desired resistance in the circuit. The current enters at A and leaves at B. Coils of 1, 2, 2, and 5 ohms resistance are connected, as shown, to the brass pieces *a b c d e*, which may be connected together by the brass plugs *k l m n*. These plugs fit tightly into

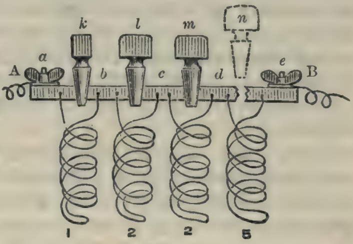


Fig. 250

circular gaps between the brasses, and it will be seen that when any one is removed the corresponding coil is thrown into the circuit. For example, when the plug *n* is removed, the coil of 5 ohms' resistance is placed in the circuit; if the plugs *k* and *n* are removed, then a resistance of 6 ohms is inserted; and it is evident that with these four coils it is possible to insert a resistance of any integral number of ohms between 0 and 10. When all the plugs are in, the only resistance between A and B is that offered by the intervening brasses and plugs, and these being massive and

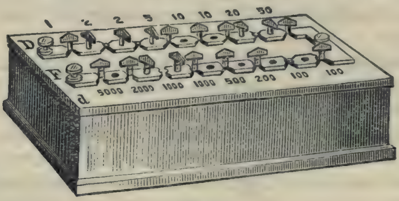


Fig. 251.

made of good conducting material, their resistance is practically *nil*, provided that the plugs fit well and the surfaces of contact are clean. Fig. 251 shows the general appearance of a resistance box capable of supplying a resistance

of any exact number of ohms between 0 and 10,000. Resistance boxes are also constructed so as to give fractional parts of an ohm. For example, a box containing four coils of ·1, ·2, ·2, ·5 ohm, in addition to the usual coils, would give any required resistance to ·1 of an ohm.

It will be noticed that the coils in a resistance box are arranged, with respect to magnitude, in the same way as a set of weights, and the resistance of any given conductor may be measured in a way exactly analogous to that employed in determining the weight of any body.

Let the given resistance be arranged in circuit with a cell or battery of constant action—say a battery of four Daniell's cells—and a delicate galvanoscope. When the current is steadily established, let the deflection of the galvanoscope needle be carefully noted. Now remove the unknown resistance, and substitute a resistance box in its place. Let the resistance in the box be adjusted until the galvanoscope deflection is exactly the same as at first. Then the resistance in the box must be equal to the resistance of the given conductor.

This simple *substitution* method of measuring resistance is, however, rather inconvenient in practice and is not often resorted to.

144. Specific Resistance. Experiment shows that the resistance of a conductor of uniform section, such as a wire or rod, varies directly with its length and inversely with its area of cross section. That is, the longer the conductor is the greater is its resistance, and the thicker it is the less its resistance. Hence, if l denote the length of a conductor of uniform cross section of area a, then

$$R \propto \frac{l}{a},$$

where R denotes the resistance of the conductor. That is,

$$R = S\frac{l}{a},$$

where S is a constant, depending in magnitude upon the material of the conductor—the greater the conductivity of any material, the smaller the value of S for that material.

The constant S for any substance is thus a *specific* constant for that substance. This constant for any substance is known as the *specific resistance* of that substance. If we consider a conductor of unit length and unit area of cross section, then

$$R = S$$

that is, the *specific resistance* of any substance may be defined as the resistance of a conductor of that substance of unit length and unit area of cross section—for example, the resistance between two opposite faces of a unit cube of the substance.

The *specific conductivity* of a substance is sometimes defined as the reciprocal of the specific resistance of the material. Accepting this definition, the *conductivity* of any conductor may be defined as the reciprocal of its resistance Thus the conductivity, k, of a conductor of resistance R is given by

$$k = \frac{1}{R}.$$

145. Resistance of Conductors in Parallel. If two conductors, a and b (Fig. 252), of resistance r_1 and r_2, are arranged *in series* in the circuit, the combined resistance of the two is simply $r_1 + r_2$. If, however, they

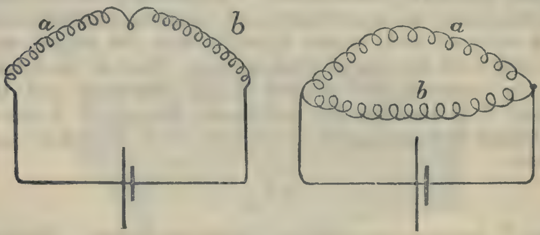


Fig. 252.

are *in parallel* or *in multiple arc*, as shown in the right-hand figure, it is evident that their combined resistance is *less* than either of them, for the current has now two paths open instead of only one. The *conductivity* of the two conductors arranged as here shown is evidently equal to the

sum of the conductivities * of the individual conductors a and b. That is, if k denote the conductivity of the combined conductors, then $k = k_1 + k_2$, where k_1 and k_2 denote respectively the conductivities of the conductors a and b. Hence we have

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2},$$

where R denotes the resistance of the combined conductors; that is

$$R = \frac{r_1 r_2}{r_1 + r_2}.$$

Similarly, if any number of conductors of resistances $r_1, r_2, r_3 \ldots r_n$ be arranged in parallel, their combined resistance, R, is given by

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_n}.$$

Considering the relation

$$R = \frac{r_1 r_2}{r_1 + r_2},$$

it is evident that if $r_1 = r_2 = r$, say, then

$$R = \frac{r}{2}.$$

That is, the combined resistance of two conductors of *equal* resistance arranged in parallel is half that of either. This result is evident at once from first principles, for, if the current has two exactly similar paths through which it may divide, then the resistance of the double path will be just one-half that of either of the single paths by itself. Similarly, the combined resistance of n *equal* resistances arranged in parallel is $1/n$th that of any one.

When a current flows through a *divided circuit* such as that formed by the conductors a and b in Fig. 249, then the current divides along the separate branches: the portion of the total current in any one branch *is proportional to the conductivity of that branch.*

* When the conductors are arranged in series they increase the *resistance* of the circuit, but when arranged in parallel they increase the *conductivity* of the circuit.

Thus, if c_1 and c_2 denote the currents in a and b respectively, then

$$\frac{c_1}{c_2}=\frac{k_1}{k_2}=\frac{1/r_1}{1/r_2}=\frac{r_2}{r_1}.$$

That is, the currents in the branches are *inversely* proportional to their resistances. For example, if the resistance of the conductor a be 2 ohms, and that of b be 3 ohms, then $\frac{2}{5}$ of the total current flows through b and $\frac{3}{5}$ through a, the *greater* current flowing in the conductor of *lesser* resistance.

146. Shunts. An important application of this principle of divided circuits is found in the use of *shunts.* It sometimes happens that the current to be passed through a galvanometer turns out to be too strong for the instrument. When this occurs, instead of changing the galvanometer for another, it is more convenient to join the terminals of the instrument by a wire of known resistance, thus allowing a known portion of the current to be shunted through the wire. The remainder of the current passes through the galvanometer, and may be regulated to any desired strength by varying the resistance of the wire across the terminals. This wire is called a *shunt*, and when a galvanometer has a shunt across its terminals, it is said to be shunted. In Fig. 253, which represents a simple circuit containing a cell and galvanometer, S represents the shunt and G the galvanometer. The gap at P represents a plug *key* similar to that shown in the figure, and by means of the key the shunt circuit may be opened or closed without disconnecting it from the galvanometer terminals. If G denote the resistance of the galvanometer, S the resistance of the shunt, and C the main current in the circuit, then the

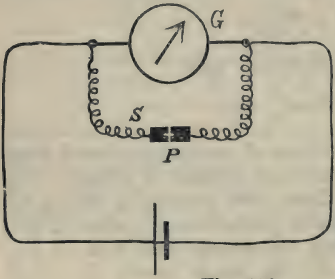
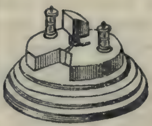


Fig. 253.



P

current *through the galvanometer* is equal to $SC/(G+S)$, and if S and G are known, the measure of this current given by the galvanometer evidently serves to measure C. When the resistance of the shunt is $\frac{1}{9}$th the resistance of the galvanometer ($S = G/9$), then the current through the galvanometer is $\frac{1}{10}$ the main current. Similarly, if $S = G/99$ or $G/999$, the current in the galvanometer will be $\frac{1}{100}$th or $\frac{1}{1000}$th of the main current. Shunts of this kind are generally supplied with the galvanometer with which they are to be used, and are called the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th shunts, or the $\frac{1}{10}$th, $\frac{1}{100}$th, or $\frac{1}{1000}$th shunts, according as resistance or current is implied. In general, $S = G/(n-1)$ where S is the shunt resistance required in order that $1/n$th of the main current may flow round a galvanometer of resistance G. Properly adjusted shunts of this kind are made up in boxes, like small resistance boxes, with convenient arrangements for using any one of the three. For rough general purposes, however, a piece of wire joining the terminals of the instrument to be shunted acts as an efficient shunt, the portion of the current diverted through it being greater the less the resistance of the wire.

The reason given above for the use of shunts is not the only one; they are used generally in practical methods of measurements for a great many different purposes.

The resistance between the terminals of a galvanometer of resistance G, when shunted by a shunt of resistance S, is given by $SG/(S+G)$, and is always less than G; hence, when a galvanometer is shunted, the resistance in the circuit is diminished, and consequently the main current is increased. When it is desired to keep the main current constant, the shunt boxes used should contain *compensating resistances*, connected to each shunt in such a way that when any shunt is used a compensating resistance equal to $G - \{SG/(S+G)\}$ is put in the circuit.

147. Variation of Resistance. Experiment shows that the resistance of a conductor varies with its temperature, the resistance being greater the higher the temperature. If R_0 denote the resistance of a given conductor at 0° C., then the variation with temperature is such that for the resistance of $t°$ we may write

$$R_t = R_0\,(1+\alpha t),$$

where α denotes the coefficient of increase of resistance with temperature.

The value of α, the coefficient of increase of resistance with temperature, is, for pure metals, found to be about ·0037, showing that if α does not vary with the temperature, the specific resistance of a pure metal at the absolute zero of temperature would be zero. As a result of the recent progress in the liquefaction of gases, very low temperatures are now available, and Fleming and Dewar have measured the specific resistance of pure metals at temperatures as low as $-200°$ C. The results of their measurements are given below, and the mean values of α given in the last column indicate generally that the specific resistance of a pure metal tends to zero value at the absolute zero of temperature.

Metal.	Specific Resistance in Absolute Units.		Mean value of α between 0° and -200°C.
	0°C.	-200°C.	
Platinum	10974	3340	·0035
Copper	1564	289	·0040
Silver	1490	390	·0037
Iron	9115	1220	·0043

Owing to this variation of resistance with temperature, it is evidently necessary when stating the specific resistance of any substance to specify the temperature for which the resistance is given.

Special cases of the variation of specific resistance are the variation of the resistance of bismuth in a magnetic field and the decrease in the resistance of selenium caused by exposure to light. The resistance of tellurium and carbon are also affected by light.

148. Ohm's Law. In its simplest form, Ohm's law states that the current produced in any portion of a homogeneous conductor lying between two equipotential cross

sections at different potentials is directly proportional to the difference of the potentials of these two sections. That is, if C denote the current produced by a difference of potential or electromotive force E, then

$$C \propto E.$$

In this form, however, the full significance of the law is not at once apparent. The *law* involved is perhaps best explained by reference to the definition of resistance given in § 143. The resistance of a conductor is there defined as the ratio of the E. M. F. applied to the conductor to the current produced in it; that is,

$$R = E/C.$$

Now, Ohm's law states that, assuming resistance to be defined in this way, the resistance of a given conductor is constant for all values of E and C, for, if C is proportional to E, then the ratio of E to C must be constant. That is, a given conductor presents the same resistance to all currents, and is independent of the E. M. F. producing the current. Hence, combining our definition of resistance with Ohm's law, we may write

$$C = \frac{E}{R},$$

where C denotes the current produced by an electromotive force E in a conductor of resistance R, and R is a constant for a given conductor. The relation, $C = E/R$, which states that the current produced in a conductor of resistance R by an E. M. Force E is given by the ratio E/R, is usually taken as the full expression of Ohm's law.

If E be expressed in volts and R in ohms, then C is given in ampères; that is,

$$C \text{ (ampères)} = \frac{E \text{ (volts)}}{R \text{ (ohms)}}.$$

This relation may be written in the form

$$E = CR;$$

that is, if a current C flow through a conductor of resistance R, then the E. M. F. driving this current *through this*

conductor is given by the product C R—or, if a current C is maintained in a circuit of resistance R, the E. M. F. in the circuit is C R. Thus, let a cell of electromotive force E and resistance r have its terminals joined by a wire of resistance R. Then the total resistance of the circuit is $(R + r)$, and if C denote the current in the circuit, we have

$$E = C(R + r).$$

Similarly, if e denote the difference of potential of the terminals of the cell, then

$$e = CR,$$

for this difference of potential is a measure of that portion of the E. M. F. of the cell spent in driving the current C through the wire joining the terminals. The application of these results supplies a simple and accurate method for the measurement of the internal resistance of a cell or battery. Let the terminals of the cell be joined to the terminals of a quadrant electrometer, and the deflection corresponding to the E. M. F. of the cell noted. Then without disconnecting the cell from the electrometer, let its terminals be joined by a wire of *known* resistance R; the presence of the electrometer will in no way interfere with the current, but the deflection indicating the difference of potential at the terminals will decrease from the value corresponding to E to a lower value proportional to e. Hence, if D and d denote these deflections respectively, we have

$$\frac{E}{e} = \frac{D}{d};$$

that is,

$$\frac{C(R + r)}{CR} = \frac{D}{d} \text{ or } \frac{R + r}{R} = \frac{D}{d},$$

and therefore

$$r = \frac{R(D - d)}{d}.$$

We have here applied the relation E = C R to a *restricted* portion of a circuit. This is a slight extension of Ohm's law, first made by Kirchhoff, and hence often called *Kirchhoff's second law.* For its simple application, however,

the restricted portion considered should be a simple conductor *containing no source of E. M. F.* When the relation $E = C R$ or $C = E/R$ is applied to a complete circuit, then R denotes the *total* resistance of the circuit, E the *total* effective E. M. F. in the circuit, and C the current in the circuit. Thus if three cells of electromotive forces e_1, e_2, e_3 and resistances r_1, r_2, r_3 be arranged, as in Fig. 254, in circuit with a galvanometer of resistance G and a wire of resistance W, then the total effective E. M. F. in the circuit is $e_1 + e_2 + e_3$, and the total resistance

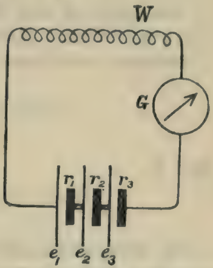


Fig. 254.

$$r_1 + r_2 + r_3 + G + W + x,$$

where x denotes the resistance of the connecting wires of the circuit. Hence, to determine the current in the circuit, we have

$$C = \frac{E}{R} = \frac{e_1 + e_2 + e_3}{r_1 + r_2 + r_3 + G + W + x}.$$

In general x is so small that it need not be considered. If, in the above arrangement, one of the cells, say that of electromotive force e_3, be reversed, then, assuming $e_1 + e_2$ greater than e_3, the total effective E. M. F. in the circuit would be $e_1 + e_2 - e_3$, and the current would be given by

$$C = \frac{e_1 + e_2 - e_3}{r_1 + r_2 + r_3 + G + W + x}.$$

It should be noticed that the reversal of the cell in the circuit does not alter the total resistance, that is, we still write $r_1 + r_2 + r_3$.

Similarly, if a battery of n cells *arranged in series*, each of electromotive force E and resistance r, give a current through a circuit in which the total *external* resistance is denoted by R, then the strength of the current is given by

$$C = \frac{nE}{R + nr}.$$

If, however, the same n cells be arranged *in parallel*, the E. M. F. of the battery is that of one cell, E; but its resistance is $1/n$th the resistance of one cell, for the cross section of the path of the current in the cell in now n times as great as before. Hence, the current produced in a circuit of external resistance R is given by

$$C = \frac{E}{R + \frac{r}{n}},$$

that is,

$$C = \frac{nE}{nR + r}.$$

By comparing these two expressions we can readily compare the efficiency of the two arrangements. It will be seen that the two values for C differ only in the fact that for the *series* arrangement the resistance of the cell, r, is multiplied by n, while in the parallel arrangement the external resistance R is multiplied by n. Now the value of C will be greatest when the denominator of the equivalent expression is least; hence, the series arrangement gives the better current when r is less than R, and the parallel arrangement is better when R is less than r. The resistance of a cell is generally small; hence we may say generally that parallel grouping should be employed when the external resistance is very small, and the series arrangement when it is comparatively large. When $r = R$ it is immaterial which arrangement is adopted, but the greater R is compared with r the greater the advantage of the series over the parallel arrangement, and the smaller R is compared with r the greater the advantage of the parallel grouping. For example, with ten cells, each having a resistance of 2 ohms and an E. M. F. of 2 volts, let us compare the current given with the two arrangements (*a*) through an external resistance of 1 ohm, (*b*) through an external resistance of 1000 ohms. Here, for the case (*a*) we have

(i) By series arrangement—

$$C = \frac{10 \times 2}{10 \times 2 + 1} = \frac{20}{21} \text{ ampère.}$$

(ii) By parallel grouping—

$$C = \frac{2}{\frac{2}{10}+1} = 1\tfrac{2}{3} \text{ ampères.}$$

And for the case (*b*) we have

(i) By series

$$C = \frac{10 \times 2}{10 \times 2 + 1000} = \frac{20}{1020} = \frac{1}{51} \text{ ampère.}$$

(ii) In parallel—

$$C = \frac{2}{\frac{2}{10}+1000} = \frac{20}{10002} = \frac{1}{500} \text{ ampère (nearly).}$$

That is, with the small external resistance the parallel arrangement gives the better current, but with the large external resistance the current given by the series arrangement is almost ten times as great as that given by the parallel grouping. The series arrangement of case (*b*) shows such a decided advantage because the external resistance of 1000 ohms is very much greater than the 2 ohms internal resistance of the cell.

If we have $m\,n$ cells, each of electromotive force E and resistance r, arranged in n rows in series, each row containing m cells grouped in parallel, then the current given by this grouping through an external resistance R is given by

$$C = \frac{n\mathrm{E}}{n \cdot \frac{r}{m} + \mathrm{R}}$$

for the arrangement practically consists of n large cells each of electromotive force E and resistance r/m. It can be shown that with a given number of cells the most advantageous grouping is that which gives the internal resistance of the arrangement equal to the external resistance. Thus, in the case just considered, to obtain the best current we must have

$$n \cdot \frac{r}{m} = \mathrm{R},$$

and combining this condition with the known value of $m\,n$, the values of n and m can be determined for any given

case. For example, to determine the best* method of grouping 24 cells each of 4 ohms resistance to give a current through the external resistance of 6 ohms, we have

$$4\,n/m = 6 \ldots (1).$$

and

$$m\,n = 24 \ldots (2).$$

From (1) we get $n = 3\,m/2$,

and substituting this value, in (2) we have—

$$m^2 = 16, \text{ or } m = \pm 4.$$

That is, the cells should be arranged in six rows connected in series, each row containing 4 cells in parallel.

The experimental verification of Ohm's law is based upon the following considerations. Let A B (Fig. 255) represent a conductor carrying a *constant* current C. Then, considering a portion, *a b*, of the conductor, the E. M. F. driving the current C through this portion, that is, the difference of potential, *e*, between the points *a* and *b* is given by

$$e = C r$$

where *r* denotes the resistance of *a b*.

But *since C is supposed to be constant*, we may write

$$e = (\text{constant}) \times r,$$

or

$$e \propto r$$

that is, the difference of potential between any two points in a conductor carrying a *constant* current is proportional to the resistance of the portion of the conductor included between these points. Now this deduction is very readily put to the test of experiment—the difference of potential between any two points may be measured by a quadrant electrometer as already explained, and with a resistance box in the circuit a number of points with different known resistances lying between them are readily available.† For

* The *best* method here means the method giving the *strongest* current. It is not the most economical method.

† Strictly speaking the resistance between different points on a uniform wire should be taken, for Ohm's law is assumed in the adjustment of the coils of a resistance box, and the experiment here described in reality verifies only the accuracy of the adjustment.

example, if Fig. 250 represent the portion of the circuit including the resistance box, then, all the plugs being out, let the quadrant electrometer be properly connected to the brasses *a* and *b*. A deflection proportional to the difference of potential between *a* and *b* is then obtained, and this difference of potential corresponds to a resistance of *one* ohm between *a* and *b*. Let this deflection be noted, and the electrometer terminals next connected to the brasses *a* and *c*. A deflection proportional to the difference of potential between the points *a* and *c* is now obtained, and this difference of potential corresponds to a resistance of *three* ohms between the points *a* and *c*.

Now, if Ohm's law be true, this second deflection should be exactly three times as great as the first, for the law states that under the conditions of this experiment the difference of potential between any two points is proportional to the resistance between those points. Here then, since the resistance between the points *a* and *c* is three times as great as that between *a* and *b*, the difference of potential, and therefore the electrometer deflection, should also be three times as great. Similarly, if the electrometer be connected, say, to the points *d* and *e*, the deflection should be five times as great as the first deflection for the points *a*, *b*,—and so on. Careful experiments of this nature entirely support Ohm's law, and show that it is, as far as can be detected, rigorously true.

In conducting this experiment, care must be taken to secure an absolutely constant current in the circuit—this may be done by using a battery of, say, a dozen Daniell cells working through a circuit of very *large* resistance. Further, the experimenter must be careful not to alter the resistance of the circuit during the experiment. Connection is readily made with the brass pieces of the resistance box if they are drilled with small holes, to take a small plug carrying a binding screw.

So far we have been accustomed to use the quadrant electrometer whenever it is required to measure the difference of potential between any two points, but in practice a high resistance reflecting galvanometer is much more

convenient to use, and, provided its resistance is high enough, it gives practically the same results as an electrometer.

Suppose, for example, that the terminals of a galvanometer are connected to the points a and b in Fig. 255. The current through the galvanometer will be proportional to the difference of potential existing between the points a and b *after the galvanometer has been joined on.* Now if the galvanometer is of low resistance, the total resistance

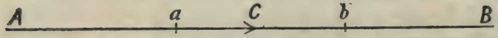


Fig. 255.

between a and b will be greatly diminished by connecting the galvanometer to these points, and therefore, by Ohm's law, the difference of potential between a and b is greatly diminished by joining on the instrument intended to measure it. If, however, the resistance of the galvanometer is very large compared with the resistance of $a\,b$, then the total resistance between a and b will be very little changed when the galvanometer connections are made, and therefore the deflection produced will be proportional to the difference of potential between a and b when these points are united only by the conductor $a\,b$. Considered in this relation, an electrometer acts as a galvanometer of infinite resistance, and therefore the resistance and difference of potential between a and b are unchanged when its terminals are connected to these points.

149. Comparison of Electromotive Forces. The quantitative comparison of electromotive forces of voltaic cells or batteries is an important part of electric measurements. We shall here notice a few only of the more important methods. Theoretically, the simplest method of comparison is by means of the quadrant electrometer—one terminal of the cell or battery is connected to one pair of quadrants and the other terminal and pair of quadrants earthed; the deflection obtained is proportional to the difference of potential between the terminals—that is, to the electromotive force of the cell, since the external circuit is not closed. A comparison of deflections obtained in this

way with different cells gives a comparative measure of their electromotive forces, and if the E. M. F. of one cell as a standard (for example Latimer Clark's cell) be known in volts, the E. M. Forces of the other cells can also be expressed in volts. A good form of quadrant electrometer is, however, somewhat troublesome to put in working order, so that in applying this method it is simpler in practice to employ a galvanometer of a very high resistance instead of the electrometer, or a condenser and ballistic galvanometer combined in the way described below.

Wiedemann's Sum and Difference method is an easy convenient method of comparing the E. M. Forces of any two cells not of too nearly equal E. M. F. The cells to be compared are arranged first *in series*, so as to send a current in the *same* direction round a circuit made up of the cells, a tangent galvanometer and a resistance box, and second *in opposition*, so as to send currents in opposite directions round the *same* circuit. The resistance of the circuit is adjusted by means of the resistance box, so that suitable deflections are obtained on the galvanometer. Then if R denote this resistance, which must be the same in both cases, and E_1 and E_2 the E. M. Forces to be compared, we have, for the first arrangement,

$$C_1 = \frac{E_1+E_2}{R} = k \tan \delta_1,$$

where δ_1 is the deflection of the galvanometer needle, and for the second arrangement

$$C_2 = \frac{E_1-E_2}{R} = k \tan \delta_2,$$

where δ_2 is the galvanometer deflection,

Whence we get

$$\frac{E_1+E_2}{E_1-E_2} = \frac{\tan \delta_1}{\tan \delta_2},$$

or,

$$\frac{E_1}{E_2} = \frac{\tan \delta_1 + \tan \delta_2}{\tan \delta_1 - \tan \delta_2}$$

A convenient arrangement of apparatus is indicated in Fig. 256.

The mercury cups *a, b, c, d,* can be connected by two short bent pieces of copper wire; and when *a c* and *b d*

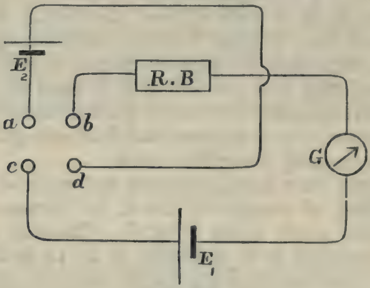


Fig. 256.

are connected the cells are in series, but when *a b* and *c d* are connected, they are in opposition.

Another important method of comparing E. M. Forces is that known as *Poggendorff's Compensation Method.* The

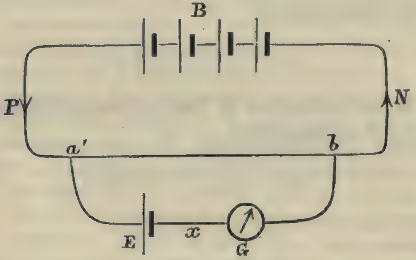


Fig. 257.

principle of this method consists in balancing the E. M. F. of the cells to be compared against the difference of potential between two points in the external circuit of a battery of considerably higher E. M. F. than that of either of the cells. Let B (Fig. 257) represent the battery and P N its external

circuit, the resistance of which is sufficiently great for the difference of potential between the terminals of the battery to be greater than the highest of the E. M. Forces to be compared. The difference of potential between any two points a, b, on the external circuit will depend upon the resistance between these points being equal to C R, where C denotes the current in the circuit and R the resistance of the portion, $a\,b$, of the circuit. If the points a b, be connected by a wire, $a\,x\,b$, a current, due to the difference of potential between a and b, will flow through it in the direction $a\,x\,b$; but if a cell, E, be included so as to tend to give a current in the opposite direction in the wire, it is possible, by properly choosing the position of the point of contact b, to obtain a balance between the E. M. F. of the cell E and the difference of potential between the points a, b. When this balance is obtained there will be no current in the branch $a\,x\,b$, and there will be no deflection of the galvanometer G placed in the circuit. Hence, if the point of contact b be moved along the main circuit until the galvanometer G shows no deflection, then we have the E. M. F. of the cell equal to the difference of potential between a, b, the points of contact—that is, $E = C\,R$. Thus, if two cells of E. M. Forces, E_1 and E_2, are to be compared, the above adjustment is made with each separately, and we have

$$E_1 = C\,R_1 \text{ and } E_2 = C\,R_2,$$

where R_1 and R_2 are the resistances in each case between the points a, b, when balance is attained. Hence we get

$$\frac{E_1}{E_2} = \frac{C\,R_1}{C\,R_2} = \frac{R_1}{R_2}.$$

If the external circuit P N consists of a long uniform wire, the resistance between any two points on it is proportional to the length of wire between the points. Hence in this case the E. M. Forces of the cells will be directly proportional to the length of the wire necessary to be included between the points a, b, to obtain balance in each case. An external circuit of this kind is sometimes called a *Potentiometer*, on account of its application to the measurement of potential in this way.

It should be noted that this method is applicable to the absolute measurement of E. M. F., for we have $E = C R$, and if C is measured in some way, say by a tangent galvanometer or voltameter, and R is known, then E can be determined absolutely. This is practically the method adopted by Lord Rayleigh in determining the E. M. F. of the Latimer Clark Cell.

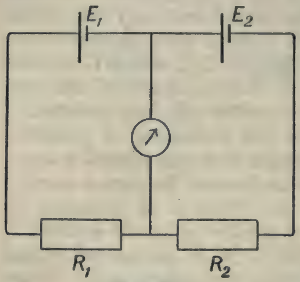


Fig. 258.

Lumsden's method of comparing electromotive force is indicated in Fig. 258. One or both of the resistances, R_1, R_2, is adjusted until the galvanometer shows no deflection. By a simple application of the method explained in Art. 152 it can be shown that when the adjustment for zero current is made, we have

$$\frac{E_1}{E_2} = \frac{R_1 + B_1}{R_2 + B_2},$$

where B_1 and B_2 are the internal resistances of the cells. As these resistances are unknown, R_1 and R_2 should be large enough to make them negligible. We then have

$$\frac{E_1}{E_2} = \frac{R_1}{R_2},$$

The E. M. Forces of two cells or batteries may also be compared by means of the ballistic galvanometer. Let a condenser of capacity C be charged first by the cell of E. M. F. E_1 and discharged through the galvanometer, giving a deflection d_1, and afterwards charged by the other cell of E. M. F. E_2, and again discharged through the galvanometer producing a deflection d_2. Then we get

$$\frac{E_1}{E_2} = \frac{E_1 C}{E_2 C} = \frac{d_1}{d_2}$$

— that is, the E. M. Forces are directly proportional to the deflections.

This is a simple and fairly accurate method of comparing electromotive forces. By means of a key known as a discharging key the above operations of charging and discharging are readily and easily performed, but the student will more readily and satisfactorily learn the practical detail of the method in a laboratory than from a text-book.

150. Comparison of Resistances. In this article we shall consider only the simpler methods based on the *Wheatstone's Bridge* arrangement.

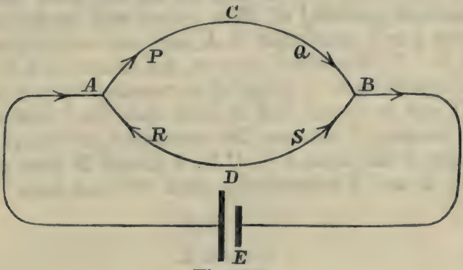


Fig. 259.

In the circuit shown in Fig. 259, the cell E sends a current round a divided circuit; and the difference of potential between the extremities of each branch of the divided circuit is the same; being, in each case, the difference of potential between the points A and B on the main circuit. Hence, in going from A to B along either of the branches A C B, A D B, the same fall of potential takes place, and points can therefore be found in each branch at which the potential is the same as at corresponding points in the other branch. Let C and D be two such corresponding points. Then the potential at C is the same as at D—that is, the fall of potential from A to C is the same as that from A to D. Now, the fall of potential between any two points *on the same conductor* is

proportional to the resistance between these points. Hence it will be evident that the fall from A to C will be the same as that from A to D when the resistance from A to C is the same fraction of the resistance of A C B as the resistance from A to D is of A D B. For example, if the difference of potential between A and B is 3 volts, and the resistance of A C B and A D B respectively 15 ohms and 9 ohms, then the fall of potential along A C B is one volt for every five ohms resistance, and along A D B one volt for every three ohms resistance; and if the resistance of A C is 10 ohms (two-thirds of A C B) then the resistance of A D must be 6 ohms (two-thirds of A D B) in order that the points C and D may be at the same potential. The common potential of these points will in this case be two volts lower than that at A, and one volt higher than that at B. It is evident that on the branches A C B and A D B any number of pairs of corresponding points, such as C, D, can be found, and in each case the following proportion must hold between the resistances involved: let the resistance of the *arms* A C, C B, A D and D B be denoted by P, Q, R and S respectively, then we must have

$$P : Q :: R : S,$$

that is

$$PS = QR.$$

Now, it is evident that if three of the four resistances P, Q, R, S be known, the fourth can be determined from this relation; or, if *the ratio* of two of them, say P : Q, be known, and another, say R, be known, then the fourth, S, can be determined; provided the four resistances P, Q, R and S are connected and adjusted by experiment in the way indicated above.

The loop A C B D, made up of the four *arms* A C, C B, A D, D B, is known as the *Wheatstone bridge* or *Wheatstone net* arrangement; it may be applied to the measurement of resistance in the following way.

Let two known resistances, P and R,* constitute the two arms A C and A D, and let the arm C B be a resistance box, and the arm D B the resistance to be

* Their resistances need not be known if their ratio is known.

measured. Then, if the terminals of a sensitive reflecting galvanometer G (Fig. 260) are connected to the points C, D, it will be possible to adjust for equality of potential of the points C and D; for, if these points are at the same potential, then no current will flow through the galvanometer—that is, there will be no deflection of the galvanometer needle. This adjustment may be made by adjusting the resistance in the resistance box which constitutes the arm C B, for since the ratio between the resistances of A C and A D is known, the adjustment will be complete when the resistance in the box bears the same ratio to the unknown resistance D B.

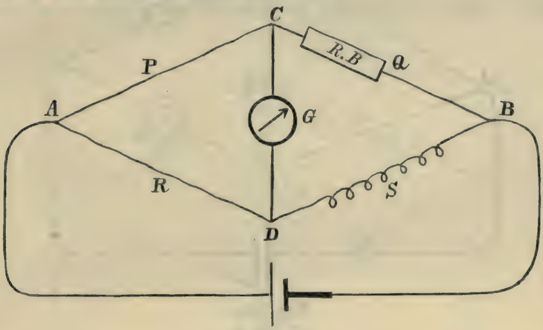


Fig. 260.

The details of the construction and practical use of the different forms of Wheatstone's bridge used in the measurement of resistance are best learnt in the laboratory, and for this reason we shall not give any further description of the arrangement.

This Wheatstone's bridge method of comparing resistance is directly adapted to the measurement of simple resistances, but it may be modified so as to be applicable in some other cases. For example, *Thomson's* method of measuring the resistance of a galvanometer by its own deflection is a modification of the method just described. The galvanometer is placed in the arm B D (Fig 261), and

the points C, D, are connected through a key K, which admits of the connection between C and D being made and broken at will. The resistance in the resistance box in C B is then adjusted until the deflection of the galvanometer is the same, whether the key K makes contact between C and D or not. This adjustment evidently depends on the same principle as the one just described: if the points C and D are at the same potential, it is evidently immaterial whether the key makes contact or not, for in either case no

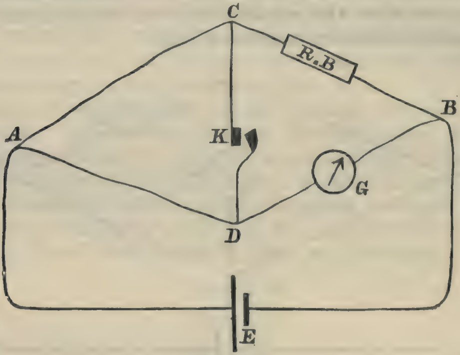


Fig. 261.

current will flow along C D and consequently the current through the galvanometer will not be changed. If, however, C and D are not at the same potential, then when the key is depressed and makes contact between C and D, a portion of the current passing through the galvanometer is shunted along C D and the deflection of the galvanometer is changed. Hence, when the resistance in C D is so adjusted that the depression of the key has no effect on the galvanometer deflection, the resistance of the galvanometer can be determined from the relation $PS = QR$; or if, as assumed above, $P = R$, then $Q = S$—that is, the resistance out in the resistance box when the adjustment is complete gives the resistance of the galvanometer.

Another application of this method is *Mance's* method of measuring the internal resistance of a cell or battery. It evidently would not do to place the cell in the arm B D and attempt to measure its resistance as if it were a simple resistance; for being a source of E. M. F., the current produced by it in the circuit would disturb the adjustment. In Mance's method the cell whose resistance is to be measured is the only one in the circuit, and it is placed in the arm B D, and the place of the cell E is taken by a key K (Fig. 262): that is, just as in Thomson's method the *galvanometer* whose resistance is to be measured is placed in the arm B D and the usual place of the *galvanometer* taken by a key, so in this method the *battery* whose resist-

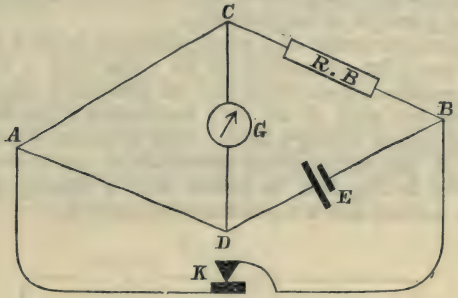


Fig. 262.

ance is to be measured is placed in the arm B D and the usual place of the *battery* is taken by a key. The adjustment is exactly as in Thomson's method,—the resistance in the arm C B is adjusted until the deflection of the galvanometer is not affected by the working of the key, and the resistance of the battery determined by calculation from the relation P S = Q R. The principle of this adjustment is slightly different to that in the two cases just described; here the adjustment does not depend upon equality of potential of the points C D, whether the key K is opened

or closed, but upon the constancy of the difference of potential between these points whether the circuit through K be made or not. The conditions of this adjustment are however the same as before, for when K is closed the resistance between the points A, B, is lessened and the difference of potential between these points changes. Hence, when the key is closed, a sudden change takes place in the difference of potential between the points A, B, and this change will not, or will, make itself felt between the points C, D, according as the relation $PS = QR$ holds or not.

For clearness in describing the above methods we have always supposed the resistance box to be in the arm B C, the unknown resistance in B D, and so on, but this arrangement is evidently not essential—any one of the resistances may be placed in any one of the arms without effecting the general principle of adjustment.

The measurement of a small resistance is most satisfactorily made by connecting it in series with a standard small resistance and comparing the differences of potential between the ends of the two resistances when the same current passes through both. The unknown small resistances, R, and the standard resistance, S, are connected, as indicated in Fig. 263, in circuit with a battery giving a steady current and a regulating resistance X.

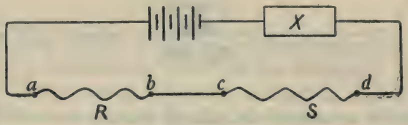


Fig. 263.

The difference of potential between the points a, b, is compared with the difference between the points c, d, by connecting the terminals of an electrometer or of a high resistance galvanometer to the points a, b, and c, d, and taking the ratio of the deflection produced. This gives

$$\frac{e_1}{e_2} = \frac{d_1}{d_2},$$

but $e_1 = \mathrm{C\,R}$ and $e_2 = \mathrm{C\,S}$, the current C being supposed to be constant. Hence

$$\frac{\mathrm{R}}{\mathrm{S}} = \frac{d_1}{d_2},$$

or

$$\mathrm{R} = \mathrm{S}\frac{a_1}{d_2},$$

and as S is a known standard resistance R is determined. This is a good method to adopt in measuring specific resistance, for the resistance of a rod of a cross section which can be accurately measured can be compared with a suitable standard low resistance.

Carey Foster's method of finding accurately the difference between two nearly equal resistances should be noticed. The method is specially useful in comparing copies of a standard resistance with the standard, for the defect of the copy can be directly determined. It is also the best method for determining the coefficient of increase of resistance with temperature. If x_0 and x_t denote the differences at 0° C. and t° C. between a given resistance and a nearly equal standard resistance maintained at a constant temperature, as determined by this method, then $x_t - x_0$ is the increase of resistance between 0° C. and t° C. and

$$\frac{x_t - x_0}{r_0 t} = a,$$

where r_0 is the resistance at 0°. This quantity r_0, can be measured, with sufficient accuracy, by the ordinary Wheatstone bridge method; it is the *change* of resistance with temperature that requires accurate measurement.

For Carey Foster's method a wire Wheatstone's bridge, with four gaps, as shown in Fig. 264, is required. The resistances, R and S, whose difference has to be determined are placed in the *end* gaps, and must be looked upon as extensions of the wire of the bridge, so that the bridge wire really extends from A to B, including R and S and the usual metre bridge wire and having only the middle section, the metre wire, graduated. The two resistances, x and y, placed one in each of the two remaining gaps are

merely two nearly equal resistances inserted so as to form permanently two arms of the bridge. The battery and galvonometer are connected as shown in the figure and an exact balance obtained in the usual way. Let a represent the position of the jockey on the wire when balance is obtained with R and S in the positions shown in the figure. The positions of R and S are now interchanged and balance

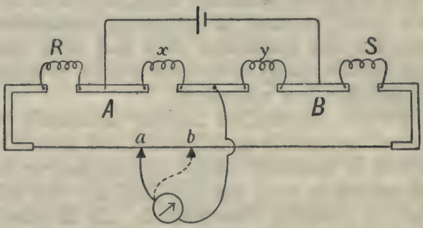


Fig. 264.

again obtained with the jockey at the point b on the wire Since the total resistance from A to B, round the wire, is constant, it is evident that the interchange of R and S would have no effect on the balance, if R and S were equal, but, if R is greater than S, for example, then, on interchange, the arm A a, now containing S instead of R, will be too small by the difference between R and S, and the B a arm now containing R will be too large by the same amount. If, however, the jockey be moved from a to b, so that the resistance of $a\,b$ is equal to the difference between R and S then it is evident that the conditions for exact balance are restored. The resistance of the length of the bridge wire between the two points on it at which exact balance is obtained for the two positions of R and S is therefore the difference between the two resistances, R and S. In the case considered in the figure, R is evidently greater than S by the resistance of the portion $a\,b$ of the bridge wire. If l denote the length of $a\,b$ and ρ the resistance per unit of length, then

$$R - S = l\rho.$$

If S is taken as a standard, then

$$R = S + l\rho.$$

In order to determine ρ the same method is adopted. For S a suitable low resistance standard is taken, and R is replaced by a copper bar of practically zero resistance. The difference between this standard, S, and the copper bar is then determined in the usual way. This difference, $S-R$, is practically $S-0$ or S, and if L be the distance between the two balance points on the wire for the two adjustments, we have

$$S = L\rho \text{ or } \rho = S/L.$$

This is the average value of ρ over the length L. It is evidently possible to calibrate the wire and find the value of ρ at any point on it by slightly varying the ratio of x to y, step by step, and repeating the measurements at each variation. In this way the length l may be moved along the wire from one end to the other and a value of ρ obtained for each position. In order to facilitate the interchange of the resistances and to eliminate errors due to connection resistances a special form of bridge is usually employed for this method.

151. Resistance of Liquids. Liquids, like solids, offer resistance to the passage of the current. The amount varies greatly with the nature of the liquid; thus turpentine, paraffin, and pure water are practically insulators, while mercury is a good conductor. In electrolytes the process of conduction is of a special character (Chap. XXIV.) but Ohm's law applies to it, and their conductivity is fairly high. The resistance of solutions is found in general to depend on their strength, diminishing as the latter increases, up to a certain point, and then increasing until saturation is attained; thus in the case of sulphate of copper the least resistance is found to correspond to $CuSO_4 + 45\ H_2O$, and in that of sulphate of zinc to $ZnSO_4 + 24\ H_2O$.

The *specific resistance* of a liquid, like that of a solid, is defined as the resistance of a "centimetre cube" thereof, that is a cube each of whose edges is one cm. long, it being understood that the current flows through it in a direction perpendicular to a pair of opposite faces

Moreover, as in the case of solids, if S denote the specific resistance and R that of a column of uniform cross section a (of any shape) and length l, we have

$$R = Sl/a$$

The most important case of liquid resistance is that of an electrolyte, and here its practical measurement is attended with difficulties arising from the back E. M. F. which generally accompanies electrolysis and which renders most of the methods employed for metals, *e.g.* the Wheatstone Bridge, inapplicable. Moreover, owing to polarisation, this back E. M F. does not remain constant for any length of time. In the following method both these difficulties are obviated:—

The solution is contained in a tall cylindrical jar of known internal cross-section, a square centimetres; at the bottom rests a platinum electrode just fitting the jar and connected by a gutta-percha-covered wire to one of the poles of an external battery; the other electrode similarly connected to the other pole is also of platinum and of the same size *but is arranged so as to be capable of sliding up and down the jar.* A tangent galvanometer and contact-breaker are placed in the circuit between one of the electrodes and the corresponding battery-pole.

Now let E denote the E. M. F. of the battery, E′ the back E. M. F. of the solution, and X the joint resistance of the battery, galvanometer, and connecting wires, all these being unknown. Also let S be the specific resistance of the solution which it is our object to determine. To do this we first place the electrodes a distance l_1 centimetres (ascertained by measurement) apart, make contact and note the galvanometer-deflection d_1; if k denote the reduction factor of the instrument we have, by Ohm's law, etc.,

$$\frac{E-E'}{X+Sl_1/a} = k \tan d_1 \ldots . (1).$$

We next change the distance between the electrodes to a new (measured) value l_2 and note the deflection, d_2, *the operation being performed quickly so as not to allow time for*

the back E. M. F. appreciably to alter. We then have in like manner

$$\frac{\mathrm{E}-\mathrm{E}'}{\mathrm{X}+\mathrm{S}\,l_2/a} = k \tan d_2 \ldots . (2).$$

Lastly, we change the distance to a third value l_3, again operating quickly, when we have

$$\frac{\mathrm{E}-\mathrm{E}'}{\mathrm{X}+\mathrm{S}\,l_3/a} = k \tan d_3 \ldots . (3).$$

Dividing (2) and (3) respectively by (1) we obtain

$$\frac{\mathrm{X}+\mathrm{S}\,l_1/a}{\mathrm{X}+\mathrm{S}\,l_2/a} = \frac{\tan d_2}{\tan d_1} \quad \text{and} \quad \frac{\mathrm{X}+\mathrm{S}\,l_1/a}{\mathrm{X}+\mathrm{S}\,l_3/a} = \frac{\tan d_3}{\tan d_1}$$

from which two equations we can eliminate X and find S.

A better method devised by Kohlrausch is a Wheatstone's bridge method. The liquid may be placed in a jar or tube fitted with electrodes as described above. The usual connections are made, but the secondary of an induction coil is used instead of a battery, and a telephone instead of a galvanometer. The adjustment is correct when the buzzing sound heard in the telephone, due to the action of the make and break of the induction coil, is reduced to zero or to a minimum. The resistance of the column of liquid is then calculated in the usual manner.

A galvanometer may be used in this method instead of a telephone, if a low tension alternating current is used and a commutator arranged to give a direct current through the galvanometer.

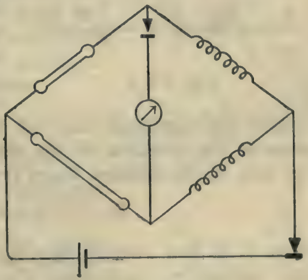


Fig. 265.

One of the best methods of measuring the resistance of an electrolyte is that known as the Stroud-Henderson method. The electrolyte is placed in two tubular cells A and B, exactly similar in all respects except length. These tubes, filled with the electrolyte, are

placed in the arms of a Wheatstone bridge as shown in Fig. 265.

The resistances P and Q are set equal, and the resistance R in the arm with the shorter cell is adjusted until balance is obtained. The currents in the electrolytic cells are equal, and the back electromotive forces due to polarisation will be almost exactly equal. Hence, when balance is obtained, the value of R gives the resistance of a column of the electrolyte of length equal to the difference in the lengths of the two tubes. This length and the area of cross section can be measured, and the specific resistance of the liquid thereby determined with considerable accuracy.

152. Kirchhoff's Laws. Kirchhoff's laws have already been stated. The first law is conveniently expressed by the relation $\Sigma(c) = 0$, where $\Sigma(c)$ denotes the algebraic sum of the currents meeting at a point in a network carrying a steady current. The second law in its simplest application is expressed by $E = CR$, where E denotes the difference of potential between two points on a conductor, C the current in the conductor, and R the resistance of the portion of the conductor between the points. If, however, the portion of the conductor between the points considered contains sources of electromotive force then the law is expressed by $E = CR + \Sigma(e)$, where $\Sigma(e)$ denotes the algebraic sum of the opposing electromotive forces. That is, E, the difference of potential between the points is able to give a current C through the resistance R *and* to balance the algebraic sum of the opposing electromotive forces in the portion of the circuit considered. Thus in Fig. 266, if the current flows from A to B we have

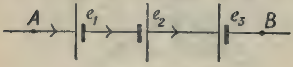


Fig. 266.

$$E = CR + e_1 - e_2 + e_3,$$

where C is the current flowing from A to B and R is the total resistance, including the resistance of the cells, between A and B. These laws may be applied to establish a number of important results. Taking the case of a divided circuit. If the current at A (Fig. 259) divides

between the two branches A C B and A D B of resistance r_1 and r_2 giving currents c_1 and c_2 in these branches, then the difference of potential between A and B is given by $c_1 r_1$ or $c_2 r_2$. Also if R denote the joint resistance of the two branches joining the points A and B the difference of potential between the points is given by $(c_1 + c_2)$ R. Hence we have

$$c_1 r_1 = c_2 r_2 = (c_1 + c_2) \mathrm{R}$$

From
$$c_1 r_1 = (c_1 + c_2) \mathrm{R}$$

we get
$$\mathrm{R} = \frac{c_1}{c_1 + c_2} r_1$$

and from
$$c_1 r_1 = c_2 r_2$$

we have
$$\frac{c_1}{c_2} = \frac{r_2}{r_1} \text{ or } \frac{c_1}{c_1 + c_2} = \frac{r_2}{r_1 + r_2}$$

and therefore
$$\mathrm{R} = \frac{r_2 r_1}{r_1 + r_2}.$$

This establishes the result obtained in Art. 145, and, by the extension of the method, it can be shown that for any number of branches in parallel arc we have

$$\frac{1}{\mathrm{R}} = \Sigma \left(\frac{1}{r}\right).$$

Again, if we have two coils joined in parallel and connected to a circuit, as shown in Fig 267, Kirchhoff's laws may be applied to determine the currents c_1, c_2, and c_3. Employing the notation indicated in the figure we have

$$c_1 + c_2 = c_3$$

and
$$c_1 r_1 - e_1 = c_2 r_2 - e_2 = - c_3 r_3,$$

each being equal to the difference of potential between A and B. From the three equations here available c_1, c_2 and c_3 can be determined. It will be found that

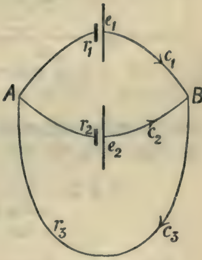


Fig. 267.

$$c_3 = \frac{e_1 r_2 + e_2 r_1}{r_1 r_2 + r_2 r_3 + r_2 r_1}.$$

If, in working out any particular case of this, or any other arrangement, the value obtained for any current should be negative in sign, the interpretation evidently is that the current flows in a direction opposite to that assumed in obtaining the equations.

The general application of this method to determine the current in each branch of a complicated network is extremely troublesome both in forming and in working out the equations. The cyclic method suggested by Maxwell enables the equations for any network to be written down easily, and by means of determinants the equations can be readily solved. Maxwell's method assigns to each mesh of the network a cyclic current of specified value which is supposed to flow round the boundary of each mesh in the same direction. The current in any branch will thus be the difference between the cyclic currents of the meshes it separates. The rule for forming the equations is, for each mesh, to multiply the cyclic current of the mesh by the resistance of the boundary, to subtract from this the product of the cyclic current of each adjacent mesh into the resistance of the branch separating the two meshes, and to equate the final result to the electromotive force in the mesh boundary. Thus, taking the case of the *Wheatstone bridge* network, and assigning cyclic currents as indicated in Fig. 268, we get

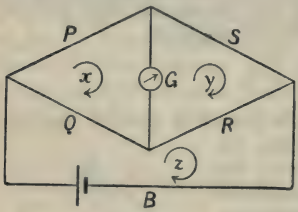


Fig. 268.

$$\begin{array}{lll} (P+Q+G)x & -Gy & -Qz = 0. \\ -Gx & +(R+S+G)y & -Rz = 0. \\ -Qx & -Ry & +(B+Q+R)z = E. \end{array}$$

By solving these equations the values of x, y, and z can be determined, and the current in any branch deduced from these values. Thus the current in the galvanometer is evidently $x-y$ or $y-x$, and the condition for no current

in the galvanometer is that $x = y$. When the current in only one branch is required, it is convenient to specify the cyclic currents in the meshes separated by this branch in such a way that their difference involves only one quantity. Thus, to determine the current in the galvanometer in Wheatstone's network, it is convenient to specify the cyclic currents as indicated in Fig. 269.

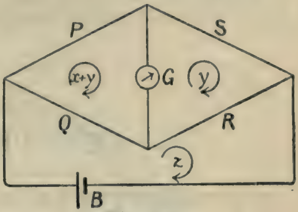


Fig. 269.

The current in the galvanometer is now denoted by x, and on forming the equations we get

$$\begin{aligned}(P+Q+G)(x+y) - Gy - Qz &= 0.\\ (R+S+G)y - G(x+y) - Rz &= 0.\\ (B+Q+R)z - Q(x+y) - Ry &= E.\end{aligned}$$

Simplifying these we get

$$\begin{aligned}(P+Q+G)x + (P+Q)y - Qz &= 0.\\ -Gx + (R+S)y - Rz &= 0.\\ -Qx - (Q+R)y + (B+Q+R)z &= E.\end{aligned}$$

Solving these for x we get

$$x = \frac{PR - QS}{D},$$

where D is a long expression involving P, Q, R, S, B and G. Without determining the value of D it is evident that $x = 0$, when $PR = QS$, that is, when $\frac{P}{S} = \frac{Q}{R}$, as explained above.

CHAPTER XXIII.

MEASUREMENT OF CAPACITY IN ELECTROMAGNETIC UNITS.

153. Standard Condensers. A common form of standard condenser is shown in Fig. 270. It consists of two sets of sheets of tinfoil insulated by thin sheets of mica, as explained in Art. 45, and enclosed in a box. The two sets of tinfoil sheets are connected, one set to each of the two brass pieces on the ebonite top of the box. These brass pieces form the terminals of the condenser, and the brass plug fitted between them keeps the condenser discharged when not in use.

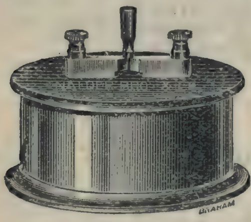


Fig. 270.

Lord Kelvin's standard condenser is shown in plan and section in Figs. 271, 272. It consists of two systems a and b of parallel metal plates, arranged, as shown in Fig. 271, so that the plates of one system alternate with those of the other. The plates of each system are bolted together by four vertical brass rods passing through the corners of the plates and exact parallelism of the plates is secured by means of accurately cut distance pieces or rings on the rods. One system of plates is fixed to the sole-plate of the condenser and the other set rests on three short glass pillars carried by screws working through the sole-plate. By means of these screws the plates of the

system carried by them can be adjusted parallel to those of the other system and so that any plate of one system is midway between the two adjacent plates of the other

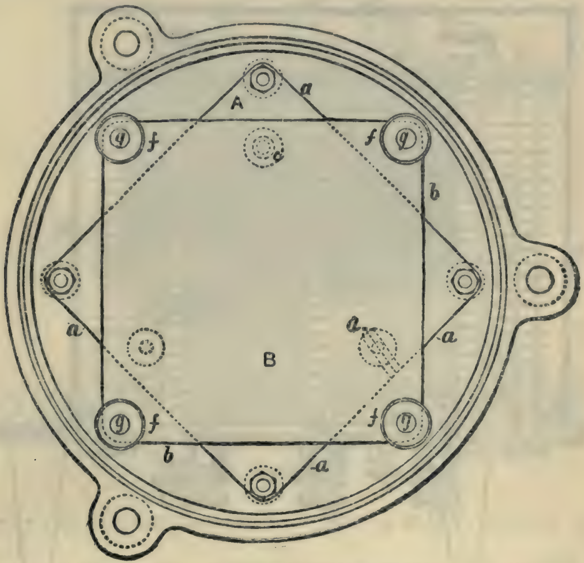


Fig. 271.

system. The plates are covered by a dust proof cover and the instrument rests on insulating vulcanite legs.

An adjustable standard condenser is shown in Fig. 273, and a plan of its connections is given in Fig. 274. It will readily be seen that when the plugs are in the back set of holes (Fig. 274) the component condensers are short circuited or connected for discharge, but when the plugs are in the front row of holes all the condensers are connected in parallel and their joint capacity is the sum of their individual capacities. It will be understood from

this that, by moving one or more plugs from the back to the front, it is possible to combine in parallel one or more

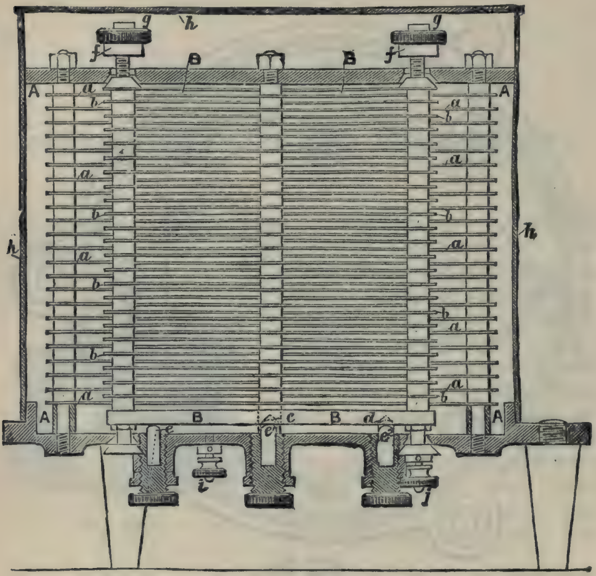


Fig. 272.

sections of the condenser. The terminals of the condenser are shown at A and B.

154. The time Constant of a Condenser. If a condenser is charged and then allowed to discharge through a very large resistance the difference of potential between its terminals falls according to a definite law, the rate of fall depending upon the capacity of the condenser and the magnitude of the resistance through which it discharges. Let C denote the capacity of the condenser, R the resistance

connecting its terminals, and V_1 the initial difference of potential between the terminals. At any instant during

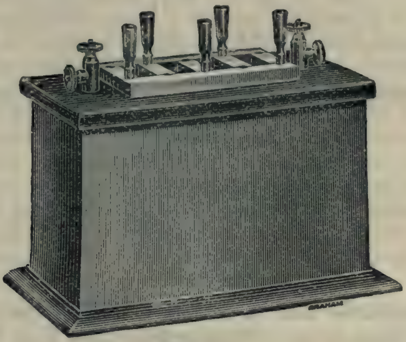


Fig. 273.

discharge the current through the resistance is evidently the time rate of decrease of the charge in the condenser That is,

$$c = -\frac{dQ}{dt}$$

where Q denotes this charge. Now $Q = C V$, where V is

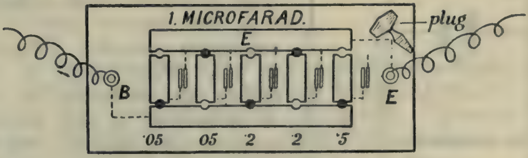


Fig. 274.

the difference of potential at the instant considered, and therefore

$$c = -C\frac{dV}{dt}.$$

Also $$e = \frac{V}{R},$$

and we get $$\frac{V}{R} = -C\frac{dV}{dt},$$

that is, $$\frac{dV}{V} = -\frac{dt}{CR}$$

Integrating this from the limit where $V = V_1$ and $t = 0$ we get $$\log\frac{V}{V_1} = -\frac{t}{CR}$$

or $$e^{-\frac{t}{CR}} = \frac{V}{V_1}$$

and $$V = V_1 e^{-\frac{t}{CR}}.$$

This indicates that in a time C R from the instant of charging the potential difference falls from V_1 to V_1/e. That is, in an interval of time measured by n C R the potential difference is reduced to $1/e^n$ of its initial value. The time C R is called the *time constant* of a condenser of capacity C discharging through a resistance R. The law of fall of potential difference with time is evidently such that as the time increases in arithmetical progression the potential difference decreases in geometrical progression, and for a series of time intervals, each equal to C R, the common ratio of the successive potential differences is $1/e$. The result of this article can evidently be used in measuring a very high resistance (R) through which a condenser of known capacity (C) is allowed to discharge slowly.

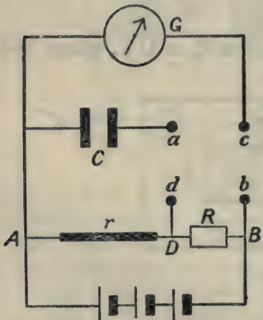


Fig. 275.

155. Measurement of capacity. The capacity of a condenser may be measured in electromagnetic units by the

following method. The Condenser is charged to a definite difference of potential, V, then discharged through a ballistic galvanometer and the throw of the galvanometer needle noted. The same difference of potential, V, or, as this is generally too great, a known fraction of V is then applied so as to give a steady current through the galvanometer. The steady deflection thus produced is noted and compared with the throw due to the discharge. The capacity of the condenser can then be calculated from the constants involved by the conditions of the experiment.

In practice the condenser may be connected as shown at C in Fig. 275, so that by connecting the points a, b, it may be charged up to the difference of potential between the points A, B, on the external circuit of a constant low resistance battery, and by connecting a, c, it may be discharged through the galvanometer C. If α be the first angular throw of the needle, corrected for damping, we have

$$Q = \frac{H\,T}{\pi\,G}\frac{\alpha}{2} \qquad \text{(Art. 135).}$$

That is $$C\,V = \frac{H}{G}\cdot\frac{T\,\alpha}{2\pi}, \quad \text{or} \quad C = \frac{H}{V\,G}\cdot\frac{T\alpha}{2\pi},$$

where V is the difference of potential between A and B.

To determine the value of H/V G here, let the points c, d, be connected so that a steady current passes from A to D through the galvanometer, and let δ denote the permanent angular deflection of the needle. Then the current c is given approximately by $$c = \frac{H}{G}\cdot\delta.$$

But if S, the resistance of the galvanometer branch, is very large compared with r, the resistance between A and D, we have $c = v/S$, where v denotes the initial difference of potential between A and D. Also, if R, the resistance between D and B, is large compared with r then

$$v = \frac{r}{R+r}V, \quad \text{and } c = \frac{r}{R+r}\,\frac{V}{S}.$$

That is $$\frac{r}{R+r}\cdot\frac{V}{S} = \frac{H}{G}\,\delta, \quad \text{or} \quad \frac{H}{G\,V} = \frac{r}{(R+r)S\,\delta}.$$

Substituting this value in the result obtained above we get

$$C = \frac{r\,T\,a}{2\,\pi(R + r)S\,\delta} = \frac{r\,T\,d_1}{2\,\pi(R + r)S\,d_2},$$

where d_1 and d_2 are linear deflections on the galvanometer scale, d_1 being corrected for damping.

This result is sufficiently accurate when r is small compared with R and S, but for exact calculation we have

$$V = \frac{R + r}{R + r + B}\,E,$$

where E is the electromotive force, and B the internal resistance of the battery, and

$$v = \frac{\rho\,E}{\rho + R + B} \quad \text{where} \quad \rho = \frac{rS}{r + S}.$$

The exact relation between v and V, assuming E to be constant, can thus be obtained, and the value of C deduced as above.

156. Comparison of Capacities. The capacity of two condensers may be readily compared by means of a ballistic galvanometer. Let the capacities of the two condensers be C_1 and C_2, and let them be, separately, first charged by a battery of constant E. M. F., E, and then discharged through the galvanometer. The quantities of electricity discharged are respectively $E\,C_1$ and $E\,C_2$, and if d_1 and d_2 are the corresponding galvanometer deflections corrected for damping, then,

$$\frac{E\,C_1}{E\,C_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}.$$

The accuracy of the result by this method evidently depends on the accuracy of observation of the deflections—and experiment shows that the errors attending this observation may be considerable. It is found that much more accurate results may in all cases be obtained by *null* or *zero* methods, in which the measurement depends upon adjusting for no deflection of the galvanometer. Two of the best known null methods for comparing capacity are given below.

The principle of the *Wheatstone bridge method* is indicated by Fig. 276. On pressing the key K the two condensers C and S are charged by the battery B. The charging current divides at A, the charge for C passing through the resistance P and the charge for S through the resistance Q. If the charge is divided in this way by P and Q in the ratio of the capacities of the condensers the potential of the corresponding terminals of the condensers C and S will rise in the same way, and no difference of potential will be established during the charging between two points *c* and *s*. Hence, if a sensitive galvanometer be placed across *c s* and the resistances P and Q adjusted until, on pressing K, there is no deflection of the galvanometer, it may be assumed that the charge divides at A in the ratio C : S, the capacities of the condensers. But the charges to C and S will, by Ohm's law, divide in the ratio Q : P. Therefore, when accurate adjustment is made we have

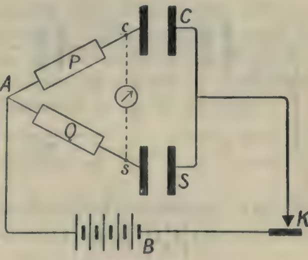


Fig. 276.

$$\frac{C}{S} = \frac{Q}{P}$$

and if S be the standard of known capacity then C is given by

$$C = \frac{Q}{P} S.$$

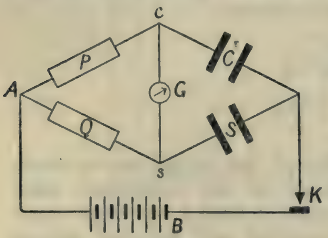


Fig. 277.

Fig. 277 shows how the *arrangement* of apparatus for this method corresponds to the Wheatstone bridge method of measuring resistance. The principle of the two methods is evidently not the same.

The method of mixtures is another satisfactory null method. The principle of the method is simple but the connections are somewhat complicated. By the arrangement shown in Fig. 278 the condenser C is charged to the difference of potential between A and B, and the condenser S to the difference of potential between the points B and C. By adjusting the resistances P and Q these two differences of potential can be made to have the inverse ratio of the capacities. When this adjustment is made we have—

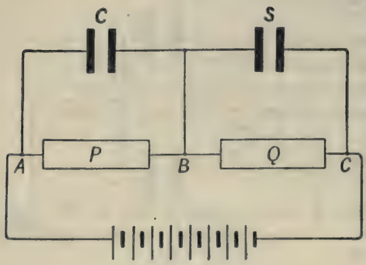


Fig 278.

$$\frac{{}_A V_B}{{}_B V_C} = \frac{S}{C}$$

or

$${}_A V_B \cdot C = {}_B V_C \cdot S$$

that is, the condensers possess equal charges, and since

$$\frac{{}_A V_B}{{}_B V_C} = \frac{P}{Q}$$

we evidently have for this adjustment

$$\frac{S}{C} = \frac{P}{Q}$$

or

$$C = \frac{Q}{P} S.$$

In order to test the equality of the charges and so make this adjustment the terminals of the condensers after charging must first be connected in such a way that oppositely charged plates are connected, and the two charges "mixed" so as to tend to neutralise each other. Then immediately this "mixing" is effected the combined condensers are discharged through a galvanometer. If the

charges before "mixing" were equal the final charge after mixing will be zero, and there will then be no deflection of the galvanometer. The operations of the method therefore consist of charging, mixing, and discharging through the galvanometer, and the required adjustment is made by adjusting P and Q until the galvanometer shows no deflection when the "mixed" condenser system is discharged through it. When this adjustment is made we have

$$C = \frac{Q}{P}S$$

when S is the capacity of the standard condenser. Fig. 279

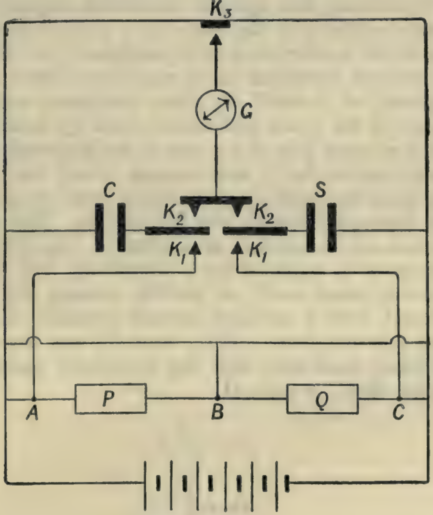


Fig. 279.

shows diagrammatically the connections for performing the three operations of the method in rapid sequence. By

pressing the K_1 keys charging is effected, the K_2 keys, when closed, arrange the "mixing," and on pressing K_3 the condensers C and S as one combined condenser are discharged through the galvanometer G. By using a Pohl's commutator without the cross wires the K_1 and K_2 contacts shown in the diagram between the condensers can each be effected at one operation.

In all these methods it is very essential that the keys and other pieces of apparatus shall give good insulation, and it is well to arrange that the whole of the apparatus is insulated from the earth, and that the wire connections do not touch each other or anything but the points to be connected.

In measuring the specific inductive capacity of any substance available in the form of a plate, but unsuitable for use in the construction of a condenser, the Wheatstone bridge method described above may be modified to compare the capacity of a small air plate condenser with a plate condenser of the given material, formed by pasting a sheet of tinfoil on each side of a plate of the material in such a way as to leave a fairly wide margin round the foil. This margin must be carefully and cleanly dried so as to prevent surface leakage between the foil sheets. The two small condensers are connected as shown in Fig. 276, but the galvanometer is replaced by a telephone, and the resistances are adjusted until, on rapidly making and breaking the current with a suitable contact breaker, no sound is heard in the telephone. If C_a and C_m are the capacities of the inner condenser, and the condenser under test respectively, then, by Art. 41,

$$C_a = \frac{A_a}{4\pi d_a}$$

and

$$C_m = \frac{K A_m}{4\pi d_m}$$

where A_a A_m denote the areas of the condenser plates, d_a and d_m the thickness of the dielectric in each condenser, and K the specific inductive capacity of the given material.

If when the Wheatstone bridge adjustment is made we have

$$\frac{C_a}{C_m} = \frac{P}{Q},$$

then

$$K = \frac{A_a}{A_m} \cdot \frac{d_m}{d_a} \cdot \frac{Q}{P}.$$

All the resistances used in capacity measurements should be non-inductive.

CHAPTER XXIV.

CHEMICAL EFFECTS OF CURRENTS.

157. Chemical Equivalence and Laws of Electrolytic Chemical Action. When dilute sulphuric acid acts on zinc, the chemical reaction which takes place may be represented by the equation

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

Here it will be noticed that one atom of zinc displaces two atoms of hydrogen, and hence we say that one atom of zinc is chemically equivalent to two atoms of hydrogen. The atomic weight of zinc is 65, and that of hydrogen is 1; therefore we may say that 65 grammes, or pounds, or tons of zinc are chemically equivalent to 2 grammes, etc., of hydrogen.

Similarly, from the equation

$$AgNO_3 + HCl = AgCl + HNO_3,$$

representing the reaction between silver nitrate and hydrochloric acid, we see that one atom of silver is equivalent to one atom of hydrogen. Combining this result with the previous one, we see that one atom of zinc is equivalent to two atoms of hydrogen, and also to two atoms of silver; that is, 65 grammes of zinc are equivalent to 2 grammes of hydrogen or to 216 grammes of silver. In this way, if the atomic weights and valency of the elements are known, we are at once able to state the mass of any element chemically equivalent to unit mass of hydrogen. The number expressing this mass is called the *chemical equivalent* of the element, and is evidently obtained by dividing the atomic weight of the element by its valency. Thus, zinc being a dyad, its chemical equivalent is $65/2 = 32{\cdot}5$; but silver being a monad, its chemical equivalent is $108/1 = 108$.

The simple **laws of chemical action** are two in number:—

(1) *The amount of chemical action in any one cell of a circuit is equivalent to that in any other cell.* This law applies to all cells in the circuit, whether battery cells or electrolytic cells.

Thus, if a current from, say, six Daniell's cells, be passed through a circuit of three voltameters arranged for the electrolysis of copper sulphate, silver nitrate, and water respectively, then the amount of copper deposited in *each* battery cell is the same as that deposited on the cathode and dissolved from the anode of the copper sulphate voltameter. Also, the amounts of zinc, copper, silver, oxygen, hydrogen, etc., liberated in the several cells are all chemically equivalent. For example, for every 108 parts of silver deposited in the silver nitrate voltameter, $32\frac{1}{2}$ parts of zinc are dissolved in *each* battery cell. By arranging a circuit of this nature, and determining quantitatively the amount of chemical action in each cell, this law may be put to the test of experiment, and will be found to be strictly true.

(2) *The amount of chemical action in any cell, in a given time, is directly proportional to the quantity of electricity which has passed through the cell in that time;* that is, if m denote the mass of a particular ion liberated in a given cell by a current C in a time t, then

$$m \propto C\,t,$$

for $C\,t$ denotes the quantity of electricity which has passed through the cell in the time t.

The mass of ion liberated in one second by a current of one ampère is called the *electrochemical equivalent* of the ion. If this equivalent for any element be denoted by w, then it is evident from this second law that the amount of the element deposited in a time t by a current C is given by

$$m = w\,.\,C\,t,$$

that is,

$$m = C\,w\,t.$$

The electrochemical equivalent of silver has been very carefully determined, and is found to be equal to 0·001118 gramme per coulomb; that is, a current of one ampère

passing through a solution of silver nitrate deposits silver on the cathode at the rate of 0·001118 gramme per second. From this determination for silver the electrochemical equivalent of any other element may be determined by calculation, for the *electrochemical* equivalents of any two elements are directly proportional to their *chemical* equivalents. Thus, if w denote the electrochemical equivalent of hydrogen, then

$$108 : 1 :: 0{\cdot}001118 : w,$$

or

$$w = \frac{0{\cdot}001118}{108} = 0{\cdot}00001035,$$

and the electrochemical equivalent of any element is readily found by multiplying this value for hydrogen by the chemical equivalent of the element. For example, the electrochemical equivalent of oxygen is

$$\frac{16}{2} \times {\cdot}00001035 = 0{\cdot}00008281 \text{ gramme per coulomb}$$

and that of zinc is

$$\frac{65}{2} \times {\cdot}00001035 = 0{\cdot}0003364 \text{ gramme per coulomb}$$

and so on.

It is evident that this law forms a basis for a method of measuring current. Thus, the ampère, the practical unit of current, may be taken as that constant current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0·001118 gramme per second. Hence, if an unknown current C be passed through a solution of silver nitrate in water, and deposit m grammes of silver in t seconds, then from the relation

$$m = C w t,$$

we have

$$C = \frac{m}{0{\cdot}001118 \times t} \text{ ampères.}$$

This definition of the ampère is that adopted in the Report to the Board of Trade by the Electrical Standards Committee, and appended to their Report is a specification giving detailed instructions for the use of a silver voltameter in the measurement of current. We quote a portion

of this specification which explains in a very clear way the different operations involved in the process.

"In employing the silver voltameter to measure currents of about one ampère, the following arrangements should be adopted. The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 cm. in diameter, and from 4 to 5 cm. in depth.

"The anode should be a plate of pure silver some 30 square cm. in area, and 2 or 3 mm. in thickness.

"This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver, which is formed on the anode, from falling on to the kathode the anode should be wrapped round with pure filter paper, secured at the back with sealing wax.

"The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

"The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

"*Method of making a measurement.* The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a dessicator. When thoroughly dry, it is weighed carefully.

"It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding screw is attached. This copper support must be insulated.

"The anode is then immersed in the solution so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are made.

"Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed.

Care must be taken that the clock used is keeping correct time during this interval.

"The solution is now removed from the bowl, and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol, and dried in a hot air bath at a temperature of about 160° C. After cooling in a dessicator, it is weighed again. The gain in weight gives the silver deposited.

"To find the current in ampères this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed and by ·001118.

"The result will be the time-average of the current, if during the interval the current has varied."

In laboratory practice it is often more convenient to use a small copper voltameter, made up with two copper plates immersed in a solution of pure copper sulphate, instead of a silver voltameter; and the method is seldom used for the direct measurement of current, but usually to determine the constant of a tangent galvanometer or other current-measuring instrument.

For this purpose the galvanometer and voltameter are arranged in series in the circuit, and precautions taken to secure a constant current. The current is then allowed to pass for a given time, and since the same current passes through the voltameter and the tangent galvanometer, say, then

$$C = \frac{m}{wt} = k \tan \delta,$$

where δ denotes the deflection of the galvanometer. Hence we have

$$k = \frac{m}{wt \, . \tan \delta},$$

and the constant k is thus determined in terms of m, w, t, and δ.

158. Electrolytic Conduction. The modern theory of electrolytic conduction is based upon the theory of dissociation. In an electrolyte, that is, in a solution susceptible of electrolysis, the dissolved substance is supposed to be either partially or completely dissociated into

its constituent *ions*. The more dilute the solution the more complete is the dissociation or ionisation, so that in a very dilute electrolytic solution practically all the molecules of the dissolved substance dissociate into free separate ions. Each of these dissociated ions carries its own appropriate charge of electricity, and therefore the chemical and general properties of an ion differ essentially from those of the same atom or group of atoms when free of charge. Thus in a dilute solution of K Cl, the K Cl molecule is dissociated into K and Cl ions. The K ion carries a positive charge and the Cl ion a negative charge, and so differ from free K and free Cl. The behaviour and properties of a particular ion are found to be independent of the other ion with which it may be associated, so that many of the properties of an electrolytic solution are additive properties determined by the properties of the ions present.

The explanation of the process of electrolysis on this basis is a simple one. When an external E.M.F. is applied so as to give a current through an electrolyte it is assumed that the influence of the E.M.F. is merely directive. It causes the free positively charged ions to travel in one direction through the solution and the free negatively charged ions to travel in the opposite direction. The charges carried by the ions are delivered up at the electrodes and the ions themselves liberated.

The experimental evidence on which this theory rests is found along a number of widely divergent paths of research. Kohlrausch and, later, Fitzgerald and Trouton have shown that the flow of current through an electrolyte is subject to Ohm's law. This implies that *in the electrolyte* no back E.M.F., due to molecular decomposition, is set up, and this is exactly what ought to obtain in a solution containing free dissociated ions, The gradient of potential set up in the liquid urges the positively charged ions in one direction and the negatively charged ions in the opposite direction without the development of back electromotive force. The back E.M.F. due to polarisation takes place at the electrodes where the ions give up their charges and not in the mass of the liquid.

The colour of electrolytic solutions is always an additive property. Thus, the colour of copper chloride molecules is yellow, a concentrated solution of the salt is green and a dilute solution blue. This is explained by assuming that in the concentrated solution dissociation is incomplete, and the green colour results from the combination of yellow due to undissociated copper chloride molecules and blue due to the copper ions, while in the dilute solution where the dissociation is complete the colour is the blue of the copper ions.

Measurements of osmotic pressure in electrolytic solutions also support the dissociation theory. Direct determination of osmotic pressure and determinations of the associated constants, the lowering of the vapour pressure and the lowering of the freezing point give abnormally high results for all electrolytes. These results are at once explained and become normal if it is assumed that the number of active molecules per unit volume of the solution is given, not by the number of molecules of the dissolved substance but by the number of free ions and molecules per unit volume.

The chemical activity of electrolytes is also found to depend upon the dissociation of the dissolved molecules.

In the experimental investigation of electrolysis the quantity which is most directly measured is the specific resistance of the electrolyte. The best method of measurement is that due to Kohlrausch, described in Art. 151. In applying the method it is usual to place the electrolyte in a glass cell with platinum electrodes, and from a measurement of resistance for an electrolyte of *known* specific resistance to obtain the *cell constant* by means of which the specific resistance of any other electrolyte may be determined. If an electrolytic solution contain m gramme equivalents per litre and the specific resistance of the solution be ρ, then $1/\rho$ is generally known as the conductivity, κ, of the solution, and κ/m is the *equivalent conductivity.* This constant is an important one in electrolysis. It is found that for solutions of salts the equivalent conductivity gradually increases as the concentration decreases, and for very dilute solutions is practically constant. That is, for very dilute

solutions, the conductivity is proportional to the concentration. In the case of acids and alkalies the equivalent conductivity increases as the concentration decreases, attains a maximum for very low concentration and then rapidly decreases. For all salt solutions the value of κ/m is practically the same; for acids and alkalies the maximum value is about three times that for neutral salts. The value of κ for a solution of K Cl containing 0·1 gramme equivalent per litre is $1{\cdot}119 \times 10^{-13}$ C.G.S. units.

The ratio of the number of active molecules in a dilute electrolyte to the number there would be if there were no dissociation can evidently be determined from the ratio of the value of κ/m for the solution to its limiting value for an infinitely dilute solution. This ratio can also be determined from the lowering of the freezing point of the solution. The table given below, due to Arrhenius, shows the agreement of the results obtained. The first column gives the ratio as determined by measurements of conductivity by Kohlrausch and Ostwald, the second gives the values as determined by Raoult from the lowering of the freezing point.

$NaOH$	1·88	1·96	·88
HCl	1·90	1·98	·90
HNO_3	1·92	1·94	·92
H_2SO_4	2·19	2·06	·60
KCl	1·86	1·82	·86
$NaCl$	1·82	1·90	·82

The third column gives the coefficients of ionisation indicating the proportion of dissociated molecules present in the solutions.

The conductivity of electrolytes increases with rise of temperature but the value of the coefficient of increase decreases with rise of temperature and increase of concentration, varying from ·035 for dilute solutions at 0°C. to ·015 for concentrated solutions at 18°C.

Another quantity which can be directly measured is the electrochemical equivalent of an element. This quantity is defined in Art. 157 as the mass of the element deposited by unit quantity of electricity. Hence if w denote the

electrochemical equivalent of hydrogen in grammes per electromagnetic unit, $1/w$ is the charge carried by one gramme equivalent of *any monovalent ion.* The value of w is $1{\cdot}036 \times 10^{-4}$, so that $1/w$ is about 9650 electromagnetic C.G.S. units. The charge carried by one gramme equivalent of any monovalent ion is therefore constant, and each individual monovalent ion may therefore be associated with a constant charge which furnishes a natural unit of electricity, called by Maxwell an atom of electricity. A divalent ion carries two such units, a trivalent ion three units, and so on.

159. Ionic Velocities. It will be seen that by the ionic theory the current through an electrolyte depends upon three things, (1) the number of ions involved, (2) the charge carried by each ion, (3) the velocity of the ions through the electrolyte. Thus, in a dilute solution of potassium chloride containing n gramme equivalents per cubic centimetre, let the charges carried by a gramme equivalent of the K and Cl ions be e and $-e$ units, and let the velocities of these ions be u and v cms. per sec. respectively. The current across one square cm. in the electrolyte will then consist of the transfer of neu units of positive electricity in one direction, and nev units of negative electricity in the opposite direction, that is, the current is measured by $ne(u+v)$ or $(n/w)(u+v)$ since $e = 1/w$. The current is also given by $\kappa \dfrac{dV}{dx}$, where κ is the conductivity of the liquid and dV/dx the gradient of potential in the direction of the current. Hence

$$\frac{n}{w}(u+v) = \kappa\frac{dV}{dx} \quad \text{or} \quad u+v = \frac{\kappa}{n}.w.\frac{dV}{dx}.$$

Here, if the concentration of the solution be expressed as m gramme equivalents per litre we have $m = n \times 10^3$ and the value of $(u+v)$ for a potential gradient of one volt per cm. $= 10^8$ e.m. units of potential per cm. is given by

$$u+v = \frac{\kappa}{m} \times 10^3 \times (1{\cdot}036 \times 10^{-4}) \times 10^8 = \frac{\kappa}{m} \times 1{\cdot}036 \times 10^7.$$

Hence $(u+v)$, the sum of the velocities of the ions, can be calculated from κ/m, the equivalent conductivity of

the electrolyte. The ratio of the velocities can be determined from measurements of the changes of concentration which take place near the electrodes during electrolysis, and by combining the two results the values of u and v can be separately determined. The table given below gives the values of $u + v$, u, and v in 10^{-6} cms. per sec. for K, Na, and Cl in solutions of KCl and $NaCl$ of different concentrations.

	KCl			NaCl		
m	$u+v$	u	v	$u+v$	u	v
0·0000	1350	660	690	1140	450	690
·0001	1335	654	681	1129	448	681
·001	1313	643	670	1110	440	670
·01	1263	619	644	1059	415	644
·03	1218	597	625	1013	390	623
·1	1153	564	589	952	360	592
·3	1088	531	557	876	324	552
1·0	1011	491	520	765	278	487

This table indicates (1) that the ionic velocity increases as the dilution increases and tends to a constant maximum value at infinite dilution; (2) that the limiting velocity of the same ion (Cl) is the same in different electrolytes, that is, the velocity is a specific constant of the ion.

The specific ionic velocities of some of the commoner ions are given below in 10^{-5} cms. per sec.

H	320	OH	162
K	66	Cl	69
Na	45	I	69
Ag	57	NO_3	64

From these velocities it is evidently possible to calculate the conductivity of dilute electrolytes. For example, for a dilute solution of $AgNO_3$, the value of $(u + v)$ is 121×10^{-5} cms. per sec., and substituting this value in the relation

$$u + v = \kappa/m \times 1{\cdot}036 \times 10^7,$$

the value of κ/m can be determined.

This exposition of ionic velocity was given by Kohlrausch in 1879. Since that date many direct experimental determinations of the velocities of ions have been made, and the results have confirmed Kohlrausch's theory.

The first determination was made by Lodge in 1885. Two vessels containing dilute sulphuric acid were connected by a tube containing a slightly alkaline agar-agar jelly solution of sodium chloride and a trace of phenol phthalein. A current was passed from one vessel to the other through the tube, and as electrolysis went on the velocity of transfer of the H ion was indicated and measured by the rate at which the phenol phthalein indication of the formation of H Cl travelled along the tube. Similar determinations of ionic velocity have more recently been made by W. C. D. Whetham and by Orme Masson. The methods adopted were slight modifications of Lodge's original method. In one experiment the rate of displacement of the boundary between equivalent solutions of potassium chloride and potassium bichromate, as indicated by the advance of the coloured ion $Cr_2 O_7$, was observed. In another a coloured cation, such as Cu from a solution of copper chloride, and a coloured anion, such as $Cr_2 O_7$ from a solution of potassium bichromate, were made to travel in opposite directions along a tube containing a jelly solution of K Cl, and the rate of progress of each ion observed.

160. Electrolytic Conduction in Gases. There is strong evidence that the passage of electricity through a gas is of the nature of electrolytic conduction. J. J. Thomson states that "chemical decomposition is not to be considered merely as an accidental attendant on the electrical discharge, but as an essential feature of the discharge without which it could not occur." In a simple gas the discharge involves dissociation of the molecules, in a complex gas decomposition may take place.

A number of phenomena give general support to this view of electric discharge in gases. Perrot's experiment, recently confirmed by Thomson, showing the decomposition of steam by the discharge from a coil, Grove's experiment showing the oxidation of a silver anode during electric

discharge in a gaseous mixture of hydrogen and oxygen, the well-known chemical changes produced in gases by electric discharge, the conductivity of the halogens when dissociated at a high temperature, and the conductivity along a path of recent discharge all suggest the electrolytic character of the conduction in gases. There is evidence too that in discharge through gases, the ions or atoms carrying the negative charge have a mass only one-thousandth that of a hydrogen atom. These ions have been assumed to be the ultimate particles of atoms and when associated with the charge appropriate to an atom have been called *electrons.* They are probably the active agents in the cathode rays. (See also Chap. XXXIV.)

161. Secondary Batteries. Viewed from the standpoint of energy, the principle of any voltaic cell is that we take certain substances which react chemically upon one another; during this process the chemical potential energy of the system diminishes, and the equivalent of the loss is obtained as electrical energy, which ultimately becomes all heat or partly heat and partly mechanical work according as the external circuit consists of a simple resistance or contains some kind of electromotor. In a *voltameter* precisely the reverse transformations occur. The salt employed is made to undergo decomposition, whereby the chemical potential energy of the system is augmented, and the increase is furnished at the expense of the electrical energy of the current employed to effect the electrolysis. If now, the latter having been performed, the external current be taken away and the plates of the voltameter connected by a wire, chemical action involving re-union between the products of electrolysis takes place, the chemical potential energy of the system diminishes and the equivalent of the loss is again obtained as electrical energy; in short *when the plates of a voltameter after electrolysis are connected it behaves as a battery.* This is the principle of "secondary batteries" or "accumulators" which are simply commercial forms of voltameter used mainly in connection with electric lighting; the process of performing electrolysis in them is known as "charging," while when subse-

quently used to furnish the current they are said to be "discharged."*

The pioneer of modern accumulators was *Grove's gas-battery.* It consists of two long glass tubes open at the bottom and closed at the top with brass caps having binding screws attached; inside each tube and extending nearly the entire length thereof is a platinum plate attached by a wire to the brass cap. The tubes are filled with very dilute sulphuric acid and immersed mouth downwards in a vessel of the same liquid, when the "battery" is complete. To "charge" it the binding screws are connected to the poles of an external battery when electrolysis occurs, hydrogen and oxygen accumulating and being partly absorbed or "occluded" by the platinum plates. The battery is then removed when the occluded gases will remain for a considerable time without escape, but as soon as the plates are connected the gases reunite and the battery is discharged. This instrument is, however, too feeble and the current furnished of too short duration for it to be of any practical value.

It should be noted that in this case the action between the materials deposited on the two plates takes place through the intervening molecules of liquid in accordance with the theory of Arts. 109 and 158. The same is true in the accumulators now to be considered.

In all accumulators used for commercial purposes the plates are of lead and the electrolyte is moderately dilute sulphuric acid (about one part by weight of H_2SO_4 to ten of water); during the process of charging the kathode remains in the metallic state, while the nascent oxygen liberated on the anode causes it to become coated with a layer of plumbic peroxide, PbO_2. After discharge both plates are found to be coated with plumbic sulphate, $PbSO_4$, and the usually received view of the action taking place during discharge is that here set forth.

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* It is a common popular notion that they are charged with electricity. Of course this is not the case; what they are really charged with is chemical potential energy.

Before discharge: $Pb \quad SO_4H_2 \quad SO_4H_2 \quad SO_4H_2 \quad SO_4H_2 \quad SO_4H_2 \quad O_2Pb$
After discharge: $PbSO_4 \quad H_2SO_4 \quad H_2SO_4 \quad H_2SO_4 \quad H_2O \quad H_2O \quad SO_4Pb$

Although the foregoing paragraph sets forth the general action of accumulators it omits several details. The earliest accumulators were devised by Gaston Planté in 1860. In these the plates prior to charging were two sheets of ordinary lead coiled together, but prevented from coming into actual contact by a layer of felt. On *once* passing the charging current the + plate became partly peroxidized while the − plate remained unchanged and being in a compact form was during discharge not at all easily attackable by the $S\,O_4$ so that the action was very poor. To remedy this the charging was made to consist of passing the current a great number of times in alternate directions; calling now the original anode A and the original kathode B, the first passage peroxidizes A leaving B unchanged, the second peroxidizes B and reduces the portion of A previously peroxidized to metallic lead *in a spongy state,* the third passage similarly reduces B while it peroxidizes at any rate some of the hitherto unattacked portions of A, and so on until eventually one of the plates contains a thick coating of the peroxide while the other consists largely of *spongy* metallic lead, and since this is more easily attacked than the compact sheet the discharging action is much more efficient. The process of charging the Planté cells was, however, very tedious, and to obviate this Faure in 1881 improved upon them by coating the plates to begin with with a layer of *red-lead* (Pb_3O_4); on charging, the red lead on the kathode became quickly reduced to spongy metal, while that on the anode was peroxidized. In this way much time was saved. In the more recent types of secondary cells the plates are made in the form of a grid, the hollows being filled with a paste of red lead; in this way they become more cohesive and less liable to fall to pieces. The cell plates are usually compound plates and arranged in "nests," the positive and negative nests locking into one another, while electrically separated by strips of wood or india-rubber.

After any form of accumulator has been discharged it

can be charged again by passing a current from an external source; the sulphuric acid thus becomes electrolysed, the nascent oxygen on the anode converts the $PbSO_4$ into $Pb\,O_2$ according to the equation

$$Pb\,S\,O_4 + O + H_2O = Pb\,O_2 + H_2\,S\,O_4$$

while the nascent hydrogen on the kathode reduces the $Pb\,S\,O_4$ to the spongy metallic state thus

$$Pb\,S\,O_4 + H_2 = H_2\,S\,O_4 + Pb.$$

The great advantages of secondary over primary batteries are (i) their low internal resistance, which is only about ·02 ohm per square foot of plate, and (ii) no appreciable action takes place in them when their terminals are unconnected. Their E. M. F. is about 2 volts per cell.

CHAPTER XXV.

HEATING EFFECTS OF CURRENTS.

162. Conservation of Energy in a Voltaic Circuit. In dealing with the conservation of energy in a circuit there are three essential points that require consideration. These are, first, the work done in the voltaic cell or battery; second, the work done in the external circuit; and third, the energy of the current. Of these the first—the chemical work done in the cell—is a source of energy in the circuit and gives rise to the other two, which must therefore be together equivalent to it.

The chemical work done in a voltaic cell in a given time is the mechanical equivalent of the heat which would be generated by the total chemical action going on in the cell in that time. Thus, in a Daniell's cell, if 32·5 milligrams of zinc are dissolved in one hour, then the chemical work done in the cell in the hour is the mechanical equivalent of the heat generated by the solution of 32·5 milligrammes of zinc in sulphuric acid, and the solution in water of the zinc sulphate so formed, *less* the heat absorbed by the deposition of 31·5 milligrammes of copper from copper sulphate, and the abstraction of the necessary copper sulphate from its solution in water.

The work done in the external circuit may be either of a chemical or magnetic nature; for example, the circuit may include a voltameter in which electrolytic chemical work may be done, or an electromagnet whose power of attracting iron may be employed in doing mechanical work. In either case the energy expended in doing work in the external circuit takes no part in maintaining the current in the circuit; only that portion of the cell's energy which is not spent in doing other work in the circuit is available

for the production of current, and the energy of the current so produced is entirely dissipated as heat in the circuit.

From what has been said it is evident that Ohm's law requires some modification, for, in accordance with this law, a battery of fixed electromotive force E, working in a circuit of resistance R, should give a constant current $C = E/R$. From what we have just said, however, it appears that the current in the circuit may be varied by varying the work to be done in the external circuit, for the less the work to be done, the greater the proportion of the battery's energy available for the production of current, and therefore the strength of the current should increase as the work done in the external circuit diminishes.

Now it is found that wherever work is done in the external circuit, "a back" E. M. F. is set up in opposition to the main E. M. F. of the battery, and in applying Ohm's law this back E. M. F. has to be considered. In the relation $C = E/R$, E denotes the *effective* or *resultant* E.M.F. in the circuit, and is therefore equal to the E. M. F. of the cell, *less* all back electromotive forces set up at points where work is done in the external circuit. When the external circuit consists only of simple metallic conductors then there is no back E. M. F. to be considered; but if a water voltameter, for example, be included in the circuit, then a back E. M. F. due to polarisation in the voltameter is set up; and if e denote this back E. M. F. and E the E. M. F. of the cell, then the current in the circuit, according to Ohm's law, is given by

$$C = \frac{E - e}{R},$$

where R denotes the resistance of the circuit.

It is important to notice that, in the case of a silver or copper voltameter, *no* back E. M. F. is set up, for in each of these cells the same metal is dissolved from the anode and deposited on the cathode, so that on the whole *no* chemical work is done and no polarisation takes place.

It is evident from this that the work done in maintaining the current for a given time may be determined by considering the work done in raising the quantity of electricity which flows through the circuit in that time, against a

difference of potential equal to the effective E. M. F. in the circuit.

Hence, if a current C flow for a time t round a circuit in which the *effective* E. M. F. is E, then a quantity of electricity, Ct, is raised through a difference of potential E, and the work done in maintaining the current for the time t is given by

$$W = C\,t\,E.$$

If C and E be expressed in electromagnetic units, and t in seconds, then W is given in ergs, but if C is in ampères and E in volts, then W is given in units of work called *Joules*, one joule being equal to 10^7 ergs.* Thus, C ampères $= \frac{1}{10}$C electromagnetic units, and E volts $= E \times 10^8$ electromagnetic units, and therefore

$$C\,t\,E \text{ joules} = \frac{C}{10}\,t \times E \times 10^8 \text{ ergs},$$

that is $$C\,t\,E \text{ joules} = C\,t\,E \times 10^7 \text{ ergs};$$

and therefore $$1 \text{ joule} = 10^7 \text{ ergs}.$$

The work done on the current *per second*, that is, the *activity* of the cell or battery in supplying the current, is evidently given by the product CE, and if C and E are expressed in practical units—ampères and volts respectively,—then the activity is given in units known as *Watts*, a watt being one joule per second, therefore equal to 10^7 ergs per second.†

In a voltaic cell the chemical reactions are essentially of an *exothermic* nature, that is, they are such as to produce under ordinary circumstances, an evolution of energy in the form of heat. In a voltaic cell this energy is not evolved as heat, but the electrical energy evolved is the equivalent of this heat energy. In a voltameter the chemical reactions are of an *endothermic* nature, that is, they are usually accompanied by an absorption of heat energy, and the presence of a voltameter in a circuit therefore means the absorption of an amount of electrical energy from the circuit equivalent to this heat energy. From

* In *ergs* because the electromagnetic system of units is based on the C.G.S. system, of which the *erg* is the unit of work.

† One horse-power = 746 watts (nearly).

these considerations the E. M. F. of a given voltaic cell or the back E. M. F. in a voltameter can be calculated if the necessary data are known. Thus if in a voltaic cell h gramme-degrees of heat are evolved by the total chemical action which takes place when one gramme of the negative element of the cell is dissolved, then Jh denotes the energy evolved in ergs, J being Joule's mechanical equivalent of heat in ergs per gramme-degree. But if E denote the required E. M. F. of the cell in volts, and w the electro-chemical equivalent of the negative element in grammes, then E/w denotes in *Joules* the energy evolved per gramme of the negative element. Hence

$$\mathrm{J}\,h = \mathrm{E}/w \times 10^7$$

or

$$\mathrm{E} = \frac{\mathrm{J}\,h\,w}{10^7}$$

in volts, or as $\mathrm{J} = 4{\cdot}2 \times 10^7$.

$$\mathrm{E} = 4{\cdot}2\,h\,w.$$

For example, in a Daniell's cell $h = 770$ gramme-degrees, roughly, per gramme, of zinc dissolved.

Therefore $\quad \mathrm{E} = 4{\cdot}2 \times 770 \times {\cdot}000{,}328$ volts;

or $\quad \mathrm{E} = 1{\cdot}06$ volts (approximately).

Similarly in the case of a water voltameter h is about 35,000 gramme-degrees per gramme of hydrogen evolved, and therefore

$$e = 4{\cdot}2 \times 35{,}000 \times {\cdot}00001038 \text{ volts;}$$

that is $\quad e = 1{\cdot}5$ volts (approximately).

163. Laws of the Development of Heat in the Circuit. We have in this way found an expression for the work done on the current, that is, the work done by the cell in maintaining the current, and we have now to consider what becomes of the energy thus supplied to the current. As already stated, the energy of the current is dissipated as heat in the circuit, and, if R denote the resistance of the circuit, then, since

$$\mathrm{W} = \mathrm{C\,E}\,t, \text{ and } \mathrm{E} = \mathrm{C\,R},$$

we have

$$\mathrm{W} = \mathrm{C}\,.\,\mathrm{C\,R}\,.\,t = \mathrm{C}^2\,\mathrm{R}\,t.$$

That is, the heat produced in a circuit of resistance R by a current C in a time t is proportional to $C^2 R t$. Hence, the laws of the development of heat in a circuit carrying a current may be formulated thus:—

(1) The heat developed in a given time in a circuit of given resistance is directly proportional to the *square* of the current strength.

(2) The heat developed by a given current in a given time is directly proportional to the resistance of the circuit.

(3) The heat developed by a given current in a circuit of given resistance is directly proportional to the time the current passes.

These laws are found to be true, *whatever* be the nature of the circuit considered. Thus, if a current, C, flow through a circuit made up of a battery of resistance B, a wire of resistance W, a galvanometer of resistance G, and two voltameters of resistance V_1 and V_2, then the heat developed in the circuit in a time t is proportional to $C^2 R t$, where R denotes the *total* resistance of the circuit, and is therefore equal to $(B+W+G+V_1+V_2)$. Further, the laws may be applied not only to the circuit considered as a whole, but also to *any portion of the circuit*. Thus, in the example given above, the heat developed in the wire is proportional to $C^2 W t$, in the battery to $C^2 B t$, and so on. This application of the laws to a portion of the circuit gives the useful rule that the amounts of heat developed in different portions of the same circuit are directly proportional to the resistances of the portions considered. For example, in the case considered above, the heat developed in the battery is to the heat developed in the wire as B, the resistance of the battery, is to W, the resistance of the wire. Hence, if it is desired to confine the development of heat as much as possible to one part of the circuit, the resistance of every other part should be as small as possible.

In applying these rules to voltameter cells in which chemical work is done, students beginning the subject may find it hard to understand how any chemical work can be done in the cell if the whole energy of the current is spent in producing heat. It has already been explained that in

general, when electrolysis takes place in a voltameter cell, the products of electrolysis accumulate on the plates of the cell and set up polarisation. The back E. M. F. due to this polarisation acts in opposition to the E. M. F. of the battery, and the current is thereby reduced below the value it would have if a simple conductor of the same resistance as the voltameter cell were substituted for it in the circuit. The energy supplied by the battery has now two things to do: first, to overcome the back E. M. F. due to polarisation; second, to maintain a current in the circuit—the energy expended in the first case appears as chemical work in the voltameter cell, and that expended in the maintenance of the current appears as heat in the circuit. Thus, if E denote the E. M. F. of a battery and e the back E. M. F. due to polarisation of an electrolytic cell in the circuit, then the current produced in a circuit of resistance R is given by

$$C = \frac{E - e}{R}.$$

The total work done by the battery per second is expressed by C E, and of this a portion C e is spent in doing chemical work in the voltameter cells, while the remainder C (E—e), that is, C^2 R, is spent in maintaining the current C in the circuit, and appears as heat in the circuit. Thus all the energy of the current is dissipated as heat in the circuit, though all the work done in the battery may not be spent in producing current—in fact, the energy of the current represents the *difference* between the work done in the battery and that done at other points in the circuit, and really represents the waste of energy in the circuit. Hence, if the chemical work to be done in a voltameter cell is equal to or greater than that which can be done in the battery, then no current will flow. This explains why a single Daniell's cell cannot decompose water—the energy necessary to decompose water into oxygen and hydrogen is greater than that supplied by the chemical action going on in the Daniell, or, in other words, the back E. M. F. due to polarisation in the voltameter cell is greater than the E. M. F. of a single Daniell.

The truth of the above laws may be verified by taking a divided circuit of two branches. One branch, A *a* B (Fig. 280), is made up of two pieces of thick copper wire united by a thin piece of brass or platinum wire coiled into a spiral; the other branch, A *b* *c* B, is made up of the thick copper wire and *two* spirals of the same thin wire—the length of wire in each spiral is exactly one-quarter that of the single spiral in A *a* B, and the total length of thick copper wire should be one-half that used in A *a* B. Prepared in this way, the resistance of A *b* *c* B is one-half that of A *a* B; and if A and B be connected to the terminals of a battery, the current flowing through A *b* *c* B will be twice that in A *a* B. Consider the relative amounts of heat developed in the spirals at *a*, *b*, and *c*. The same current flows through *b* and *c*, and these spirals are of equal resistance; hence the rate of development of heat will be the same in each. Again, comparing either *b* or *c* with *a*, the current in *a* is one-half that in *b* or *c*, and therefore, if its resistance were the same, the heat developed in it would be only one-quarter that developed in *b* or *c*; but the resistance of the spiral being four times greater than that of *b* or *c*, the heat actually developed in it per second must be exactly the same as for *b* and *c*. Hence, if the laws of heating are correct, heat should be produced at the same rate in each of the spirals, *a*, *b*, and *c*. To test if this is so, let the spirals be immersed in exactly equal amounts of water in three exactly similar calorimeters. Under these conditions the temperature of the water in each calorimeter should rise at precisely the same rate, and if the temperatures be noted and compared from time to time, this will be found to be the case.

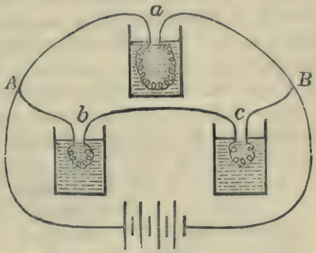


Fig. 280.

If H denote the amount of heat developed in time t in

a conductor of resistance R by a current C, we have $JH = C^2Rt$, when J denotes Joule's equivalent. This gives $J = C^2Rt/H$ or $J = ECt/H$, and therefore by measuring C R and H or E C and H the value of J may be found. For an experiment of this kind a suitable conductor is immersed in a calorimeter and a current passed through it. Then by measuring C and R or E and C electrically and H calorimetrically for a given time t the value of J can be found with considerable accuracy.

One of the most important applications of heating effects of current is the *incandescent lamp.* This consists of a glass globe (Fig. 281), completely exhausted of air and containing a fine filament of *carbon* of high resistance attached to terminals of platinum wire fused through the glass. When a strong current is sent through the filament the carbon becomes white hot, and, being in a vacuum, does not burn, but glows with a steady bright light.

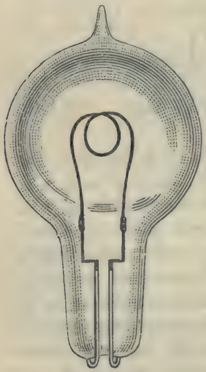


Fig. 281.

In *arc lamps* a strong current passes across a small gap between the ends of two carbon rods and the incandescent matter in the gap, together with the incandescent ends of the rods, is the source of light. The difference of potential between the ends of the carbon rods (about 50 or 60 volts) is not nearly high enough to spark across an air gap between them; but if the carbons are put in contact for an instant, the heat developed at the point of contact is great enough to heat the ends of the rods to incandescence; and if they are then separated a little the incandescent matter in the gap conducts the current across. The ends of the rods are thus kept glowing, and the bright *arc* between them persists as long as the current passes. (See also Art. 64.)

CHAPTER XXVI.

ELECTROMAGNETIC INDUCTION.

164. Fundamental Experiments. In the preceding chapter it has been stated that when *work is done* in the external circuit of a battery a back E. M. F. is set up in opposition to the main E. M. F. of the battery. From this it follows that if by any means we can *do work* in the external circuit, then an E. M. F. in the same direction as the E. M. F. of the battery will be established in the circuit. Now if we are dealing with chemical work, we can do work in the external circuit by replacing the voltameter, which absorbs energy in the circuit, by a voltaic cell, which supplies energy, and consequently establishes an E. M. F. in the circuit in the same direction as the E. M. F. of the battery. If we are dealing with the mechanical work done by an electromagnet in the circuit, then we may do work in the external circuit by doing mechanical work *against* the attracting or repelling force of the magnet. In doing work in this way an E. M. F. is established in the circuit, acting in the same direction as the E. M. F. of the battery, and the current is increased.

These effects with an electromagnet are neatly shown by the apparatus of Fig. 282. The circuit contains an electromagnet, M, and a small incandescent lamp, L. When the magnet is allowed to do work in attracting the keeper K, the back E. M. F. produced diminishes the current, and the lamp momentarily ceases to glow, or glows less brightly. If, however, work be done against the attracting force of the magnet by quickly withdrawing K, then the E. M. F. produced increases the current, and the lamp glows out more brightly for an instant. In the same way, if another magnet be employed instead of the bar K, the forces of

attraction and repulsion will be stronger, and the effects are more marked. If the results of this experiment be carefully noted, it will be seen that an E. M. F. is produced in the circuit only while work is being done on or by the

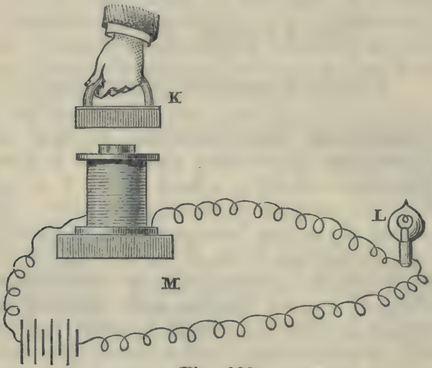


Fig. 282.

magnet; hence, with an electromagnet in the circuit there is no *continuous* back E. M. F. as there is with a voltameter, for in a voltameter chemical work is always going on, and therefore the back E. M. F. is continuous, but with an electromagnet the back E. M. F. appears only when work is done, and its magnitude depends upon the rate at which work is done.

It is not, however, necessary to work with an electromagnet to get effects similar to these. Let an insulated wire be coiled round a permanent bar magnet, and its ends joined to the terminals of a reflecting galvanometer. Then if the magnet be made to do work by attracting a piece of iron up to it, an E. M. F. is set up in the circuit and the current produced in the circuit causes a deflection of the galvanometer needle. Similarly, if work is done by withdrawing the iron from the magnet, then an E. M. F. in the opposite direction is produced, and the resulting current deflects the galvanometer needle in the opposite

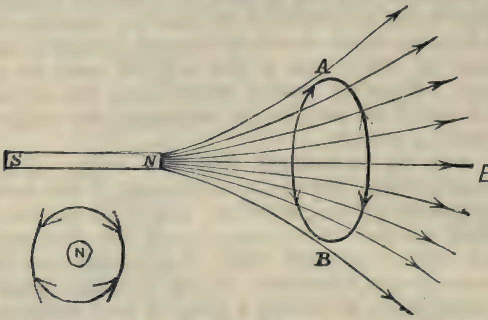
direction. Further, from what has been said above in connection with the electromagnet, it is evident that the current produced in the first case flows round the circuit in a direction opposite to that of a current which would magnetise the magnet with its existing polarity—that is, in a direction which would be opposed to that of the main current in the circuit if the magnet were an electromagnet. Similarly, in the second case, the current produced would be such as to magnetise the magnet with its existing polarity.

It will be understood that the energy of currents produced in this way is the equivalent of the work done by or against the magnetic force due to the magnet. Such currents are said to be *induced*, and the phenomena connected with them are dealt with under the head of electromagnetic induction. When a magnet does work, for example, in attracting a piece of iron towards it, we know that the work is done by means of the intervening medium. The strained medium possesses energy, and if the state of strain be altered in any way, then some energy may be freed to do work, or additional energy may be absorbed from work done.

Hence, in dealing with induction it is simplest to deal with the state of the medium surrounding the circuit in which the induced currents are produced. Considered in this way, it is found that induced currents may be produced in any circuit by altering the flow of magnetic induction through that circuit, that is, a current is produced in any circuit whenever a change takes place in the number of lines of induction running through the area bounded by the circuit.

For example, in Fig. 283, the number of lines of force which run through the coil of wire A B from the north pole of the magnet N S may be changed by *any* relative motion of the magnet and the coil. Thus, if A B be moved to the right away from N S, or if N S be moved to the left away from A B, then the number of lines of force passing through the coil will be decreased, and a current will be induced in it in a direction indicated by the full arrowheads in the figure. Similarly, if the coil and

magnet be made to approach each other by the motion of either, then the number of lines of force running through the coil is increased, and an induced current



End view from E Fig. 283.

traverses the coil in the direction indicated by the dotted arrowhead, that is, in a direction opposed to that of the current induced by the decrease in the number of lines of force. Further, if the magnet N S be suddenly reversed,

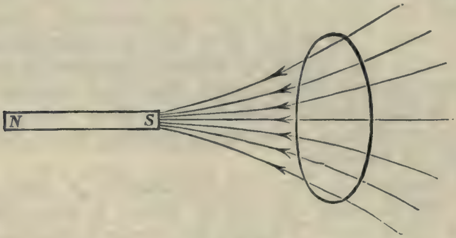


Fig. 284.

as in Fig. 284, so as to bring its south pole next the coil, the flow of force through the coil will also be reversed, and a strong induction current will be produced in the coil. The number of lines of force passing through the

coil are now numerically the same as before, *but their sign is opposite*, so that by the reversal of the magnet the number of lines of force running through the coil has been *reduced* from a *positive* value down to a numerically equal *negative* value.

If the magnet and coil be moved relative to one another as before reversal, the same effects will be produced, but the directions of the induced current will be opposite to those indicated in Fig. 283, when the north pole of the magnet was next to the coil.

The direction of the current in the coil under given conditions may be determined by the following simple rule. If a person look through the circuit along the positive direction of the lines of force, that is, in the direction in which a north pole would be urged, then, for a *decrease* in the number of lines running through the circuit, the induced current flows round the circuit in the same direction as the hands of a clock move. It is of the greatest importance to understand that these induced currents are momentary, and last only while the flow of force through the circuit is changing.

The experiment illustrated in Fig. 283 usually takes a more elaborate form, suitable for the experimental verifi-

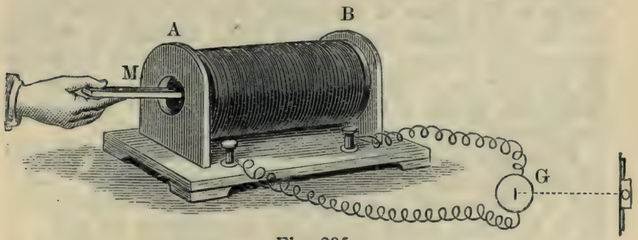


Fig. 285.

cation of the phenomena of induction. The single coil of wire is replaced by a long coil of wire, wound on a large bobbin, A B (Fig. 285), the ends of the wire being attached to binding screws on the baseboard to which the bobbin is

fixed. The terminals of a reflecting galvanometer, G, are connected to these screws, and the coil of the galvanometer thus completes the circuit of the coil on A B. If a strong magnet M be now moved relatively to A B—for example, by thrusting it into the interior of the coil, and then withdrawing it—well marked induction currents are produced, and are indicated by the deflection of the galvanometer in the circuit. Thus, if the north pole of the magnet be thrust into the coil, the spot on the scale is momentarily deflected in one direction, and then comes to rest; on withdrawing the magnet a deflection in the opposite direction is produced; and if the south end of the magnet be inserted and withdrawn, reverse effects are obtained.

Since, in order to produce induction currents in a circuit, it is only necessary to change in any way the flow of magnetic force through the circuit, it is evident that any relative displacement of the magnetic system and the circuit, or the introduction of any magnetic body into the system, may induce a current in the circuit. For example, in Fig. 283, if a piece of soft iron be brought up to A B from the side remote from the magnet, then an increased number of lines of force will be drawn through the coil up to the iron, and a current will therefore be induced in the coil; or, if the coil A B be made to rotate round a diameter at right angles to the plane of the paper, then it is evident that, as the coil rotates, the flow of force through it will be continually changed, and induction currents will be set up in it; and the student will find it a useful exercise to determine, by aid of the rule given above, the direction of the induced current at any stage in the revolution.

From this it will be understood how the magnetic field of the earth may be made to yield induction effects. Let a coil of wire wound on a large circular frame be constructed so as to rotate freely round one diameter as an axis. If this coil be placed with its axis vertical, and caused to rotate, the number of lines of force passing through the coil will vary as the coil spins, and currents will be induced in the coil. Since the axis of the coil is vertical, the plane of the coil will always be vertical, and therefore the vertical component of the earth's magnetic field need not be

considered, for no vertical lines of force can in this case cross the circuit of the coil. Hence, under the conditions here considered, we shall have to do only with H, the horizontal component of the earth's field. Imagine the coil to start rotating from a position at right angles to the magnetic meridian, and the east side of the coil to rotate towards the south. When at right angles to the magnetic meridian, the greatest possible number of lines of force crosses the circuit of the coil, for the plane of the coil is at right angles to the horizontal component of the earth's field, and H lines of force cross each unit of area enclosed by the coil. As the coil rotates the number of lines passing through the coil decreases until, when parallel to the magnetic meridian, the number is diminished to zero. The total flow of force across the coil when at right angles to the meridian is given by n A H, where n denotes the number of turns of the coil and A the area enclosed by the coil; for in the case of any one turn the number of lines of force passing across it is A H, if A denote the area of the single coil; and as this is true for every one of the n turns, it is evident that the total flow of force across the coil is n A H. Hence, during the first 90° of a revolution, starting from a position at right angles to the magnetic meridian, the flow of force across the coil *decreases* from n A H to zero, and the direction of the induced current will be from west to east (for the starting position) along the upper half of the coil. The direction of the induced current is readily determined by the application of the rule given above. Suppose a person to stand on the south side of the coil, so as to look, as specified in the rule, through the coil along the lines of force—that is, towards the north; then, as we are here dealing with a *decrease* of the flow of force through the coil, the induced current will flow round the coil in the same direction as the hands of a clock move—that is, from west to east, along the upper half of the coil. During the next 90° of rotation the flow of force will evidently *increase* from zero to n A H, and the direction of the induced current will be from east to west (for the final position), along the upper half of the coil—that is, the current will still be in the same direction *in the coil*; for although its direction *in*

space is reversed, the position of the coil is similarly reversed by revolution, so that the direction of the current in the coil is the same during the first 180° of revolution, starting from the position in which the flow of force through the coil is a maximum. During the next 180° of revolution the same changes in flow of force will take place as described above, and a current will be induced *in the coil* in the *opposite* direction to that induced during the first half-revolution, owing to the lateral reversal of the coil by that half-revolution. Hence during each complete revolution the direction of the induced current changes as the coil passes through the position at right angles to the direction of the lines of force in the field. If the terminals of the coil be connected, one to each half of a ring cut into two halves across a diameter, a continuous current in one direction may be obtained by fixing this split ring on the axis of the coil and drawing off the current by springs resting on the rotating ring and so placed that each spring changes from one half of the ring to the other at the same time as the reversal of current in the coil takes place. Such an arrangement would be a simple form of dynamo.

If the coil be fixed with its axis horizontal and parallel to the magnetic meridian, then the horizontal component of the earth's magnetic field will not enter into the varying flow of force across the coil, for the position of the plane of the coil is such that it is always parallel to the horizontal lines of force. Hence, under these conditions, the vertical component of the earth's field only need be considered.

If the terminals of the coil are connected with a ballistic galvanometer, and the throw of the needle observed, (1) when the axis of rotation is vertical, and the coil is quickly rotated through 180°, starting from a position at right angles to the magnetic meridian; (2) when the axis is horizontal and parallel to the magnetic meridian, and the coil is quickly rotated through 180°, starting from a horizontal position—then the angle of dip can be ascertained with some accuracy from the data obtained. For in the first case the change in the flow of force given by $2nAH$ is proportional to H, and in the second case the change given by $2nAV$ is proportional to V. Hence (see next

article) the quantity of electricity flowing through the galvanometer during the half-revolution is in the first case proportional to H, and in the second case to V; so that if d_1 and d_2 denote the throws of the galvanometer needle in the two cases, we have

$$\frac{V}{H} = \frac{d_2}{d_1}.$$

But from Art. 94

$$\frac{V}{H} = \tan \delta, \text{ and therefore } \tan \delta = \frac{d_2}{d_1}.$$

The arrangement described above is sometimes called an *Earth Inductor* or *Delezenne's Circle.*

Another simple arrangement for obtaining induced currents by motion of a conductor in a magnetic field is known as *Faraday's Disc* or *Barlow's Wheel.*

A circular disc or wheel of copper is fixed so as to rotate freely round a central axis at right angles to its plane. If made to rotate with its plane at right angles to the direction of the lines of force in a magnetic field, and a metal spring be made to press lightly on its edge, a continuous current will be found to flow from the spring to the centre of the wheel, and can be drawn off by making connection with the spring and the metal axis of the wheel. In some forms of the arrangement the edge of the wheel or disc (which is sometimes serrated like a circular saw) passes through a small trough of mercury in one part of the revolution, and connection is thus made with less friction than with a spring pressing on the edge.

165. Faraday's Laws of Electromagnetic Induction. In the preceding article several general laws have been given which may, perhaps, for convenience, be again referred to here. It has been found

(1) That a current is induced in a circuit if the number of lines of magnetic force running through the circuit is changed.

(2) That the induced current is transient, and lasts only while the change in the flow of magnetic force through the circuit is taking place.

(3) That the direction of the induced current is related to the change in the flow of force by the rule given in Art. 164.

In addition to these general laws, there are two more precise laws which we must notice. The intensity of an induced current is found to depend on the *rate* at which the number of lines of force passing through the circuit is changed. Thus, in the experiment illustrated by Fig. 285, the induced currents will be very feeble if the magnet is moved slowly, but strong if it is moved very quickly. From investigation of this question it is found

(4) That the electromotive force of the induced current in a circuit is measured by the change per second in the number of lines of force passing through the circuit. For example, if the lines of force initially running through the circuit number N, and in t seconds they are reduced, say, to N′, then the average E. M. F. of the current induced during the change is given by

$$E = \frac{N - N'}{t},$$

and if R denote the resistance of the circuit, the average intensity of the current is given by

$$C = \frac{E}{R} = \frac{N - N'}{R\,t}; \text{ or, } C\,t = \frac{N - N'}{R},$$

where $C\,t$ denotes the *quantity* of electricity induced in the circuit in time t. This last result shows that for a given change in the flow of force through a circuit the *quantity* of electricity set in motion is inversely proportional to the resistance of the circuit and is independent of the rate of change.

The application of this law will be more fully realised by considering a simple example. Take the case of the *Earth Inductor* described in the preceding article: the total change in the flow of force through the coil during each *half* revolution is $2\,n\,A\,H$; hence, if the coil makes m revolutions per second, the *average* change per second in the flow of force during each half revolution is $4\,m\,n\,A\,H$, and this is the *average* difference of potential between the ends of the coil. Also if R denote the resistance of the coil and the external circuit between its terminals, then $4\,m\,n\,A\,H\,/\,R$ gives the *average* current in the circuit during each half revolution. If A is in square centimetres

and H in dynes per unit pole, the quantity $4\, m\, n\, \mathrm{A}\, \mathrm{H}$ is in absolute C. G. S. electromagnetic units of potential, and can be expressed in volts by dividing it by 10^8; then, if R is in ohms, the current in the circuit can be calculated in ampères.

These laws of induction are frequently expressed somewhat differently, in the following form. If any conductor (not forming a continuous circuit) be moved in a magnetic field so as to cut across lines of force, then an electromotive force, measured by the rate at which the lines are cut, is induced in the conductor. Hence, if a continuous circuit be moved in any way in a magnetic field, different portions of it will, in general, cut across the lines of force at different rates, and the electromotive force induced in the circuit will be the algebraic sum of the electromotive forces induced in each element of the circuit considered as a separate conductor. It is evident that if the algebraic sum of the electromotive forces induced in the circuit is zero—that is, if the algebraic sum* of the lines of forces cut in unit time by each element of the circuit is zero—then the total number of lines of force passing through the circuit is constant. Hence it follows that no current is induced in the circuit so long as the number of lines of force crossing the circuit is constant, and also that the induced electromotive force is measured by the rate of change in the number of lines of force passing through the circuit.

Thus, in the case of Faraday's disc, consider an infinitely thin radial strip of the disc passing from the centre to the point of contact with the spring. As this strip rotates through a small angle, θ, the area it cuts out is $\frac{1}{2}\,\theta\, r^2$, where r is the radius of the disc, and θ is taken in circular measure, and the number of lines of force cut by the slip is $\frac{1}{2}\,\theta\, r^2\, \mathrm{H}$, where H denotes the intensity of the magnetic field in which the disc rotates. If the disc makes n revolutions per second, then the time it takes to describe an angle θ is $\theta\, /\, 2\,\pi\, n$ seconds, and the rate of cutting lines of force is $(\frac{1}{2}\,\theta\, r^2\, \mathrm{H})\, /\, (\theta / 2\,\pi\, n) = \pi\, n\, r^2\, \mathrm{H}$, and this measures the

* Cut lines passing into the circuit and lines passing out of the circuit must be taken with opposite signs.

E. M. F. or difference of potential between the centre and the circumference of the rotating disc.

Another law of importance is *Lenz's law.* It states—

(5) That when a current is induced in a circuit by the relative motion of the circuit and a magnet (or its equivalent), the reaction of the induced current is always such as to tend to stop the motion which produces it.

For example, in Fig. 285, if the magnet be held as in Fig. 283 with its north pole next the coil, then, when the magnet is moved *away* from the coil, the direction of the induced current is such that the end A of the coil is equivalent to the *south* pole of a magnet, and therefore attracts the receding north pole of the magnet. The reaction of the induced current thus tends to stop the motion which gives rise to it.

Another illustration of the action of Lenz's law is found in Arago's well-known experiment. A disc of copper is made to rotate in a horizontal plane immediately below a delicately balanced magnetic needle, the axis of rotation of the disc being vertically below the pivot of the needle. As the disc rotates it is found that the needle is gradually deflected in the same direction as the rotation, and, if the rate of rotation is sufficiently high, finally takes up a motion of rotation in the same sense as the disc, but at a slower rate. This result is explained by the fact that currents are induced in the copper disc by its rotation relative to the magnet, and the reaction between the disc and the needle is (in accordance with Lenz's law) such as to tend to stop the motion of the disc, but the needle being movable and not fixed the result of this reaction is that the needle is itself set in motion. The direction of the induced current in the disc is such that a current always flows along the diameter of the disc vertically below the needle in such a direction as to deflect the needle in the same direction as the rotation of the disc. The current flowing along this diameter spreads out at one end, and curving round the sides of the disc unites again at the other end; hence if the disc has a number of radial slits cut in it, the inductive effect on the needle is much less marked.

In the same way, if the copper disc is fixed, and the

needle made to oscillate, the oscillations quickly die away and the needle comes to rest, as the result of the interaction of the induced currents and the oscillating needle. This *damping* effect of a copper plate under a magnetic needle is sufficiently well marked to be of practical use.

Similarly, if a disc of copper be made to spin rapidly between the poles of an electromagnet in a plane at right angles to the lines of force, it is much more difficult to spin when the current is on than when it is off; and, if the disc be put in rapid motion with the current off, it is quickly brought to rest when the current is put on. The interaction of the induced currents and the poles of the magnet is such as to stop the motion of the disc.

It has already been explained that a solenoid, or coil of wire carrying a current, is equivalent to a magnet; hence if, in the experiment illustrated by Fig. 285, the magnet M be replaced by a coil of wire wound on a core or bobbin small enough to be thrust into the interior of the coil A B, currents may be induced in the outer coil by moving the inner coil while it carries a *constant* current, or by varying the strength of the current while the coil remains at rest in the interior A B. For example, if the current be stopped by breaking the circuit of the inner coil, then a current is induced in the outer coil in the same direction as that induced by the complete withdrawal of the coil when it carries a constant current. Similarly the current induced when the current is started is in the same direction as that obtained by thrusting the coil carrying the steady current into the interior of A B. If, while the coil is inside A B, the current be reversed, then the flow of magnetic force through A B is also reversed; and the induced current is the same as that which would be obtained if the coil carrying the steady current were suddenly withdrawn from A B and immediately reinserted with its ends reversed.

To determine the direction of the induced current in any given case, it must be remembered that the "north pole" of the coil is that end round which the current appears, when the coil is viewed from that end, to circulate in a direction opposite to that of the hands of a watch. The application of this fact to what has been said above gives

the following simple rule for the direction of the induced current: When the current in the inner coil is stopped or decreased, or when the coil carrying a steady current is withdrawn, *the direction of the induced current in the outer coil is the same as that of the current in the inner coil*, and consequently when the current in the inner coil is first made or increased, or when the coil carrying a steady current is inserted into the outer coil, the direction of the induced current in the outer coil is opposite to that of the current in the inner coil.

It is of the utmost importance for the student to realise that these induced currents are in *all* cases only momentary and exist only while the variations of the magnetic field to which they are due are in progress.

166. The Induction Coil. Ruhmkorff's *Induction Coil* is an arrangement of two coils for the production of induced currents of high electromotive force. The coils are arranged one inside the other, the inner, or *primary*, coil consisting of a small number of turns of thick wire, and the outer, or *secondary*, coil of a very large number of turns of thin wire. The coils are fixed in position and the induced currents in the secondary coil are obtained by arranging that a strong current in the primary coil shall be automatically made and broken a large number of times per second by an automatic "make" and "break" contact in the primary circuit. The induced currents thus obtained in the secondary coil are of very high potential, and the terminals of the coil may be made to yield sparks and "shocks" in the same way as the prime conductor of a frictional electrical machine.

Figure 286 shows a common form of induction coil: the primary and secondary coils, one inside the other, form the cylinder A B, but to strengthen the inductive action of the primary on the secondary, an iron core made up of a bundle of soft iron wires is fitted as the axis of the primary coil. This core increases the inductive effect by increasing along the axis of the coil the magnetic field due to the current in the coil, so that when the current is made or broken the change in the flow of force through the coil is greater than it otherwise would be. The automatic

"make" and "break" is shown at C; the terminals of the primary coil are seen at *a*, *b*; the current entering, say at *a*, flows beneath the baseboard to the base of the pillar *c*,* and from thence to the spring hammer, *d*, across the screw contact, *s*, between *c* and *d*. The end of the screw *s* is tipped

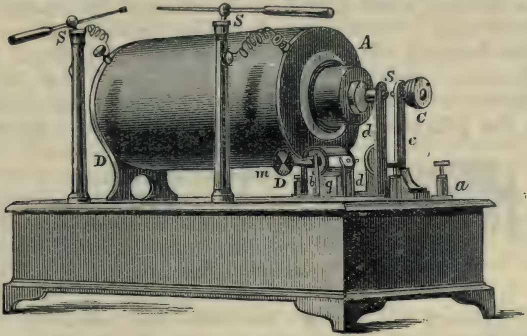


Fig. 286.

with platinum, and rests on a small disc of platinum attached to the spring *d*, so that, as these platinum surfaces always remain clean, the contact is always good. The current then passes round the coil, and returns to the terminal *b*. The action of the "make" and "break" will now be evident: when the current first passes, the iron core of the primary coil becomes an electromagnet and attracts to it the mass of soft iron which forms the head of the spring hammer *d*; this results in breaking the contact with *s*, and the current being now interrupted the iron core ceases to be an electromagnet, the hammer *d* now springs back into contact with *s*, and again makes the current. In this way the current is continuously made and broken at a very rapid rate. The arrangement shown at D is a commutator by which the current in the primary circuit may be reversed at will. The essential parts are

* For simplicity the connections to the commutator D (described below) are not here referred to

the cylinder *l*, which can be rotated by the screw *m*, and the two upright springs *p* and *q*; the cylinder is of ebonite, and has two strips of brass parallel to its length let into its surface, and 180° apart. When it is desired to start the current these strips are brought into contact with the springs, *p* and *q*, and the direction of the current may be reversed by turning the handle *m* through 180° and thus interchanging the strips in contact with the springs. From the section shown in Fig. 287 it will be seen how the commutator acts—the terminals, *a*, *b* (Fig. 286), are connected to the pillars *a′*, *b′* (Figs. 286, 287) which support the brass axis of the cylinder. This axis, as the figure shows, is divided, and one end is connected to one of the brass strips on the cylinder and the other end to the other strip. The springs *p* and *q* are directly connected to the ends of the primary coil through the make and break C. From this description of the connections it will at once be evident how the current in the primary coil is reversed by rotating the cylinder *l* through 180°, so as to interchange the springs in contact with the brass strips.

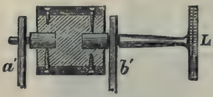


Fig. 287.

The terminals of the secondary coil are shown at S S carrying discharging rods, between the ends of which sparks can be made to pass when their distance apart is not too great.

167. Self-Induction. It has already been explained that a change in the flow of magnetic force through a coil of wire produces an induced current in the coil. Now, when a current is first started in a coil of wire a magnetic field due to this current is suddenly established in the medium around the coil, and consequently there is a sudden change in the flow of magnetic force through the circuit. The result of this is that a current is induced in the coil, and from the law given above the direction of this induced current will be opposed to that of the primary current to which it is due. Similarly when the current in the circuit is broken, the flow of force due to it through the coil is suddenly destroyed, and an induced current in the same

direction as the primary current is produced. As these induced currents are the result of very rapid changes in the flow of magnetic force through the circuit in which they are induced, they are of very high electromotive force. This is readily seen when the current in a coil of several turns is broken: a spark will be seen to leap across the small gap made at the point of interruption of the current, showing that the E. M. F. of the induced current is great enough to spark across the gap in the circuit.

These induction currents are said to be due to the *self-induction* of the circuit because they are the result of the magnetic effects attendant on the making and breaking of the current in a circuit on that circuit itself, and not on a secondary circuit, as in the experiments of Art. 164.

These induced currents due to self-induction are a source of trouble in the action of induction coils. When the current is made in the primary coil, the induced current opposed to it retards the rise of the primary current,* and consequently diminishes the inductive effect on the secondary. Similarly, when the current is broken, the induced current in the same direction as the primary sparks across the interruptor, thus making the break less sudden and definite than it otherwise would be. The sparking would also spoil the surfaces of contact of the interruptor. To prevent these effects, a condenser, made of sheets of tin-foil and paraffined paper, is connected with the primary circuit; one pole of it is connected to the pillar of the screw *s* (Fig. 286), and the other to the spring hammer *d*. The action of this condenser may be briefly explained thus. When the current is made, the condenser is at the same time charged, and thus the current takes a comparatively long time to attain its maximum intensity, and the inductive effect attendant on the make (both on its own circuit and on the secondary circuit) is very greatly reduced. Again, when the current is broken the self-induced current, having to charge the

* When a current is made or broken in a circuit, it does not attain its full intensity nor become reduced to zero instantaneously, but a finite time is required for the change to be effected, and it is during this small interval of rapid change that the induction currents are produced. See Art. 172.

condenser, is not able to spark across the break, and thus the current is very suddenly broken. Further, the electricity so stored in the condenser, being unable to leap across the break in the circuit, at once discharges round the primary coil, but in the opposite direction to that of the primary current—that is, in the opposite direction to that in which it would have passed had it been able to spark across the break. In this way the extra-current at break is made a help instead of a hindrance. The secondary induced current at make, we see, is thus practically suppressed, while that at break is intensified, so that in an ordinary induction coil the secondary induced currents are those due to the breaks of the primary current.

168. Standard Inductors. In measuring the intensity of a field of magnetic force or induction the usual practice is to determine the quantity of electricity set in motion in a suitable test coil or inductor, by suddenly removing it from the field or by rotating it through 180° in the field, as described in the case of the earth inductor. The test coil is connected in circuit with a ballistic galvanometer and by noting the throw of the galvanometer needle under definitely arranged conditions both comparative and absolute measurements of induction may be made. Comparative measurements are readily made when it is possible to use the same test coil in the same galvanometer circuit for each observation. The fields compared are then directly proportional to the observed throws of the galvanometer needle in each case. The comparison of V and H for the determination of the dip, as given in Art. 164, is an example of this method of comparison. If, however, we wish to compare two magnetic fields of very different intensities a different method has to be adopted. In comparing, for example, the horizontal component of the earth's field with the field between the poles of a strong electromagnet it is necessary to use a different inductor for each field, and the comparison therefore involves the constants of the inductors. The effective area of the inductor for the strong field may be small but the effective area of the earth inductor must be large on account of the low intensity of the field. If A denote the mean area of the earth inductor

coil and N the number of turns, then N A is the effective area of the coil, and if rotated through 180° from a position with its plane at right angles to the flow of induction through it, then, as in Art. 164, 2 N A H denotes the change in the flow of induction during the time of rotation, and $2\,N\,A\,H/R_1$, where R_1 is the resistance of the circuit in which the inductor is placed, denotes the quantity of electricity set in motion in the circuit. Similarly, if $n\,a$ denote the effective area of the small inductor, and F the strength of the field between the poles of the electromagnet, then $n\,a\,F$ will be the change in the flow of induction through the coil, produced by suddenly withdrawing it from the field* after adjusting it in position with its plane at right angles to the flow of induction in the field. The quantity of electricity set in motion in the circuit by this change in the induction through the coil is $n\,a\,F/R_2$, where R_2 is the resistance of the circuit in which the coil is placed. Now if the two inductors are connected in the *same circuit*, in series with the ballistic galvanometer, we have $R_1 = R_2 = R$, the resistance of this circuit, and if d_1 and d_2 are the throws of the galvanometer needle in each case we have

$$\frac{2\,N\,A\,H}{R} \Big/ \frac{n\,a\,F}{R} = d_1/d_2$$

or

$$\frac{2\,N\,A\,H}{n\,a\,F} = \frac{d_1}{d_2},$$

that is,

$$\frac{F}{H} = \frac{2\,N\,A\,d_2}{n\,a\,d_1},$$

and

$$F = \frac{2\,N\,A\,d_2}{n\,a\,d_1} H.$$

Now H, the horizontal component of the earth's magnetic field, can be absolutely determined in accurate measure at any place, and may therefore be taken as a standard field serving to determine the field F in absolute units. The earth inductor used in this experiment may be looked upon as a standard inductor, serving to standardise the

* It is difficult to rotate a *small* coil through 180° with any accuracy.

observations taken with the small inductor, acting as a test coil in the unknown field.

Another field which may be used as a standard is the uniform field in the interior of a long uniformly wound solenoid carrying a known current. If C denote the current in absolute units, then, as in Art. 138, the field in the interior of the coil is given by $4\pi n C$, where n is the number of turns per unit length of the coil. The inductor for use with this field usually consists of a few turns of thin and well insulated wire wound round the outside of the solenoid near its middle point. A better plan is to wind it round a truly cylindrical rod of wood or glass tube running through the solenoid. The induction throw is obtained by reversing or breaking the current in the solenoid. If $n' a'$ denote, as above, the effective area of the inductor, and C the current in absolute units, then, on reversing the current, the change in the flow of induction through the inductor coil is $8\pi n C n' a'$, and the quantity of electricity set in motion is $8\pi n C n' a' / R$, where R is the resistance of the circuit in which the inductor is placed. If C is measured accurately by means of an ammeter in the solenoid circuit, this inductor may be used for standardising the observations of a test inductor placed in the same circuit with it.

It should be noticed that the results obtained by experimenting with an inductor relate to the *induction* in the field and not to the magnetic force directly. If, however, the observations are made in the same medium, and if the permeability of the medium does not vary with the intensity of the magnetic field, then the induction at any point is directly proportional to the magnetic field at that point, and the ratio of any two induction values therefore determines the ratio of the corresponding field intensities. That is, if $B_1/B_2 = k$, then, since $B_1 = \mu H_1$ and $B_2 = \mu H_2$, ***where μ is a constant***, we have

$$\frac{\mu H_1}{\mu H_2} = k$$

or $H_1/H_2 = k$. The value of μ for air, and for all weakly magnetic substances, is practically constant.

A standard inductor may be used for determining the constant of a ballistic galvanometer. Thus, with a standard solenoid inductor we have

$$Q = \frac{8 \pi n C n' d'}{R}$$

as explained above. If the discharge of this quantity of electricity through the galvanometer gives a deflection α, we have, as in Art. 135,

$$Q = \frac{H T}{\pi G} \sin \frac{\alpha}{2}.$$

Hence, $$\frac{8 \pi n C n' d'}{R} = \frac{H T}{\pi G} \sin \frac{\alpha}{2},$$

and therefore $$\frac{H}{G} = \frac{8 \pi^2 n C n' d'}{T R \sin \frac{1}{2}\alpha},$$

and taking $$\sin \frac{\alpha}{2} = \frac{d}{4 D},$$

as in Art. 135, we have

$$\frac{H}{G} = \frac{32 \pi^2 n n' d' D C}{T R d}.$$

Similarly, if an earth inductor be taken as a standard, we get

$$\frac{2 N A H}{R} = \frac{H T d}{4 \pi G D}$$

and, as the H on each side of this relation is practically the same value, we get

$$G = \frac{R T d}{8 \pi N A D},$$

where G is the constant of the galvanometer coil.

CALCULATIONS.

THE electromagnetic unit of current is that current which, flowing in an arc of unit length and unit radius, produces a magnetic field of unit intensity at the centre of the arc. (Art. 129.)

The intensity of the magnetic field at the centre of a thin coil of n turns carrying a current C is given by

$$I = \frac{2\pi n C}{r}, \qquad \text{(Arts. 126, 129.)}$$

where r denotes the mean radius of the coil.

In a tangent galvanometer with a thin coil of n turns and mean radius r the relation between the current, C, and the deflection, a, which it produces, is given by

$$C = \frac{rH}{2\pi n}\tan a,$$

where H denotes the horizontal intensity of the magnetic field in which the needle lies—usually this is the horizontal component of the earth's magnetic field. The factor $\frac{rH}{2\pi n}$ is known as the *working constant* or *reduction factor* of the galvanometer and is frequently denoted by k, the above relation being written $C = k\tan a$. (Art. 131.)

Similarly in a sine galvanometer under the same conditions we have

$$C = \frac{rH}{2\pi n}\sin a,\text{*}$$

or

$$C = k\sin a, \qquad \text{(Art. 132.)}$$

where k denotes the reduction factor of the galvanometer.

* The needle of the galvanometer is here supposed to be short like that of a tangent galvanometer.

By Ohm's Law we have

$$C\ (\text{amperes}) = \frac{E\ (\text{volts})}{R\ (\text{ohms})}.$$

$$\begin{aligned} 1 \text{ volt} &= 10^8 \text{ C.G.S. units.} \\ 1 \text{ ohm} &= 10^9 \quad ,, \qquad ,, \\ 1 \text{ ampere} &= \frac{1}{10} \quad ,, \qquad \text{unit} \\ \text{or } 10 \text{ amperes} &= 1 \quad ,, \qquad ,, \end{aligned}$$

(Arts. 140-148.)

A *coulomb* is the electromagnetic unit of quantity of electricity, and is the quantity conveyed by a current of one ampere in one second.

If R denote the resistance of a conductor of length l and area of cross section a made of material of specific resistance S, then

$$R = S\frac{l}{a}. \qquad \text{(Art. 143.)}$$

Hence the specific resistance of any substance is the resistance of a conductor of that substance of unit length and unit area of cross section.

The resistance of a conductor, in general, increases with temperature. If R_t denote the resistance of a given conductor at $t°$ C., R_0 its resistance at 0° C., and a the coefficient of increase of resistance with temperature, then as an approximate relation we have

$$R_t = R_0\ (1 + a t). \qquad \text{(Art. 147.)}$$

[For most pure metals the value a is about ·0038.]

The joint resistance of two conductors of respective resistances r_1 and r_2 joined in parallel arc is given by

$$R = \frac{r_1 r_2}{r_1 + r_2}, \qquad \text{(Art. 145.)}$$

and the ratio of the currents in the two conductors is the inverse of that of their resistances; that is,

$$\frac{C_1}{C_2} = \frac{r_2}{r_1}. \qquad \text{(Art. 145.)}$$

The current given by a battery of n cells *in series*, each of electromotive force E and internal resistance r, through an external resistance R, is given by

$$C = \frac{nE}{nr + R}.$$

With the same cells *in parallel* the current is given by

$$C = \frac{E}{\frac{r}{n} + R} = \frac{nE}{r + nR}$$

(Art. 148.)

If we have $m\,n$ cells, each of electromotive force E and resistance r, arranged in n rows in series, each row containing m cells grouped in parallel, then the current given by this grouping through an external resistance R is given by

$$C = \frac{n\,E}{\frac{n\,r}{m} + R}. \qquad \text{Art. 148.)}$$

To obtain the strongest current from this arrangement we must have

$$\frac{n\,r}{m} = R.$$

That is, $$n\,r = m\,R. \qquad \text{(Art. 148.)}$$

If the resistance between two points on a simple conductor be denoted by R, and the current in the conductor by C, then the difference of potential between those two points is given by—

$$e = C\,R. \qquad \text{(Art. 148.)}$$

[This relation is the algebraic expression of Kirchhoff's second law.]

The *chemical equivalent* of an element is the number obtained by dividing the atomic weight of the element by its valency.

The *electrochemical equivalent* (per *ampère*) of an element is the mass of that element liberated in one second by a current of one ampère. The electrochemical equivalent of any element is obtained by multiplying the electrochemical equivalent of hydrogen (0·00001035 gramme) by the chemical equivalent of the given element.

The relative masses of different elements liberated by the electrolytic action of the *same* current are directly proportional to their chemical equivalents. (Art. 157.)

The mass of an ion liberated in a time t by a current C is given by

$$m = C\,w\,t,$$

where w denotes the *electrochemical equivalent* of the ion. (Art. 157.)

If a current C flow for a time t round a circuit in which the effective E. M. F. is E, then the work done in maintaining the current for the time t is given by

$$W = C\,t\,.\,E. \qquad \text{(Art. 162.)}$$

If C and E be expressed in electromagnetic units, and t in seconds, then W is given in ergs.

From this we have that the *activity* of the battery supplying the current is given by C E. (Art. 162.)

Here, if C be expressed in *ampères* and E in volts, then the activity is given in *watts.*

The watt is a unit of power or activity equal to 10^7 ergs per second, or such that 746 watts equal one horse-power. (Art. 162.)

The heat produced in a conductor of resistance R by a current C in a time t is given by

$$H = \frac{C^2 R t}{J}, \qquad \text{(Art. 163.)}$$

where J (Joule's equivalent) denotes the quantity of work equivalent to unit quantity of heat.

[In C. G. S. units $J = 4{\cdot}2 \times 10^7$ ergs.]

If C be expressed in *ampères*, R in *ohms*, and t in *seconds*, then this relation reduces to

$$H = \frac{C^2 R t}{4{\cdot}2} = C^2 R t \times 0{\cdot}24 \text{ gramme-degree.}$$

[The process of reduction is as follows

$$C \text{ (ampères)} = C \times 10^{-1} \text{ C. G. S. units.}$$

$$R \text{ (ohms)} \quad = R \times 10^9 \quad ,, \quad ,,$$

$$H = \frac{C^2 R t}{J} = \frac{C^2 \times 10^{-2} \times R \times 10^9 \times t}{4{\cdot}2 \times 10^7} = \frac{C^2 R t \times 10^7}{4{\cdot}2 \times 10^7} = \frac{C^2 R t}{4{\cdot}2}.]$$

Examples IV.

1. The coil of a tangent galvanometer consists of 10 turns of fine wire on a narrow ring of 22 cm. radius. Find the intensity of the magnetic field at the centre of the coil when a current of one ampère passes through it.

The intensity of the magnetic field at the centre of the coil is given by

$$I = \frac{2 \pi n C}{r}.$$

Here $n = 10$; $C = 1$ ampère $= \frac{1}{10}$ C. G. S. unit; and $r = 22$ cm.

Therefore

$$I = \frac{2 \times 22 \times 10 \times 1}{7 \times 22 \times 10} = \frac{2}{7} \text{ dyne.}$$

2. Calculate the reduction factor of the tangent galvanometer referred to in the preceding question, and find the deflection which a current of 0·21 ampère would produce when passed through the instrument.

The reduction factor of the galvanometer is given by

$$k = \frac{r H}{2 \pi n}.$$

Hence, taking H = 0·18 dyne, we have

$$k = \frac{22 \times \cdot 18 \times 7}{2 \times 22 \times 10} = \cdot 063, \text{ for C.G.S. units.}$$

Also if α denote the deflection produced by a current of 0·21 ampère, that is, 0·021 C.G.S. units, then from

$$C = k \tan \alpha,$$

we have

$$\cdot 021 = \cdot 063 \tan \alpha,$$

or

$$\tan \alpha = \frac{\cdot 021}{\cdot 063} = \frac{1}{3}.$$

That is, α is an angle whose tangent is $\frac{1}{3}$, and is therefore an angle of about 18° 26′.

3. A sine galvanometer with a short needle is used as a tangent galvanometer, and when a given current is passed through it a deflection of 30° is produced. Find the deflection which the same current should produce if the instrument were used as a sine galvanometer.

Here, when the galvanometer is used as a tangent galvanometer, we have

$$C = k \tan \alpha_1,$$

and when used as a sine galvanometer we have

$$C = k \sin \alpha_2.$$

And from the conditions of the question we have

$$k \tan \alpha_1 = k \sin \alpha_2$$

or

$$\tan \alpha_1 = \sin \alpha_2.$$

But

$$\alpha_1 = 30°;$$

therefore,

$$\tan 30° = \sin \alpha_2,$$

or

$$\frac{1}{\sqrt{3}} = \sin \alpha_2.$$

That is, α_2 is an angle whose sine is $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ (= ·577), and is therefore an angle of about 35° 15′.

4. Find the specific resistance of copper at 0° C., given that the resistance at 15° C. of a copper wire 1 metre long and 0·0376 cm. in diameter is 0·157 ohm.

Here, applying the relation

$$R = S\frac{l}{a},$$

we get

$$S = R\frac{a}{l}.$$

And from the data of the question

$$R = 0{\cdot}157 \text{ ohm.}$$

$$a = (\pi r^2) = 3{\cdot}1416 \times (0{\cdot}0188)^2 = 0{\cdot}00111 \text{ sq. cm.}$$

$$l = 1 \text{ metre} = 100 \text{ cm.}$$

Therefore,

$$S = \frac{0{\cdot}157 \times 0{\cdot}00111}{100} = {\cdot}00000174 \text{ ohm.}$$

This is the specific resistance of copper at 15° C.

To determine the specific resistance at 0° C. we must know the coefficient of increase of resistance with temperature. For copper the value of this coefficient is 0·00387.

Hence in the relation

$$R_t = R_0\,(1+a\,t),$$

we have

$${\cdot}00000174 = R_0\,(1 + {\cdot}00387 \times 15),$$

$$= R_0\,(1{\cdot}058).$$

$$\therefore\ R_0 = \frac{{\cdot}00000174}{1{\cdot}058} = {\cdot}00000165 \text{ ohm.}$$

That is, the specific resistance of copper at 0° C. is ·00000165 ohm or 1650 C.G.S. units.

5. A battery made up of five Bunsen's cells, each of E. M. F. 1·8 volts and internal resistance ·2 ohm, has its terminals joined by two wires, A and B, arranged in parallel. The resistance of A is 6 ohms and that of B is 3 ohms; find the strength of the current in each wire.

Here, the total E. M. F. in the circuit is

$$5 \times 1.8 = 9 \text{ volts.}$$

The internal resistance of the battery is

$$5 \times {\cdot}2 = 1 \text{ ohm.}$$

The external resistance of the circuit is given by

$$\frac{6 \times 3}{6+3} = \frac{18}{9} = 2 \text{ ohms.}$$

Therefore the *total* resistance of the circuit is 3 ohms, and by Ohm's Law the total current in the circuit is given by

$$C = \frac{E}{R} = \frac{9}{3} = 3 \text{ ampères.}$$

In the external circuit this current of 3 ampères is divided between A and B in the inverse ratio of their resistance.

That is,

$$\text{Current in A} = \frac{3}{9} \times 3 = \frac{1}{3} \times 3 = 1 \text{ ampère,}$$

and

$$\text{Current in B} = \frac{6}{9} \times 3 = \frac{2}{3} \times 3 = 2 \text{ ampères.}$$

6. A battery is made up of 10 Daniell's cells joined in series. A very high resistance galvanometer is connected to the terminals of the battery and shows a deflection of 156 divisions. Without disconnecting the galvanometer, the terminals of the battery are now joined by a wire of 10 ohms resistance, and the galvanometer deflection falls to 52 divisions. Find the average resistance of the battery cells.

Here, as in Art. 148, if E denote the E. M. F. of the battery and the potential difference of the terminals, then

$$E = C\,(10 + B),$$

and

$$e = C \times 10,$$

where C denotes the current in the circuit when the terminals are joined by the wire, and B the internal resistance of the battery. Hence we have

$$\frac{E}{e} = \frac{10 + B}{10}.$$

But

$$\frac{E}{e} = \frac{156}{52} = \frac{3}{1}; \qquad \text{(Art. 148.)}$$

therefore,

$$\frac{10 + B}{10} = \frac{3}{1},$$

or

$$B = 20 \text{ ohms.}$$

Hence the average resistance of a battery cell is

$$\frac{20}{10} = 2 \text{ ohms.}$$

7. The same current passes through a copper and a silver voltameter arranged in series in the circuit of a battery of 10 Daniell's cells. If 10·8 grammes of silver are deposited in the silver voltameter in a given time, find the amount of copper deposited in the copper voltameter, and the amounts of copper and zinc respectively deposited and dissolved in the battery in the same time.

The amount of the elements deposited in *each cell in the circuit* will be proportional to the chemical equivalents of the elements. The chemical equivalents of silver, copper, and zinc are (approximately)

$$Ag = 108. \quad Cu = 31{\cdot}5. \quad Zn = 32{\cdot}5.$$

Hence we have for each cell

$$Ag : Cu : Zn :: 108 : 31{\cdot}5 : 32{\cdot}5.$$

That is,

$$10{\cdot}8 : w_1 : w_2 :: 108 : 31{\cdot}5 : 32{\cdot}5$$

Therefore

$$w_1 = 3{\cdot}15$$

and

$$w_2 = 3{\cdot}25$$

That is, in each cell in the circuit 3·15 grammes of copper are de posited and 3·25 grammes of zinc are dissolved during the time that 10·8 grammes of silver are deposited.

Therefore,

Copper deposited in voltameter cell	=	3·15 grammes.
,, ,, ,, battery (10 cells)	=	31·5 ,,
Zinc dissolved ,, ,, ,,	=	32·5 ,,

8. In an experiment to determine the reduction factor of a tangent galvanometer it was found that, in 30 minutes, 1·1081 grammes of copper were deposited on the negative plate of a copper sulphate voltameter. The deflection of the galvanometer was constant at 35° during the experiment. Determine from these data the required constant, taking the electrochemical equivalent of copper to be ·0003275 gramme per ampère. From the relation

$$m = C w t$$

we have

$$C = \frac{m}{w t}.$$

Also

$$C = k \tan \delta,$$

where k denotes the reauction factor of the galvanometer

Hence

$$\frac{m}{w t} = k \tan \delta.$$

That is,

$$k = \frac{m}{wt \, . \tan \delta}. \qquad \text{(Art. 157.)}$$

Substituting the data of the question in this relation we get

$$k = \frac{1{\cdot}1081}{{\cdot}0003275 \times 30 \times 60 \times \tan 35^\circ}.$$

From tables we get $\tan 35^\circ = {\cdot}700$.
Therefore,

$$k = \frac{1{\cdot}1081}{{\cdot}0003275 \times 30 \times 60 \times {\cdot}700} = 2{\cdot}68.$$

This is the value of the reduction factor which, when used in the relation $C = k \tan \delta$, gives C in *ampères.*

9. The same current is passed through a silver voltameter and a platinum wire of 10 ohms resistance. In twenty minutes ·6708 gramme of silver is deposited in the voltameter; find the rate at which heat is developed in the wire.

Here from

$$m = Cwt \text{ or } C = \frac{m}{wt},$$

we have

$$C = \frac{{\cdot}6708}{{\cdot}001118 \times 20 \times 60} = {\cdot}5 \text{ ampère.}$$

Hence the heat developed in the wire *per second* is

$$H = C^2R \times {\cdot}24 = ({\cdot}5)^2 \times 10 \times {\cdot}24 = {\cdot}6 \text{ calorie.}$$

10. Water is heated by a spiral of platinum wire immersed in it. The resistance of the wire is 74·6 ohms and it carries a current of 10 ampères; find, in horse-power, the rate at which energy is spent in heating the water.

Here the rate at which energy is expended in heating the platinum wire (and therefore the water) is given by

$$\text{Activity} = C^2R = (10)^2 \times 74{\cdot}6 \text{ watts} = \frac{100 \times 74{\cdot}6}{746} = 10 \text{ horse-power.}$$

11. Describe the action of an electric current on a magnet, and explain how it is made use of in a galvanometer.

There are 25 turns of wire in a galvanometer coil, the mean radius of which is 150 cm. Assuming the value of H to be 0·18, find the current which will deflect through 45° a magnet placed at the centre of the coil. If the resistance of the circuit including the battery be 3 ohms, find the electromotive force required to produce the current.

12. Show that the strength of a current passing round a tangent galvanometer is proportional to the tangent of the angle of deflection.

A circuit is formed containing galvanometer, battery, and connecting wires, the total resistance of the circuit being 4·85 ohms. The galvanometer shows a deflection of $48\frac{1}{2}°$. When a piece of platinum wire is introduced into the circuit the deflection falls to 29°. From these data determine the resistance of the platinum wire, given $\tan 48\frac{1}{2}° = 1·130$, and $\tan 29° = 0·554$.

13. Explain fully the meaning of Ohm's law. What is the resistance of a conductor?

A Menotti cell of 40 ohms internal resistance is connected by thick wires with the terminals of a tangent galvanometer, formed by a single ring of stout copper wire. The deflection is 45°. Three similar cells of the same resistance are then connected in series with the first. What is the deflection of the galvanometer needle? Would any other arrangement of the four cells give a stronger current, and why?

14. A length of uniform wire, of resistance 12 ohms, is bent into a circle, and two points at a quarter of the circumference apart are connected with a battery whose resistance is 1 ohm and electromotive force 3 volts; find the currents in the different parts of the circuit.

15. A battery of 4 ohms internal resistance is sending a current through an external resistance of 6 ohms. The terminals of the battery are connected to the electrodes of a reflecting electrometer, and the deflection on the scale is 100 divisions. What will the deflection be when, everything else remaining the same, the external circuit is broken?

16. Three voltaic cells *A*, *B*, *C*, whose respective electromotive forces and resistances are as follows, namely

	A	*B*	*C*
Electromotive force	1·07	1·54	1·9 volts,
Resistance	·72	2·3	·1 ohms,

are connected in series, and the circuit is completed by a wire of resistance 5·9 ohms. Determine the strength of the current produced.

If the cell *B* were removed and replaced in the circuit with its terminals inverted, what would be the strength of the current?

17. State the law of the division of an electric current in two branches of a circuit, when part of the current is shunted.

A tangent galvanometer of 4 ohms resistance forms part of a circuit whose total resistance is 80 ohms. The galvanometer is then shunted by a wire whose resistance is 4 ohms. Find the ratio of the currents in the galvanometer before and after it is shunted.

18. Define carefully ***strength of electric current***, ***electromotive force***, and ***resistance***. What resistance should a wire have which, when connected across the terminals of a galvanometer whose resistance is 3663 ohms, would let one-hundredth part of the whole current pass through the galvanometer, and $\frac{99}{100}$ths through itself?

19. What is the current in a circuit which consists of a battery of 10 similar cells arranged in series and a wire of 24 ohms resistance? (Each of the cells has an electromotive force of 1·8 volts, and an internal resistance of ·3 of an ohm.)

20. Eight cells, each with half an ohm internal resistance, and 1·1 volts E.M.F., are connected, (*a*) all in series, (*b*) all in parallel, (*c*) in two parallel sets of four cells each: calculate the current sent in each case through a wire of resistance ·8 ohm.

21. An electrical current may pass from A to B by either of two wires A C B, A D B, the resistances of which are 3 and 7 respectively. What will be the resistance of a single wire which replaces A C B and A D B in such a way as not to produce any alteration in the current in the rest of the circuit?

22. A wire is formed into a circle, 1 foot in diameter, and two points, A and B, a quarter of the circumference apart, are connected to the poles of a battery of E. M. F. 3 volts and resistance 5 ohms. If 1 foot of the wire have a resistance of 6 ohms, find the current in the battery and in the two parts of the wire.

23. Explain the effects on the current from a battery of the size of the plates, their closeness together, the number of cells in series, and the number in parallel (*a*) when the external resistance is small, (*b*) when it is large.

24. A current from three Daniell's cells in series is passed through solutions of nitrate of silver, of sulphuric acid, and of sugar of lead, all in series, platinum electrodes being used. How much hydrogen is liberated, how much sulphuric acid decomposed, and how much lead, copper, and silver deposited, by the time half a gramme of zinc has been fairly dissolved in each cell? If all this occurs in 30 minutes, what is the strength of the current, being given that unit current (one ampère) deposits 4 grammes of silver in an hour?

N.B.—Atomic weights: H = 1, O = 16, S = 32, Cu = 63·5, Zn = 65, Ag = 108, Pb = 207.

25. A current produces 35 c.c. of mixed hydrogen and oxygen per minute in an electrolytic cell, and at the same time produces 155 units of heat per second in a wire; if the current be increased until it produces 105 c.c. of mixed gases per minute, calculate how much heat will now be produced per second in the wire.

26. The same current produces heat in a wire, and decomposes a solution of copper sulphate. What is the relation between the heat produced and the copper deposited, and how may it be verified?

27. If the current of a battery of 10 Grove's cells connected in series is sent simultaneously through two voltameters, containing respectively a solution of cupric sulphate and a solution of silver nitrate,

and placed one after another in the circuit, show how much copper and how much silver will be deposited while 3·25 grammes of zinc are dissolved in the entire battery. Also show how much copper and silver would be deposited during the solution of the same quantity of zinc, if the battery were arranged in two parallel series, each of 5 cells, instead of in a single series of 10 cells.

[Zn = 65·0; Cu = 63·4; Ag = 108.]

28. The internal resistance of a voltaic battery is equal to the resistance of three metres of a particular wire. Compare the quantities of heat produced, both inside and outside the battery, when the poles are connected by one metre of this wire, with the quantities produced in the same time when they are connected by 27 metres of the same wire.

29. Ten voltaic cells, each of internal resistance 2 and electromotive force 1·5, are connected (*a*) in a single series, (*b*) in two series of five each, the like ends of the two series being joined together. If the terminals are in each case connected by a wire of resistance 10, show what is the strength of the current in the wire in each case, and compare the rates of consumption of zinc.

30. A battery of electromotive force e and resistance R is employed in sending a current through a wire of resistance r, and there are no other resistances in the circuit. How much work will be done per second by the battery, and how much heat will be generated in the wire?

31. What is meant by the electrochemical equivalent of a substance? If a current of 1 ampère decompose per minute 0·0056 gramme of water, what quantity of silver will a current of 0·5 ampère separate from a solution of silver nitrate in an hour?

[108 grammes of silver are chemically equivalent to 1 gramme hydrogen.]

32. State the laws of electrolysis. A current deposits 5 grammes of copper per hour in a voltameter and also produces 5 units of heat per minute in a wire; if it be increased so as to deposit 10 grammes of copper per hour, how much heat will it produce in a minute in the wire?

CHAPTER XXVII.

ELECTROMAGNETIC INDUCTION.

169. Coefficient of Self-Induction. When a current flows through a coil it produces a flow of magnetic induction through the coil. This flow of induction is proportional to the current when the permeability of the surrounding medium is constant. In this case if F denotes the flow of induction through the coil and C the current in the coil then

$$F = L C,$$

where L is a constant. This constant is the co-efficient of self-induction of the coil.

If the permeability of the surrounding medium varies with the intensity of magnetisation, then L is not a constant, but varies with the permeability of the medium and therefore varies with C.

When F varies on account of the variation of C then we evidently have the variation

$$\frac{dF}{dt} = L\frac{dC}{dt}, \text{ that is, } E = -L\frac{dC}{dt},$$

This relation supplies another definition of L, and shows that the induced electromotive force is proportional to the rate of change of current in the coil. The minus sign indicates that when $\frac{dC}{dt}$ is positive the induced electromotive force, E, opposes the existing current.

170. Coefficient of Mutual Induction. If we have two separate circuits the variation of a current in one will set up an induced electromotive force in the other. Or, we may say that when a current exists in one there is a flow

of induction through the other, and any variation of this flow of induction gives an induced electromotive force in that circuit. Let the two circuits be denoted by A and B and let C and C′ be the currents in these circuits. Let the flow of induction *through* A due to the current C′ *in* B be denoted by M C′, where M is a constant involving the permeability of the medium. Then, as in Art. 83, for two magnetic shells, the mutual energy of the two circuits is M C C′. Similarly, if the flow of induction *through* B due to the current C *in* A is denoted by M′ C, the mutual energy of the circuits is given by M′ C C′. Hence, we have $M\,C\,C' = M'\,C\,C'$ or $M = M'$. That is, the flow of induction through A for unit current in B is the same as the flow of induction through B for unit current in A. This constant M is the coefficient of mutual induction for the two circuits. If, then, we have two circuits for which the coefficient of mutual induction is M, the flow of induction through one for current C in the other is M C, and if in a very short time t the current changes to C″, the *rate* of change of the flow of induction through the circuit is

$$\frac{M\,C - M\,C'}{t} \quad \text{or} \quad \frac{M\,(C - C')}{t}.$$

This measures the induced electromotive force, and the relations just explained are most conveniently written as

$$F = M\,C,$$

and

$$\frac{dF}{dt} = M\frac{dC}{dt} \text{ or } e = M\frac{dC}{dt}.$$

The coefficient of mutual induction, M, is evidently defined by its function in either of these relations.

171. Quantity of Electricity set in Motion by Inductive Action. Let the flow of induction through a circuit be denoted by F. If at any instant we have an infinitely small change dF in this flow in the infinitely small time dt then, neglecting sign, we get as before

$$e = \frac{dF}{dt}.$$

If R denote the resistance of the circuit, the current during the time $d\,t$ will be given by

$$c = \frac{e}{R} = \frac{1}{R} \cdot \frac{d\,F}{d\,t},$$

and the quantity of electricity set in motion during this time will be

$$c \, . \, d\,t = \frac{d\,F}{R}.$$

It follows that, for any finite change of the flow of induction from F to F′, the *quantity* of electricity inductively set in motion is given by

$$q = \frac{F - F'}{R}.$$

This shows that q is independent of the time of variation of F and varies inversely as R, the resistance of the circuit. This explains why, in induction experiments with the ballistic galvanometer, the throw of the needle is the same whatever the time of variation of the flow of induction through the coil, provided this time is small compared with the time of swing of the needle. Also as q varies inversely as R, it is evident that in order to get an appreciable throw of the galvanometer needle the resistance of the circuit, including the galvanometer, must be small, for the smaller it is the larger q will be. On the other hand, when a ballistic galvanometer is used to determine the quantity of electricity discharged through it by a condenser, the quantity of electricity is *fixed* and *small*, and the throw of the needle will be increased by the multiplying effect of a coil of many turns without being reduced by the resistance of the coil. For these reasons a low resistance ballistic galvanometer is best to use for magnetic induction experiments and a high resistance instrument *must* be used for condenser work.

172. Theory of the Rise and Fall of a Current in a Circuit. Let E denote the E. M. F. of the cell in the circuit, R the resistance, and L the self inductance of the circuit. Then, during the variable

state when the current is rising to its full value, we have, in Art. 152,

$$E - L\frac{dC}{dt} = CR.$$

or, dividing by R and transposing, we get

$$E/R - C = \frac{L}{R} \cdot \frac{dC}{dt},$$

that is,
$$\frac{dC}{(E/R - C)} = \frac{R}{L}\,dt.$$

But
$$d(E/R - C) = -dC.$$

Hence, we have
$$\frac{d(E/R - C)}{(E/R - C)} = -\frac{R}{L}\,dt.$$

Integrating this from the lower limits, where t and $C = 0$,

we get
$$\log_e \frac{E/R - C}{E/R} = -\frac{R}{L}t,$$

C denoting the current at the end of the time t from the starting of the current.

This gives

$$e^{-\frac{R}{L}t} = \frac{E/R - C}{E/R},$$

or
$$e^{-\frac{R}{L}t} = 1 - \frac{CR}{E},$$

or
$$\frac{CR}{E} = \left(1 - e^{-\frac{R}{L}t}\right).$$

That is
$$C_t = E/R\left(1 - e^{-\frac{R}{L}t}\right)$$

where C_t denotes the current at the end of the time t from the instant of closing the circuit.

In this relation E/R evidently denotes, in accordance

with Ohm's law, the final value of the current. If we denote this by C we have

$$C_t = C\left(1 - e^{-\frac{R}{L}t}\right).$$

or

$$\frac{C_t}{C} = 1 - e^{-\frac{R}{L}t}.$$

From this it is evident that, when t is equal to L/R, 2 L/R, 3 L/R, etc., the ratio of the actual current to the maximum value attainable is given by $1 - 1/e$, $1 - 1/e^2$, $1 - 1/e^3$, etc. That is, at the ends of the time L/R, 2 L/R, 3 L/R, etc., the current value is ·6321, ·8647, ·9502, etc., of the final attainable value. The quantity L/R is usually

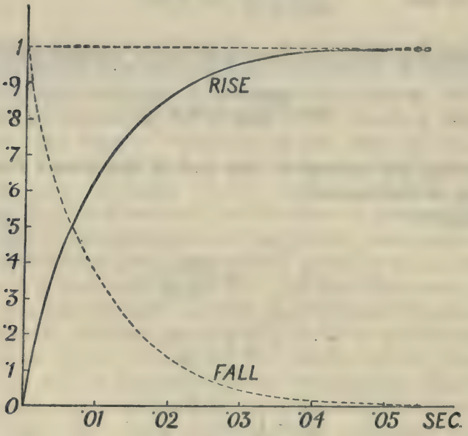


Fig. 288.

called the *time constant* of the circuit. It is evident from this that the current will require an infinite time to attain its full value, but that, as L/R is usually very small, it rapidly attains a value very nearly equal to its final value. This is shown in Fig. 288, which gives the curve of

rise of a current in a circuit, where $R = 2$ ohms and $L = \frac{1}{50}$ henry, and for which, therefore, the time constant

L/R is $$\frac{\frac{1}{50} \times 10^9}{2 \times 10^9} = \frac{1}{100}.$$

In this circuit the current rises to ·6321 of its full value in ·01 sec., to ·8647 of its full value in ·02 sec., to ·9502 of its full value in ·03 sec., and so on, evidently attaining practically its full value in a small fraction of a second. It will readily be understood that when L is great and R relatively small the time constant of the circuit will be large, and the current may take a considerable time to establish itself. On the other hand, when L is small compared with R the current rises very quickly. In all circuits where iron cores exist L will necessarily be large, and if these circuits are required to respond quickly to the make and break of a current it is evident that, in order to keep the ratio L/R small, R must be as large as possible consistently with obtaining a sufficiently strong working current.

When the circuit is broken the E.M.F. of the cell disappears from the relation given above, and we get

$$C R = -L \frac{dC}{dt}$$

or $$\frac{dC}{C} = -\frac{R}{L} dt.$$

Hence, if C be the value of the current at the instant of breaking circuit, then the current, C_t, at a time t afterwards is given by

$$\log \frac{C_t}{C} = -\frac{R}{L} t,$$

that is $$e^{-\frac{R}{L}t} = \frac{C_t}{C}$$

or $$C_t = C e^{-\frac{R}{L} t}$$

Here it is evident that in times L/R, 2L/R, 3L/R, etc. from the break of the circuit, the current falls to $1/e$, $1/e^2$,

$1/e^3$, etc., of its initial value, that is to ·3679, ·1353, ·0498, etc., of the initial value.

The dotted curve of Fig. 288 shows the fall of the current in the circuit for which the full line gives the curve of rise of the current.

This investigation assumes that the circuit is broken instantaneously. In practice this is not the case. Usually the resistance of the circuit is increased, it may be rapidly, but not instantaneously to an infinite value, and in most cases the resistance of the circuit is varied even after the metallic circuit is broken by the reduced resistance of the air gap along the path of the extra current spark at the break of the circuit.

During the rise or fall of a current C the change in the flow of magnetic induction through the circuit is L C, and, by Art. 171, the quantity of electricity set in motion by this change is L C/R or E L/R². This is the quantity carried by the "extra currents"; at make the flow of electricity is diminished by this amount, and at break the additional flow takes place after the circuit is broken. It should be noticed that this quantity is equal to that carried by the steady current E/R in a time L/R, the time constant of the circuit. The field energy associated with the extra current is evidently equal to $\frac{1}{2}$ L C², as explained in Art. 174.

173. Inductive Resistances in parallel. It has already been shown (Art. 145) that a steady current, dividing at the point A (Fig. 289) along the two resistances R_1 and R_2, divides in the inverse ratio of the resistances, and we have

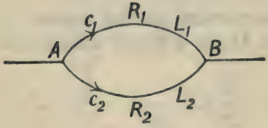


Fig. 289.

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} \quad \text{or} \quad C_1 R_1 = C_2 R_2.$$

This is true for *steady* currents whatever the value of L_1 and L_2, the inductances of the resistances, but it does not hold for alternating currents, or when the currents are increasing or diminishing in strength. Let x and y denote the currents in the two branches at any instant during a

varying state. Then at that instant the difference of potential between A and B is given by

$$x\,R_1 + L_1 \frac{dx}{dt} \quad \text{or} \quad y\,R_2 + L_2 \frac{dy}{dt},$$

that is,
$$x\,R_1 + L_1 \frac{dx}{dt} = y\,R_2 + L_2 \frac{dy}{dt}.$$

If we apply this relation to the first instant of the variable state at the starting of the currents, where x and y have both zero value, we get,

$$L_1 \frac{dx}{dt} = L_2 \frac{dy}{dt} \quad \text{or} \quad \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{L_1}{L_2},$$

that is, the rates of increase of the currents at the instant of starting are in the inverse ratio of the self inductances of the branches. At the instant the full values of the currents are established $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are both of zero value and the relation reduces to the usual simple form, $c_1\,R_1 = c_2\,R_2$, where c_1 and c_2 are the final steady values of x and y.

It must be noted, however, that if the current at A starts from a steady value and returns to the *same* value, after a period of variation, the total quantity of electricity that passes during the period of variation divides between the branches in the same way as a steady current. For example, the discharge current from a condenser or the currents due to induction which start and end at zero values come under this rule, so that the quantity of electricity discharged from a condenser or set in motion by an inductive impulse divides among branching conductors in the inverse ratio of the resistances.

The truth of this rule may be deduced from the relation

$$x\,R_1 + L_1 \frac{dx}{dt} = y\,R_2 + L_2 \frac{dy}{dt}.$$

For this gives

$$R_1\,x\,dt + L_1\,dx = R_2\,y\,dt + L_2\,dy,$$

and $x\,.\,dt$ and $y\,.\,dt$ evidently denote the quantity of electricity passing during the infinitely short interval of time

$d\,t$, when the strengths of the currents are x and y respectively. Assuming the currents to vary from values c_1 and c_2 to C_1 and C_2 and integrating between these limits, we get

$$R_1 \int_o^t x\,d\,t + L_1 \int_{c_1}^{C_1} d\,x = R_2 \int_o^t y\,d\,t + L_2 \int_{c2}^{C_2} dy.$$

Now, if $C_1 = c_1$ and $C_2 = c_2$, evidently $\int_{c_1}^{C_1} d\,x$ and $\int_{c2}^{C_2} dy$ both vanish, and we have

$$R_1 \int_o^t x\,.\,d\,t = R_2 \int_o^t y\,.\,d\,t.$$

But $\int_o^t x\,.\,d\,t = q_1$, the quantity of electricity which has passed through R_1 in the time t, and $\int_o^t y\,.\,dt = q_2$, the quantity which has passed through R_2 in the same time. Hence, $R_1\,q_1 = R_2\,q_2$, or

$$\frac{q_1}{q_2} = \frac{R_2}{R_1}.$$

This result is applied in proving the methods of measuring inductance given in Chapter XXVIII.

174. Energy in the Magnetic Field of a Current. It has been shown in Art. 128 that when a circuit, carrying a current C, is moved in a magnetic field so that there is a small change $d\,F$ in the flow of induction through the circuit, the work done during the motion is $C\,.\,d\,F$. Hence, if E be the E. M. F. of the battery or cell giving the current, and R the resistance of the circuit, we have, for the energy given out by the battery in a very short time $d\,t$, the expression $E\,C\,d\,t$. Also, the work done against electromagnetic forces during the displacement of the circuit is $C\,.\,d\,F$, and the energy dissipated in the circuit in the time $d\,t$ is $C^2 .\,R\,.\,d\,t$. We, therefore, have

$$E\,C\,.\,dt = C^2\,R\,.\,dt + C\,.\,d\,F.$$

Dividing this relation through by $C\,.\,d\,t$ we get

$$E = C\,R + \frac{d\,F}{d\,t}$$

or

$$E - \frac{dF}{dt} = CR$$

and

$$\frac{E - \frac{dF}{dt}}{R} = C.$$

This shows that, as the result of the electromagnetic work attending the variation of the flow of induction through the circuit, the *impressed* E. M. F., E, is *reduced* by a quantity $\frac{dF}{dt}$ $\left(\text{or } \textit{increased} \text{ by a quantity } -\frac{dF}{dt}\right)$, so that $E - \frac{dF}{dt}$ is the effective E. M. F. and $\frac{dF}{dt}$ is the *back* electromotive force, and is also the measure of the E. M. F. induced in the circuit by the change dF in the flow of induction through the circuit. The rule given arbitrarily in Art. 165, can in this way be deduced from the principle of conservation of energy in the circuit.

As we have seen in Art. 169, when the induced E. M. F. is due to self induction we have $F = LC$ and

$$\frac{dF}{dt} = L\frac{dC}{dt},$$

and therefore

$$e = -\frac{dF}{dt} = -L\frac{dC}{dt}.$$

If we consider a circuit of resistance R, containing a cell of electromotive force E, the steady value of the current is, by Ohm's law, E/R, but on account of self-induction there is always at the make and break of the circuit a variable state, lasting, in most circuits, for a very short time, during which the current at make increases from zero value up to its final value, and at break decreases from its highest value again to zero. The energy distribution during this variable state is important. At any instant during the rise of the current at make let C be the current in the circuit. Then, for an infinitely small increase dC in C, there will

be an increase $L . dC$ in the flow of induction through the circuit, and we, therefore, have

$$EC . dt = C^2 R . dt + LC . dC,$$

for the energy equation during the infinitely short time dt. This equation evidently means that the energy given out by the battery, $EC . dt$, is greater than the energy, $C^2 R . dt$, dissipated as heat in the circuit by an amount $LC . dC$, which is associated with the establishment of the magnetic field due to the current, and must therefore be looked upon as the electromagnetic energy stored in the medium in which the magnetic field exists. As the current rises the amount of this energy increases until the steady state is attained. It will be explained later that the energy from the cell really travels from the cell *through the medium* to the circuit, where in general it is dissipated, but at the starting of the current some of this energy, instead of passing to the circuit to be dissipated, remains in the medium as the energy of the magnetic field, and during this short period the energy dissipated is less than the energy given out by the cell by the amount which remains in the medium. Similarly, when the circuit is broken, the current very rapidly decreases to zero value, and the energy of the magnetic field in the medium rushes into the circuit, where it is dissipated as heat, giving rise to what has already been described as the extra current at break.

From the equation given above the electromagnetic energy stored in the medium during a very small change dC in the current is $LC . dC$, where L is the coefficient of self-induction of the circuit. Hence, if a current increase from 0 to C in a circuit of self-inductance L, the total energy in the medium will be

$$\Sigma_0^C LC . dC \text{ or } L\int_0^C C . dC,$$

that is, $\frac{1}{2} LC^2$. This result is very similar to the result $\frac{1}{2} QV$ or $\frac{1}{2} VC^2$ of Art. 25, and may be established in the same simple way. We may consider the flow of induction LC introduced within a circuit in which, as the flow of induction is established, the *average* value of the current

is $C/2$, and therefore the work done is $LC \cdot C/2$ or $\frac{1}{2}LC^2$.

If it be remembered that the energy of a condenser is really the electrostatic energy of the dielectric of the condenser (Art. 33), it will be seen that there is a real analogy between the two results compared above. One measures the electromagnetic energy in the magnetic field set up in the medium surrounding a circuit carrying a current, the other measures the electrostatic energy in the electric field set up in the dielectric of the condenser. The energy of a condenser may be written as

$$E = \frac{1}{2}QV = \frac{1}{2}VC^2 = \frac{1}{2}\frac{Q^2}{C}$$

where Q, V, and C, have the usual interpretations.

Similarly, the medium energy of a circuit carrying a current may be written as

$$E = \frac{1}{2}FC = \frac{1}{2}LC^2 = \frac{1}{2}\frac{F^2}{L},$$

where F denotes the flow of induction through the circuit, and is equal to L C, where L denotes the self inductance of the circuit, and C the current in the circuit.

Some idea of the process of transfer of energy through the medium from a cell to its circuit may be obtained by considering the charging and discharging of a condenser. Let the parallel plate condenser shown in Fig. 290 be charged by connecting its plates to the poles of a cell. When connection is made unit tubes of force 11, 22, 33, etc., each carrying unit quantity of positive electricity at one end and unit quantity of negative electricity at the other end, travel towards the condenser and quickly fill up the medium between the two plates. When the charging *begins* the difference of potential between P and N (Fig. 291), the terminals of the cell, is equal to the E. M. F. of

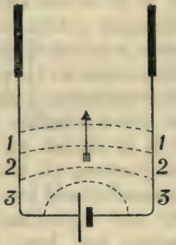


Fig. 290.

the cell, and no difference of potential has yet been established between A and B. It follows therefore that during the initial stages of the charging the difference of potential between two points, a and b, decreases as the points are taken farther from P and N, and nearer to A and B. The energy in a tube of force being proportional to the difference of potential between its ends, the unit tubes of force between P and N will tend, therefore, to move from P N towards A B in order to take up positions of minimum potential energy, and each tube, as it passes towards A B, undergoes loss of electrostatic energy, the amount lost being partly dissipated as heat in the conductors P A and N B, and partly transformed into electromagnetic energy of the magnetic field surrounding the conductors. This electromagnetic energy may be considered as of the nature of kinetic energy associated with the motion of the tubes of force, the electrostatic energy of which being assumed to be of the nature of potential energy. As the charging goes on the difference of potential between A and B rises until finally it becomes equal to the E. M. F. of the cell, and points on P A and on N B are at the same potentials as P and N respectively. When this state is attained the charging is complete and the tubes of force cease to travel through the medium from the cell to the condenser. The energy which has passed from the cell to the condenser through the medium between P A and N B is stored up in the electrostatic field between these conductors, all the electromagnetic energy produced being retransformed into electrostatic energy. During the transfer and attendant transformation a certain amount of energy has been dissipated as heat in conductors P A and N B, so that during the charging the cell has given out more energy than is stored in the electrostatic field.

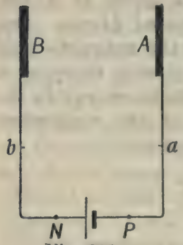


Fig. 291.

Let us now consider the discharge of the condenser when

the plates A and B are disconnected from the cell and connected by a high resistance.

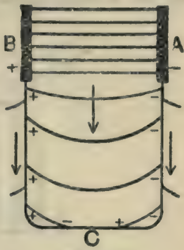


Fig. 292.

Directly connection is made the unit tubes of force between A and B (Fig. 292) travel towards C, so as to reduce their energy by reducing the difference of potential between their ends. As each tube travels towards C, the energy it loses is partially dissipated as heat and partially transformed into electromagnetic energy in the medium surrounding the conductor A C B, and finally, when the ends of the tube meet, the tube disappears and all the energy in it is transformed into heat and electromagnetic energy, the latter being finally dissipated as heat in the circuit. As the discharge goes on, tube after tube disappears, and finally the whole energy of the condenser's field is dissipated and the condenser is discharged.

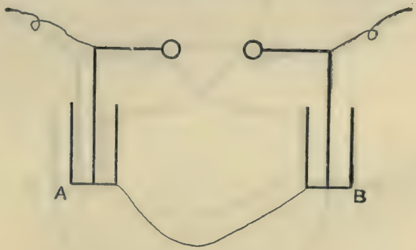


Fig. 293.

Figs. 293 to 298 are given by Prof. J. J. Thomson to illustrate the behaviour of tubes of force in the discharge of the Leyden jar condenser arrangement shown in the figures.

In the case of a simple circuit P A B N (Fig. 299) made up of a cell and a conductor, the process of transfer

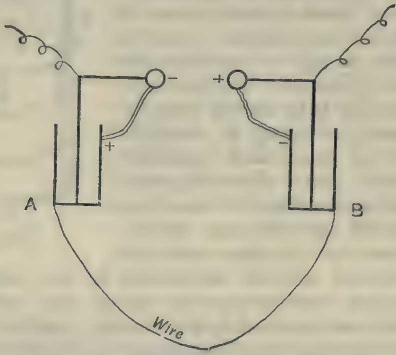


Fig. 294.

and dissipation of energy by means of the medium is practically the same as described above.

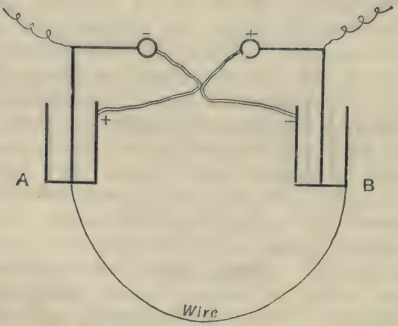


Fig. 295.

When the current first starts some electrostatic energy is converted into electromagnetic energy as the magnetic field

of the current is established, and when the circuit is broken this electromagnetic energy is dissipated as heat in the

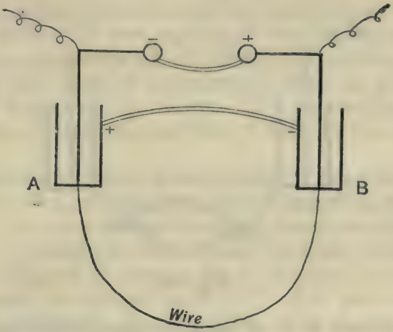


Fig. 296.

circuit, but when the current is steady there is a steady dissipation of electrostatic energy as heat in the circuit.

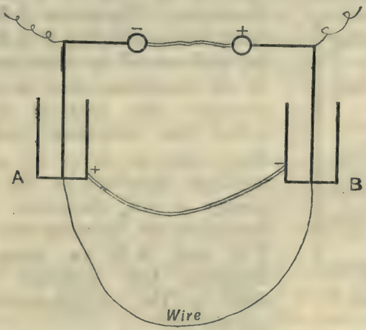


Fig. 297.

Tubes of force pass through the medium from the cell out along the circuit, and each tube, as it shortens to nothing,

as at A and B, gives up all its energy to be dissipated as heat in the circuit. There is therefore in the case of a steady

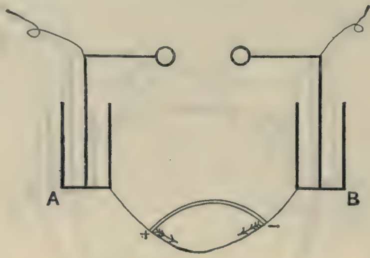


Fig. 298.

current a steady flow of tubes of electrostatic energy from the cell out to the medium, the ends of each tube travel along the conductor of the circuit and the energy of each tube, diminishing as it travels along, is ultimately completely dissipated as heat in the circuit. It must be remembered that Fig. 299 is only of a diagrammatic character. The tubes of force passing from the conductor P A to N B must be understood to start out from PA in all directions, and to curve round through the medium to N B on which they terminate, and on which they close in from all sides. The medium involved in the transfer of energy is not, therefore, confined to that directly between the conductors PA and NB as shown in the diagram, but includes the whole field in the neighbourhood of the circuit.

Fig. 299.

It will be indicated later that in an electric field such as we have described, where the tubes of force are not in equilibrium, but in motion, we have, as a result, a magnetic field—the magnetic field associated with the so-called

currents in the conductors in the field. If we draw on the field a series of equipotential surfaces and also a corresponding series of surfaces of equal electric force, the lines of intersection of these surfaces can be shown to be lines of magnetic force. If the motion of the tubes of force in the field has attained a steady state these surfaces will be fixed, and the lines of intersection, that is, the lines of magnetic force, will be fixed. This is another way of saying that a steady current has a definite fixed magnetic field.

175. Force between Two Current Circuits. For any two closed circuits, for which the co-efficient of mutual induction is M, the force exerted between them can be deduced from the expression for the mutual potential energy of the circuits. If C and C′ denote the currents in the circuit, the mutual energy is given by $M\,C\,C'$. For any very small linear displacement dx, let dM be the corresponding change in M, then $C\,C' \,.\, dM$ is the change in the mutual energy, and $C\,C' \,.\, \frac{dM}{dx}$ is the force acting between the circuits. The force is one of attraction or repulsion according to the direction of the currents. A special case, important for its application to the measurement of current, is the case of two parallel circular coils arranged co-axially with their planes at right angles to the common axis and carrying the same current C. The force of attraction or repulsion between them is $C^2 \frac{dM}{dx}$, where $\frac{dM}{dx}$ is evidently the rate of variation of M with distance along the axis at any point. This force can be measured directly by weighing, and if $\frac{dM}{dx}$ is known C^2 can be measured. This is the principle of the Kelvin current balance (Art. 176). This method of measurement, since it gives C^2 and not C directly, can evidently be applied to the measurement of alternating currents and the value of C so obtained will be the "root of the mean square" value.

176. Electrodynamometers. Currents may be measured by measuring the force acting between two coils, one of

which is fixed and the other movable. If the same current is passed through both coils the force or couple moment exerted between the coils is, as explained in Art. 175, equal to $C^2 \frac{dM}{dx}$ or $C^2 \frac{dM}{d\theta}$, where dM is the small change in M, the co-efficient of mutual induction, corresponding to a small linear displacement dx or angular displacement $d\theta$. The value of $\frac{dM}{dx}$ or $\frac{dM}{d\theta}$ is not a constant, but varies with the change of the relative position of the coils. Hence, in the construction of electrodynamometers it is necessary to arrange either that, when finally adjusted for reading, the coils are always in the same relative position or that the conditions of displacement are such that $\frac{dM}{dx}$ or $\frac{dM}{d\theta}$ is readily expressed in terms of the displacement.

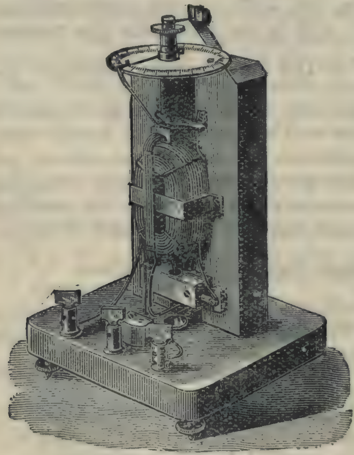


Fig. 300.

In Siemen's dynamometer, shown in Fig. 300, the two coils, one fixed and one moveable, are set with their planes at right angles. When a current is passed through them the moveable coil rotates round its vertical axis and tends to set parallel to the fixed coil. By means of the torsion head, however, the coil can be brought back to its initial position, and the amount of torsion necessary to do this is proportional to the moment of the couple tending to cause deflection and therefore proportional to the square of the current strength. That

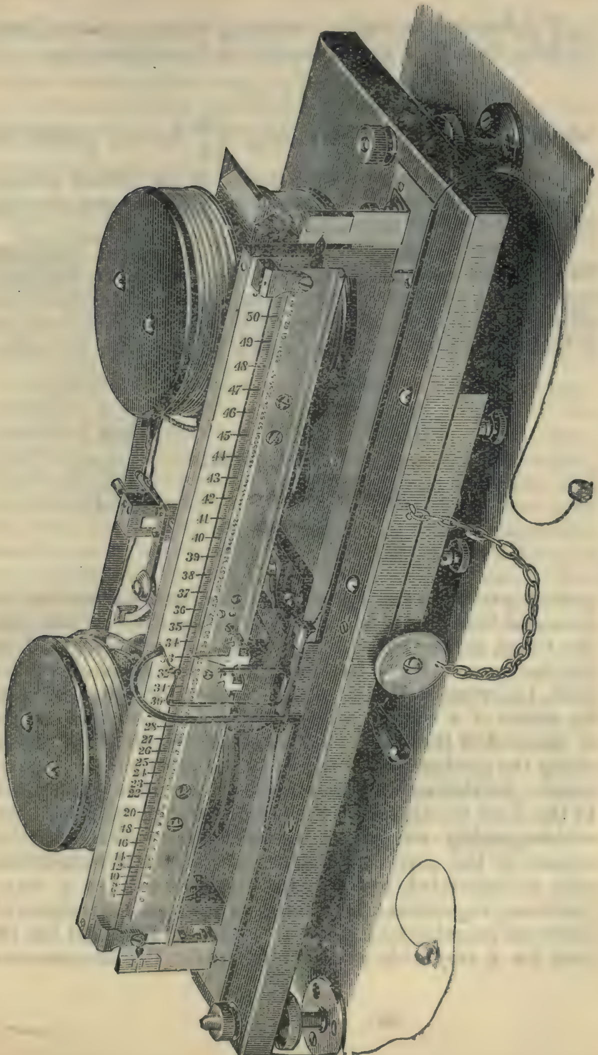


Fig. 301.

is, if θ denotes the torsion necessary to bring the coil back to zero, then C is proportional to $\sqrt{\theta}$, and therefore

$$C = k\sqrt{\theta},$$

where k is a constant involving $\frac{dM}{d\theta}$ for the position of adjustment and the torsion constant of the torsion spring. The amount of torsion is read off on the graduated circular scale shown in the figure.

In Kelvin's current balance, shown in Fig. 301, there are two movable coils and four fixed coils, as shown diagrammatically in Fig. 302. The two movable coils are fixed, one at each end of the balance beam, so that each coil hangs in a horizontal position in the space between the two com-

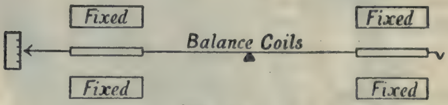


Fig. 302.

ponent parts of each fixed coil. The connections are so arranged that when the current is passed through the coils the forces acting between the coils on each side tend to tilt the beam in the same direction. The beam can, however, be brought to its initial position of exact balance by means of a counterpoise placed in the V-shaped trough at one end of the beam, and a sliding weight which moves along the graduated bar shown in the front of the instrument. In this way the movable coils are always adjusted to the same position, and the balancing moment due to the counterpoising weights is therefore proportional to the square of the current. The constant of the instrument may be calculated, but is usually determined by a standardising experiment. By using counterpoising weights of different magnitudes a current balance of this kind can be used for a very wide range of current. The instruments

are made for several specified ranges of current, the coils for any particular range being adapted to carry the maximum current of the range without undue heating.

In the two forms just described the adjustment for the measurement of the current brings the coils always into the same relative position, so that $\frac{dM}{d\theta}$ or $\frac{dM}{dx}$ is a constant, being the constant for that particular position.

In Weber's dynamometer, however, the movable coil is deflected, and the current measured by means of the deflection.

In Weber's dynamometer, as shown in Fig. 303, the large outer coil is fixed, and the small inner coil, suspended by a bifilar suspension, is movable. Initially the coils are set at right angles, but when a current is passed through them the small movable coil tends to set parallel to the fixed coil, and therefore suffers deflection until the opposing couple due to the suspension balances the deflecting couple. The inner coil is very small relatively to the outer one, and is adjusted in position so that the centres of the two coils are coincident. The two wires of the bifilar suspension serve as leads for the current. When the axes of the coils are exactly at right angles the value of M is zero. But if the inner coil be deflected through an angle θ then the flow of force through it, for unit current in the outer coil, is $G n A \sin\theta$, where G is the constant of the outer coil, n the number of turns of the inner coil, and A its mean area of cross section. This then is the change in M for a small angular displacement θ, and $\frac{dM}{d\theta}$, when θ is very small, is $G n A \frac{\sin\theta}{\theta}$, or $G n A$. The moment of the deflecting couple is therefore $C^2 G n A$, and if $T\alpha$ denote the moment of the opposing suspension couple we have

$$C^2 G n A = T\alpha,$$

or

$$C^2 = \frac{T}{G n A}\alpha.$$

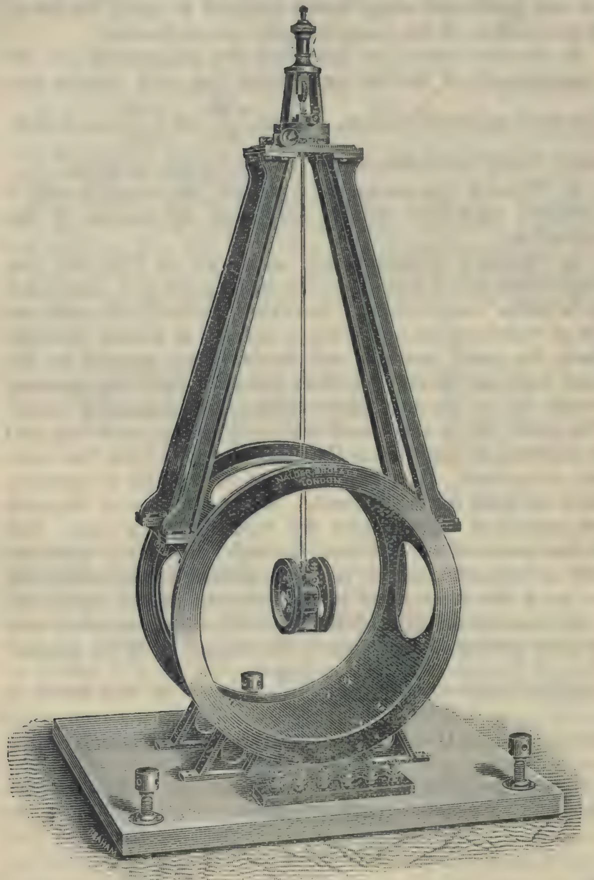


Fig. 303.

That is,

$$C = \sqrt{\frac{T}{G n A}}\ \sqrt{a}.$$

or

$$C = k\sqrt{a},$$

where k is a constant which may be calculated or determined by experiment.

Since all electrodynamometers measure C^2 they may be used for the measurement of alternating currents as explained in Chapter XXIX.

Electrodynamometers may also be used as Wattmeters. If in a circuit the current is C ampères and the electromotive force E volts, then the power or activity in the circuit is E C watts. If, therefore, the current C is sent through one coil of an electrodynamometer, usually the fixed coil, and a current, c, *proportional to* E, through the other coil then the indication of the instrument, being proportional to $C c$, is directly proportional to E C, or if w denote the power, we have, in the case of Weber's dynamometer, for example,

$$w = k\,\theta,$$

where k is a constant to be determined. The current c, proportional to E, may be obtained as in a voltmeter by connecting the terminals of the source of E through a very high resistance to the dynamometer coil.

CHAPTER XXVIII.

MEASUREMENT OF INDUCTANCE.

177. Measurement of Inductance. The accurate measurement and comparison of self and mutual induction is a problem of some difficulty. We shall here notice only a few of the typical methods.

The Wheatstone bridge arrangement for measuring resistances has been applied to the measurement of inductance, and furnishes some of the most satisfactory methods of measurement. In all Wheatstone bridge methods the method of measurement starts from the fact that, in the usual Wheatstone bridge adjustment for the comparison of resistances, the balance for a steady current is made by pressing first the battery key and then, when the current is established, the galvanometer key. If the galvanometer key were put down first and then the battery key the presence of inductive resistances in the arms of the bridge would, during the variable state of the currents, disturb the balance of the bridge, and the galvanometer would indicate this by a sudden throw of the needle at the instant of closing the battery key. If on closing the keys in this order, after first adjusting for exact balance with steady currents in the usual way, there is no throw of the needle, then, either there is no inductive resistance in the arms or the induction effects in the arms of the bridge balance each other in the galvanometer. The conditions for this balance of induction effects depend upon the inductances and resistances in the bridge arms, and may, therefore, be applied to compare inductances suitably placed in the bridge circuit. If there is only one inductive resistance in the circuit, the throw of the galvanometer needle due to the self-induction of this resistance may be compared with the

throw produced by the discharge of a known quantity of electricity through the galvanometer, or by the permanent deflection caused by a known steady current when passed through the galvanometer.

178. Measurement of Self-Inductance. In Rayleigh's modification of Maxwell's original method of measuring self-induction the throw of the galvanometer needle in testing for the balance of transient currents, after obtaining exact balance for steady currents, is compared with the permanent deflection produced by the steady current determined through the galvanometer by disturbing the balance for steady currents. This disturbance of balance is effected by making a small change in the resistance of one arm of the bridge.

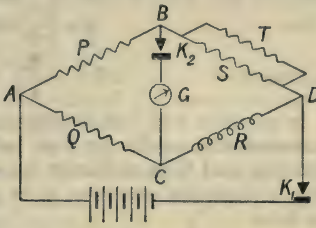


Fig. 304.

For this method the coil whose inductance is to be measured is connected up, as shown at R in Fig. 304, in one arm of a Wheatstone bridge arrangement of non-inductive resistances. If the resistance of the coil is very small it is advisable to insert a non-inductive resistance in the same arm with it.

The simplest arrangement is to make $P = Q$, and to facilitate exact balancing for steady currents the resistance in the arm BD should be two resistance boxes arranged in parallel. Approximate balance for steady currents is obtained between the resistances P, Q, R, and S, then S, being adjusted to a value a little above the value necessary for exact balance, the resistance T is then adjusted until exact balance is obtained with a resistance $ST/(S+T)$, equal to R, in arm BD. The keys are now worked in the order K_2, K_1, so as to test the balance for transient currents. The inductance in the arm CD will now cause the discharge of a quantity of electricity through the galvanometer, and there will be a sudden throw of the galvanometer needle.

This throw is noted; let the scale deflection be d_1, indicating an angular deflection, δ_1, of the needle. The balance for steady currents is now disturbed by increasing T so that the value of $\mathrm{S\,T}/(\mathrm{S}+\mathrm{T})$ is increased by a small amount ρ. On testing for balance of steady currents there will now be a permanent deflection of the galvanometer. Let this be a scale deflection d_2, indicating an angular deflection, δ_2, of the needle.

If L denote the self-inductance of the coil and C the current established in the arm C D on testing for transient current balance by closing K_1 after K_2 then, q, the quantity of electricity discharged through the galvanometer is proportional to L C, that is, $q = k\,\mathrm{L\,C}$, where k is a constant.

But
$$q = \frac{\mathrm{H\,T}}{\pi\,\mathrm{G}} \sin\frac{\delta_1}{2}$$

and therefore
$$k\,\mathrm{L\,C} = \frac{\mathrm{H\,T}}{\pi\,\mathrm{G}} \sin\frac{\delta_1}{2}.$$

In practice it is best to obtain the induction throw of the galvanometer needle by first balancing exactly for steady currents and then suddenly *reversing* the current. When this is done the quantity, q, is proportional to $2\,\mathrm{L\,C}$ and we have
$$k\,\mathrm{L\,C} = \frac{\mathrm{H\,T}}{2\,\pi\,\mathrm{G}} \sin\frac{\delta_1}{2}.$$

Also, when the resistance in the arm B D is increased by an amount ρ the potential difference for that arm is increased by an amount $\mathrm{C}'\rho$, where C' is the current in the arm after the increase is effected and the balance for steady currents disturbed. Hence, the permanent current determined through the galvanometer may be said to be due to this increment of potential difference in the arm B D and is therefore proportional to $\mathrm{C}'\rho$. That is c, the current through the galvanometer is given by $c = k\,\mathrm{C}'\rho$, where k is, on account of the symmetry of the bridge, the same constant as for q above.

But
$$c = \frac{\mathrm{H}}{\mathrm{G}} \tan\delta_2,$$

and therefore
$$k\,\mathrm{C}'\rho = \frac{\mathrm{H}}{\mathrm{G}} \tan\delta_2.$$

From the two results thus obtained, we get

$$\frac{k\,L\,C}{k\,\rho\,C'} = \frac{T}{2\,\pi} \cdot \frac{\sin\frac{\delta_1}{2}}{\tan\delta_2}\,.$$

And
$$L = \frac{T\,\rho}{2\,\pi} \cdot \frac{C'}{C} \cdot \frac{\sin\frac{\delta_1}{2}}{\tan\delta_2}\,,$$

since, δ_1 and δ_2 are small

$$\frac{\sin\frac{\delta_1}{2}}{\tan\delta_1} = \frac{d_1}{2\,d_2}\,,$$

and, when ρ is very small, C and C′ are approximately equal. Hence, we get

$$L = \frac{T\,\rho}{2\pi} \cdot \frac{d_1}{2\,d_2}\,.$$

When necessary the exact value of C′/C can be calculated in terms of the resistances involved.

Another typical method of measuring self-inductance is to determine it in terms of the capacity of a condenser by balancing the throw due to inductance in one arm of the Wheatstone bridge against the throw due to the action of a condenser in another part of the bridge circuit. For this purpose the bridge circuit is arranged as shown in Fig. 305.

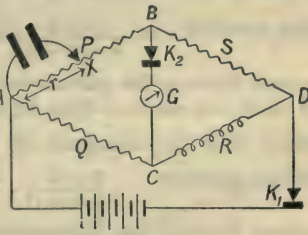


Fig. 305.

The arrangement is the same as that given above with the addition of a standard condenser of capacity C, having one terminal connected at A and the other terminal movable so that it can be connected at any point X on the resistance P in the arm A B.

Imagine the keys K_1 and K_2 to be closed, and that exact balance obtains for steady currents with the condenser

connected between the points A and X. The presence of the condenser will not in any way affect the conditions of balance in the steady state, but if, while exact balance obtains, the key K_1 is suddenly opened the condenser will discharge round the circuit and through the galvanometer from C to B. The discharge through the galvanometer due to self-induction in R will, however, be from B to C. Hence by adjusting the position of the point X in the resistance P it is evidently possible under suitable conditions to charge the condenser to a difference of potential such that the portion of its charge discharged through the galvanometer shall be exactly equal to the quantity sent through the galvanometer by the induction effect in coil in the arm C D.

If C denote the capacity of the condenser, r the resistance of A X, and c_1 the steady current in A B, then the charge in the condenser is $C c_1 r$. When discharge takes place the quantity passing through the circuit external to A X is

$$\frac{r}{Y} \cdot C c_1 r \quad \text{or} \quad \frac{C c_1 r^2}{Y},$$

where Y is the resistance of the circuit $\mathrm{AB}\overset{\diagup \mathrm{D} \diagdown}{\text{——}}\mathrm{CA}$. Of this quantity

$$\frac{R+S}{R+S+G} \cdot \frac{C c_1 r^2}{Y} = q_1$$

passes through the galvanometer. Similarly the quantity of electricity set in motion by self-induction in R is $L c_2/Z$, where Z is the resistance of the circuit $\mathrm{DB}\overset{\diagup \mathrm{A} \diagdown}{\text{——}}\mathrm{DA}$ and of this quantity

$$\frac{P+Q}{P+Q+G} \cdot \frac{L c_2}{Z} = q_2$$

passes through the galvanometer. On working out the values of Y and Z and simplifying, we get

$$\frac{q_1}{q_2} = \frac{C c_1 r^2 (R+S)}{L c_2 (P+Q)}.$$

But

$$\frac{c_1}{c_2} = \frac{Q}{P} = \frac{R}{S}.$$

Hence

$$\frac{q_1}{q_2} = \frac{C r^2}{L} \cdot \frac{R}{S} \cdot \frac{R+S}{P+Q}.$$

That is, since by the condition for steady balance $SQ = PR$,

$$\frac{q_1}{q_2} = \frac{C r^2}{L} \cdot \frac{R(R+S)}{P(R+S)},$$

or

$$\frac{q_1}{q_2} = \frac{C r^2}{L} \cdot \frac{R}{P}.$$

Therefore, when there is a balance for transient currents and $q_1 = q_2$, we have

$$C r^2 R = L P,$$

or

$$L = \frac{C r^2 R}{P}.$$

The balance for transient current should hold at make as well as break of the current. Hence the usual method of adjustment is to change the position of X until there is no movement of the needle on making and breaking or reversing current at K_1 while K_2 is closed. It will be clear from the relation

$$L = \frac{C r^2 R}{P}$$

that as the maximum value of r is P the minimum value of C, with which the adjustment for balance is possible, is L/P R.

By a modification of this method due to Professor

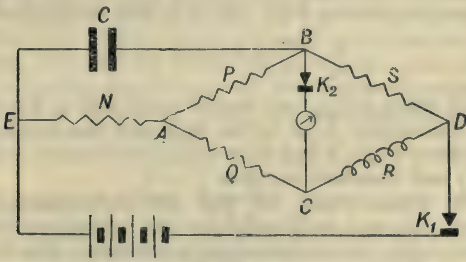


Fig. 306.

Anderson it is possible to avoid the movable contact at X. The diagram of Fig. 306 shows the modified arrangement of the

apparatus. An extra non-inductive resistance, N, is introduced at A E and the condenser is permanently connected between the points E and B. The value of N does not affect the condition for steady balance of the bridge, so that by altering it after the steady current balance is obtained this balance is not disturbed and the charge in the condenser can be adjusted so as to give the balance for transient currents. The operations of this method consist therefore of first obtaining exact balance for steady currents and then adjusting N until there is also exact balance for transient currents.

Adopting the same notation as above it is evident that F, the charge in the condenser, is $[N(c_1 + c_2) + P c_1] C$, and the portion of this tending to pass through the galvanometer is evidently

$$\frac{R + S}{R + S + G} \cdot \frac{P}{Y} \cdot F = q_1.$$

Also as above

$$q_2 = \frac{P + Q}{P + Q + G} \cdot \frac{L c_2}{Z},$$

and after substitution for Y and Z and simplification, we get

$$\frac{q_1}{q_2} = \frac{F P (R + S)}{L c_2 (P + Q)},$$

and on substituting for F and reducing as before, we get.

$$\frac{q_1}{q_2} = \frac{C}{L} (N R + N S + P R).$$

so that when $q_1 = q_2$ we have $L = C (N R + N S + P R)$. This determines L in terms of C and the non-inductive resistances P, R, S and N as adjusted for exact balance of steady and transient currents.

179. Comparison of two self-inductances. The method proposed by Maxwell for this comparison is simple in theory but very troublesome in practice. The two coils, whose self-inductances are to be compared, are placed each in series with a non-inductive resistance in the adjacent arms of a Wheatstone bridge arranged as shown in the diagram of Fig. 307

The method of adjustment consists in obtaining a balance for steady currents with different values of R and S, the total resistances in the arms C D and B D respectively, until the ratio of these values is so adjusted that there is also exact balance for transient currents.

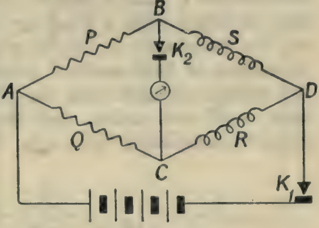


Fig. 307.

If L_1 and L_2 denote the self-inductances of the coils in BD and C D, and c_1 and c_2 the currents in A B D and A C D, then the electromotive impulses in these coils are $L_1 c_1$ and $L_2 c_2$ respectively, and, from the position of the coils relative to the galvanometer, the electromotive impulses through the galvanometer are in opposite directions and in the ratio $L_1 c_1/L_2 c_2$. But $c_1/c_2 = R/S$ and therefore $L_1 c_1/L_2 c_2 = L_1 R/L_2 S$, so that when there is balance for transient currents and these impulses are equal we have $L_1 R = L_2 S$, or $L_1/L_2 = R/S$. Hence, although balance for steady currents is obtained when $P/Q = S/R$, whatever the values of S and R may be, the balance for transient currents is obtained only when $S/R = L_1/L_2$. It is necessary therefore to vary the ratio of S to R, adjusting P and Q for balance for steady currents for every value of the ratio until finally, on testing for balance of the transient currents, exact balance is obtained. When this troublesome double adjustment is made we have

$$\frac{L_1}{L_2} = \frac{S}{R} = \frac{P}{Q}.$$

A modification of this method, proposed by Niven, allows the comparison to be made without repeated adjustment of balance for steady currents.

The bridge resistances are arranged as shown in Fig. 308, and an additional adjustable non-inductive resistance N is connected between the points b and c, the point b being the

junction of s and σ the inductive and non-inductive resistances in the arm B D, and the point c the junction of the resistances r and ρ in the arm C D. The coil r in C c is also shunted with a plug key K, so that, when the plug is in, the resistance in C c is practically nothing. The first operation of this method consists in obtaining a balance for steady currents with the plug in the key K, the resistance σ of zero value, and the value of N infinite. This adjustment gives $P/Q = s/\rho$. Then K is unplugged and the

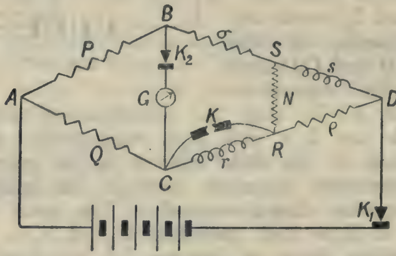


Fig. 308.

resistance σ adjusted until balance for steady currents is again obtained. This gives $P/Q = (\sigma + s)/(r + \rho)$ and we therefore have $P/Q = s/\rho = \sigma/r$, that is, b and c are for steady currents at the same potential. For the third operation, the balance for transient currents is tested and the resistance N adjusted until exact balance is obtained. When this balance is obtained it can be shown, by the method worked out for similar cases above, that

$$\frac{L_1}{L_2} = \frac{P(N + \rho + s)}{N Q}.$$

180. The Measurement of Mutual Inductance. The mutual induction for two coils may be measured in terms of the capacity of a condenser by the following method. The two coils P and S (Fig. 309), P the primary and S the secondary, are connected, as shown in the diagram, in circuit with a condenser of capacity C, the non-inductive resistances R and Q and the galvanometer G. By means

of a key at K the current in P can be made, broken, or reversed, and an inductive impulse equal to Mc or 2Mc thereby set up in S, where M is the co-efficient of mutual induction for the coils and c is the current in R and P. The condenser also, with its terminals connected to the points A and B, becomes charged or discharged, or has its charge reversed according as the current is made, broken, or reversed. By the arrangement of the circuit the two discharges through the galvanometer due to the inductance of S and the capacity of C are in opposite directions, and may be adjusted to equality by adjusting R or Q until the galvanometer shows no deflection on working the key at K. When this adjustment is made we have

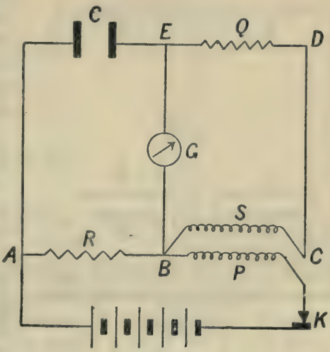


Fig. 309.

$$\mathrm{M} = \mathrm{C\,R}(\mathrm{Q} + \mathrm{S}).$$

Proof.—The charge in the condenser is CcR, and the portion of this which passes through the galvanometer is

$$\frac{\mathrm{Q} + \mathrm{S}}{\mathrm{Q} + \mathrm{S} + \mathrm{G}} \cdot \mathrm{C}c\mathrm{R}.$$

The electromotive impulse set up in S is Mc, and the quantity of electricity set in motion by it is

$$\frac{\mathrm{M}c}{\mathrm{G} + \mathrm{S} + \mathrm{Q}}$$

This quantity all passes through the galvanometer. Hence we have

$$\frac{\mathrm{Q} + \mathrm{S}}{\mathrm{Q} + \mathrm{S} + \mathrm{G}} \cdot \mathrm{C}c\mathrm{R} = \frac{\mathrm{M}c}{\mathrm{Q} + \mathrm{S} + \mathrm{G}},$$

or

$$(\mathrm{Q} + \mathrm{S})\mathrm{C\,R} = \mathrm{M},$$

where M is the mutual inductance to be measured.

181. Comparison of Mutual Inductances. The two pairs of coils whose mutual inductances are to be compared are arranged as shown in Fig. 310. The arrangement is exactly that of Lumsden's method of comparing electromotive forces, and is here used to compare the electromotive impulses $M_1 c$ and $M_2 c$ produced in the coils S_1 and S_2 by making or breaking the current c in the battery circuit. Hence, when R_1 and R_2 are adjusted to give no deflection of the galvanometer on making or breaking circuit at K, we have

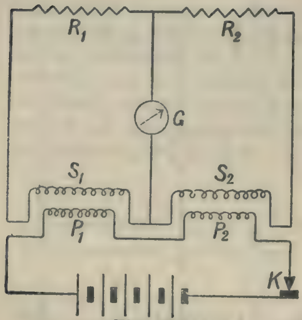


Fig. 310.

$$\frac{M_1 c}{M_2 c} = \frac{R_1 + S_1}{R_2 + S_2},$$

and therefore

$$\frac{M_1}{M_2} = \frac{R_1 + S_1}{R_2 + S_2}.$$

CHAPTER XXIX.

ALTERNATING CURRENTS.

182. Alternating Currents. Consider the case of a thin coil rotating in a uniform magnetic field about an axis at right angles to the direction of the field. When the plane of the coil makes an angle α with the plane at right angles to the field the flow of induction through the coil is $n\,A\,H\cos\alpha$, where H denotes the strength of the field, A the area of the coil, and n the number of turns. If the coil rotates with uniform angular velocity ω, and time is measured from the instant of passing through the position at right angles to the field, then we may write $\alpha = \omega t$, where t denotes the time in which the angle α has been described. The flow of induction through the coil is therefore $n\,A\,H\cos\omega t$, and the *rate* of change of this with time is $-n\,A\,H\,\omega\sin\omega t$. The induced electromotive force in the coil is therefore, Art. 165, given by $n\,A\,H\,\omega\sin\omega t$. That is, the induced electromotive force is a simple harmonic function of the time, and may be assumed to give rise to a current varying in the same periodic manner. The maximum value of this electromotive force is $n\,A\,H\,\omega$, which obtains when $\omega t = \frac{\pi}{2}$, or when the plane of the coil is parallel to the direction of the field. If we write this maximum value as E_o we have

$$E = E_o \sin\omega t.$$

This variation of the electromotive force may be represented graphically by a sine curve where the maximum ordinate has the value $E_0 = n\,A\,H\,\omega$.

If the circuit of the coil were without inductance the current at any instant would be given by

$$C = \frac{E}{R} = \frac{E_0}{R} \sin \omega t.$$

That is, the current would also be harmonic with the same period and phase, but of different amplitude, and could be represented by a sine curve where the maximum ordinate of the curve is E_o/R.

No circuit, however, is without self-inductance, and its effect must therefore be considered.

In order to do this it is simplest perhaps to suppose that a simple harmonically varying or alternating current exists in a circuit, and to find what must be the nature of the electromotive force in the circuit in order to produce such a current. Let the current in the circuit be denoted by

$$C = C_o \sin \omega t.$$

This gives

$$\frac{dC}{dt} = C_o \omega \cos \omega t,$$

and, if L be the self-inductance of the circuit, the induced electromotive force due to self-induction given by $L \frac{dC}{dt}$ is equal to $L C_o \omega \cos \omega t$. Now by applying Ohm's Law to the circuit we get

$$E = CR + L\frac{dC}{dt}.$$

that is,

$$E = C_o R \sin \omega t + L C_o \omega \cos \omega t,$$

or

$$E = C_o [R \sin \omega t + L \omega \cos \omega t].$$

This may be written

$$E = C_o \sqrt{R^2 + L^2 \omega^2} \sin (\omega t + \phi),$$

if

$$\frac{R}{\sqrt{R^2 + L^2 \omega^2}} = \cos \phi \quad \text{and} \quad \frac{L \omega}{\sqrt{R^2 + L^2 \omega^2}} = \sin \phi.$$

That is, if

$$\tan \phi = \frac{L\omega}{R}.$$

Here the maximum value of E is $C_o\sqrt{R^2+L^2\omega^2}$, and if this be denoted by E_o, we have

$$E = E_o \sin(\omega t + \phi).$$

That is, when the current varies harmonically in a circuit the electromotive in the circuit also varies harmonically, with the same *frequency*, and the maximum value of the electromotive force is $\sqrt{R^2+L^2\omega^2}$ times the maximum value of the current. The phase of the electromotive force is, however, in advance of that of the current, or the phase of the current lags behind that of the electromotive force by $\phi/2\pi$ of a complete period, the angle ϕ being such that $\tan\phi = L\omega/R$, where R is the resistance, and L the self-inductance of the circuit, and $\omega/2\pi$ the *frequency* of the alternations of the current and the electromotive force in the circuit. Hence, if the electromotive force in the circuit be given by

$$E = E_o \sin \omega t.$$

Then for the current we have, in accordance with the result just stated,

$$C = \frac{E_o}{\sqrt{R^2+L^2\omega^2}} \sin(\omega t - \phi)$$

so that C_o, the maximum value of the current is given by

$$C_o = \frac{E_o}{\sqrt{R^2+L^2\omega^2}}.$$

The quantity $\sqrt{R^2+L^2\omega^2}$ is usually called the *impedance* of the circuit. In a circuit of negligible inductance it is practically equal to R, the resistance of the circuit, but in a circuit of small resistance and high inductance it is practically equal to $L\omega$. Hence in a current for which $L\omega$ is high the value of the maximum current may be very low even when R is small. Thus, in a circuit of resistance 5 ohms, self-inductance ·045 henry and frequency 50 the

maximum value of the current for a maximum electromotive force of 105 volts is

$$\frac{105}{\sqrt{25+200}} \text{ ampères} = \frac{105}{15} \text{ ampères} = 7 \text{ ampères.}$$

A steady electromotive force of 105 volts in a circuit of 5 ohms resistance would give a steady current of 21 ampères so that in this case the effect of self-inductance for an alternating current is to reduce the maximum current possible to one-third of the steady value for a continuous or non-alternating current.

183. Graphic representation. The relation between the *impressed*, the *effective*, and the self-induced electromotive forces expressed by the equation

$$\mathrm{E} = \mathrm{C\,R} + \mathrm{L}\frac{d\,\mathrm{C}}{d\,t}$$

can be represented graphically. Each of the quantities varies harmonically and may therefore be represented by the projection of the radius of a circle revolving with a period equal to that of the electromotive force alternations. The self-induced electromotive force, $-\mathrm{L}\dfrac{d\,\mathrm{C}}{d\,t}$ lags a quarter of a period behind the effective electromotive force C R, for when one is at its maximum the other is at zero value. If in Fig. 311, therefore, O E represents the maximum value of the effective electromotive force and O N the maximum value of the induced electromotive force, then O P drawn parallel to N E will represent the impressed electromotive force.

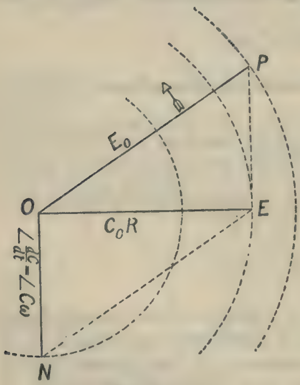


Fig. 311.

For, since O P E N is a parallelogram the projection of O E on any straight line through O is equal to the sum

of the projections of O P and O N. Hence, if the figure O P E N be supposed to revolve round O in the direction of the arrow, the projections of O P, O N and O E on the line O X (Fig. 312) give at any instant the instantaneous values of E, $-\mathrm{L}\dfrac{d\mathrm{C}}{dt}$ and C R and as geometrically the projection O e always equals O p + O n the construction expresses the relation

$$\mathrm{E} - \mathrm{L}\frac{d\mathrm{C}}{dt} = \mathrm{CR}.$$

$$\mathrm{E} = \mathrm{CR} + \mathrm{L}\frac{d\mathrm{C}}{dt}.$$

The angle P O E evidently represents the angle ϕ of the above formula and indicates the lag of the current alternations behind those of the electromotive force.

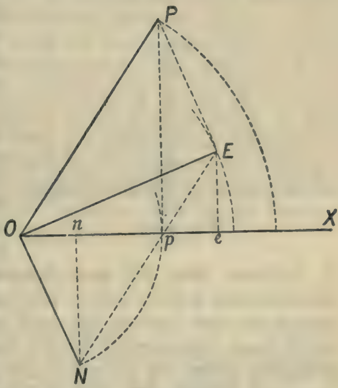


Fig. 312.

184. The "square root of the mean square" Current. An alternating current of suitable frequency acts, in cases where the effect does not depend upon the direction of the current, very much like a continuous current. Incandescent electric lamps can, for example, be lighted by an

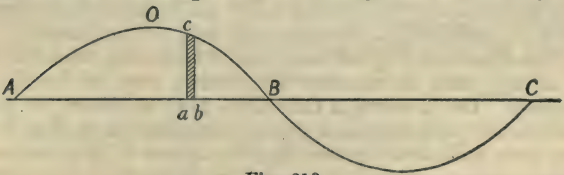


Fig. 313.

alternating current of the proper strength and frequency. As, however, the strength of an alternating current varies from instant to instant it is necessary to specify how the

strength of such a current is measured. The mean or average value of the current for a half period is not used for this purpose but a value known as the *square root of the mean square* current is used. Let the sine curve A B C (Fig. 313) represent the current for a complete period. Since the two half periods are symmetrical and of opposite sign any *average* or *mean* value for the whole period must evidently have zero value. To get a mean arithmetical value it is necessary, therefore, to consider only the half period. Since the abscissae of the curve represent time and the ordinates current it follows that the area between the curve AOB and and the line A B evidently represents the *quantity* of electricity carried during the half period. For in any very short time represented by $a\,b$ the quantity that passes is evidently given by $a\,c \times a\,b$ the area of the shaded strip standing on $a\,b$, and this is true for every short interval into which the half period represented by A B can be divided. Hence the area A O B A represents the total quantity of electricity carried by the current during the half period, and the average value could be obtained by dividing this quantity by the time of the half period. For a current varying harmonically this average value can be found to be equal to $2/\pi\ C_0$, where C_0 is the maximum value of the current.

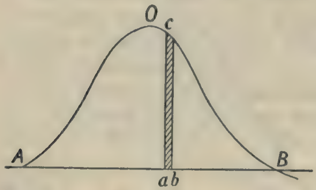


Fig. 314.

The square root of the mean square value, the value which is always used to measure the current, may be found as follows. Let a curve A O B (Fig. 314) be drawn for the half period with abscissae representing time as in Fig. 313 but with ordinates representing the *square* of the current at any instant. Then the area of the strip $a\,b\,c\,d$ represents the value of $C^2\,d\,t$, where $d\,t$ is the very short time represented by $a\,b$ and C is the square of the current strength represented by $a\,c$ or $b\,d$. The area of curve A O B A therefore represents $\Sigma\,C^2 . d\,t$ for the half period and the value of

$\Sigma C^2 . dt$ divided by the time of the half period gives the *mean square* value for the current and the square root of this gives the square root of the mean square value. The reason for taking this value will readily be understood. The heat developed by a current C in a time dt in a resistance R is $C^2 R . dt$ or $R . C^2 dt$, hence the heat developed in a resistance R by an alternating current during a half period is $R \Sigma C^2 dt$, and if $\mathbf{C}$ be the square root of the mean square current for the half period t then $\mathbf{C}^2 t = \Sigma C^2 . dt$. The heat developed in the resistance R by the alternating current whose square root of mean square value is $\mathbf{C}$ is therefore the same as the heat developed by a steady continuous current of strength $\mathbf{C}$ in the same time. That is the alternating current is measured by the strength of the steady current which would produce the same heating effect. When a current varies harmonically

$$\mathbf{C} = \left[\frac{C_0^2}{T}\int_0^T \sin^2 \omega t . dt\right]^{\frac{1}{2}} = \frac{C_0}{\sqrt{2}},$$

where 2 T is the period of alternation.

Similarly $\mathbf{E}$, the square root of the mean square electromotive force is defined in the same way and is equal to $E_0/\sqrt{2}$ when the variation is truly harmonic. The apparent resistance of the circuit given by $\mathbf{E}/\mathbf{C}$ is evidently equal to E_0 / C_0 that is to the *impedance* already defined.

185. Power. The rate of doing work or the *activity* of a current is, in case of a steady current given by E C, where E is the E.M.F. and C the current. In an alternating current both E and C vary harmonically and they differ in phase. From the relations given above we have

$$E = E_0 \sin \omega t$$

$$C = C_0 \sin (\omega t - \phi).$$

Hence the activity at any instant is given by

$$E C = E_0 C_0 \sin \omega t \sin (\omega t - \phi) = \tfrac{1}{2} E_0 C_0 [\cos \phi - \cos (2 \omega t - \phi)]$$

During a half period the angle $2 \omega t$ and therefore $(2 \omega t - \phi)$ changes by 2π, and the mean value of its cosine for the half period is, therefore, zero, hence the power or activity of the alternating current in which the

alternations follow the simple harmonic law may be given as

$$\tfrac{1}{2}\, E_o\, C_o \cos\phi$$

for a time equal to any integral number of half periods. Since

$$\tfrac{1}{2}\, E_o\, C_o = \frac{E_o}{\sqrt{2}} \cdot \frac{C_o}{\sqrt{2}} = \mathbf{E}\,\mathbf{C}$$

we may write

$$\text{Power} = \mathbf{E}\,\mathbf{C} \cos\phi.$$

Also since

$$\cos\phi = \frac{R}{\sqrt{R^2 + L^2\,\omega^2}}.$$

$$\text{Power} = \mathbf{E}\,\mathbf{C}\,\frac{R}{\sqrt{R^2 + L^2\,\omega^2}} = \frac{\mathbf{E}^2\,R}{R^2 + L^2\,\omega^2}.$$

It must be remembered that these are theoretical results based on the assumption that the electromotive force and current vary *harmonically*. In practice, alternating currents, although periodic, do not vary harmonically. The phenomena of hysteresis and permeability of the iron cores introduce additional factors into the results, but the principles involved and the general nature of the results are not affected.

If we plot a power curve for a given alternating current by plotting the values of E C as ordinates to times as abscissae, as in Fig. 315, we notice that the work done by

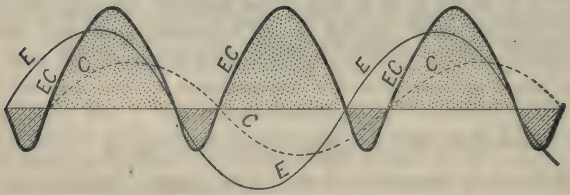


Fig. 315.

the current, as represented by the *area* of the curve, is partly positive and partly negative, that is, during certain

short periods occurring periodically the current generator gives out power, and at other periods it absorbs power. The shaded areas in the figure represent the power given out, the dotted areas the power taken in, and the difference of the two sets of areas during a complete period represents to total work done during the period.

186. Choking Coils. In the case of a coil with an iron core the self inductance L is not a constant, for the flow of induction through the coil for a current C in it is given by $L \mu C$, where L is a constant depending on the geometry of the coil and μ is the permeability of the iron which varies within wide limits (Art. 102) with the intensity of the magnetic field, and is therefore a function of C. On account of hysteresis also a certain amount of energy is dissipated as heat in the iron during each period, for during a complete period the iron goes through a complete cycle of magnetisation, and the energy dissipated is represented by the area of the hysteresis loop for the cycle (Art. 100).

It is evident, however, that for a coil with an iron core the self-inductance $L \mu$ must generally be large for μ may be very great (Art. 102). Hence, if an alternating current be sent through a low resistance coil with a soft iron core the retardation of the current relative to the electromotive force in the coil will be practically a quarter period, for

$\cos \phi = \dfrac{R}{\sqrt{R^2 + L^2 \omega^2}}$ or, if R be small and L large,

$\cos \phi = R/L\omega$ and $R/L\omega$ being very small ϕ is nearly $\pi/2$. This shows that $\mathbf{E}\,\mathbf{C} \cos \phi$, the power absorbed by the coil is very small although on account of its large impedance, $\sqrt{R^2 + L^2 \omega^2}$, it admits of the passage of only a small current through it. Such a coil is usually called a *choking coil* on account of its effect on the current and choking coils are largely used in alternating current circuits for the purpose of adjusting the current to any required value without waste of energy such as takes place when a regulating resistance is placed in the circuit of a continuous or alternating current. An interesting experimental demonstration of the retardation of phase produced by an iron core is due to Professor Elihu Thomson. Let

two coils, one of which is fixed and the other freely suspended be arranged with their planes vertical, parallel and with coincident axes, and let a thick iron core pass horizontally through them as shown in Fig. 316. If then an alternating current be sent through the fixed coil the suspended coil will be seen to be strongly repelled, showing that the alternating current induced in it is approximately half a period behind that in the fixed coil. The alternations of the current in the fixed coil induce an electromotive force in the free coil and the phase of this induced electromotive force will, on account of the large self-inductance due to the iron core, be very nearly a quarter of a period behind that of the primary current. Then again the current resulting from this induced electromotive force will for the same reason be nearly a quarter of a period behind the electromotive force and therefore nearly *half a period* later in phase than the primary current. That is, the two currents will, at every instant, be in opposite directions and will repel each other.

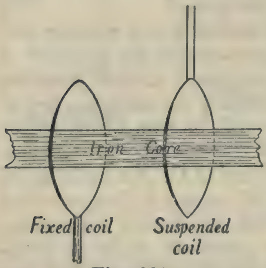


Fig. 316.

187. Skin Conduction. In the case of a steady current in a conductor the current is distributed uniformly over the cross section of the conductor. When a current first starts in a conductor, however, it is first established in the outer layer and takes a small but finite time to travel from layer to layer inwards to the interior of the conductor. The current energy dissipated in the conductor as heat comes out of the medium *external* to the conductor and it is therefore to be expected that the process of dissipation should begin in the outer layers and penetrate quickly, but not instantaneously into the substance of the conductor. Hence in the case of a rapidly alternating current the alternations of the current are impressed first on the "skin" or outer layer of the conductor and the period of alternation may be too short

to allow the current to penetrate to any appreciable depth into the substance of the conductor. This is found to be the case in practice—alternating currents tend to flow in the outer layers of the conductor and the more rapid the alternations the thinner the skin of the conductor to which the current is confined. Hence in providing a conductor of a given cross section for an alternating current to carry any sudden discharge or rush of electricity it is best for the conductor to take the form of a thin tube. Another factor, the permeability of the material of the conductor also enters into the question. As the current penetrates towards the axis of the conductor the magnetic field associated with it also invades the material and if the material be of high permeability as in the case of iron this may cause an appreciable increase in the energy of the electromagnetic field (Art. 106) and so be in opposition to the principle that the energy in the field tends to a minimum. For this reason the surface distribution of an alternating current is very noticeable in iron conductors.

188. Capacity and Inductance in series: When a harmonically varying electromotive force $E_0 \cos \omega t$ is applied to a circuit containing an inductive resistance of ohmic resistance R, self-inductance L, and a condenser of capacity F, in series, we have

$$E_0 \cos \omega t = C R + L \frac{dC}{dt} + V,$$

where C is the current in the circuit and V the potential difference of the condenser at a given instant. The difference of potential, V, will be a harmonically varying quantity with the same period as the electromotive force and may therefore by written

$$V = V_0 \cos (\omega t - a).$$

The quantity of electricity in the condenser is F V and the current in the circuit being

$$\frac{d\,(F\,V)}{d\,t}$$

we have

$$C = F \frac{dV}{dt} = - F \omega V_0 \sin (\omega t - a)$$

and

$$\frac{dC}{dt} = - F\omega^2 V_0 \cos(\omega t - \alpha).$$

Without following out the general result which may be obtained by substituting these values of C and $\frac{dC}{dt}$ in the relation given above, it can be seen that when

$$L\frac{dC}{dt} = -V,$$

and the effect of the capacity in the circuit neutralises the effect of the inductance; we have, as the condition for this result

$$L F \omega^2 V_0 \cos(\omega t - \alpha) = V_0 \cos(\omega t - \alpha).$$

That is

$$L F \omega^2 = 1$$

or

$$L F = \frac{1}{\omega^2}.$$

As $\omega^2 = \frac{2\pi}{T}$, where T is the period of alternation of the applied electromotive force, we have

$$L F = \frac{T^2}{4\pi^2} \text{ or } T = 2\pi\sqrt{LF}.$$

This, as shown in Art. 191, is the period for electric oscillations in the circuit. Hence, when the period of the applied electromotive force is the same as that of the circuit for electric oscillations, the interaction of capacity and inductance is such as to give exact neutralisation, and the general relation given above reduces to

$$E_0 \cos \omega t = C R.$$

or

$$C = \frac{E_0 \cos \omega t}{R}.$$

That is the current and the applied electromotive force are in the same phase, and the current has the same value as in a circuit of resistance R, free from inductance and capacity

189. Transformers. Transformers are used in modern electrical practice for converting an alternating current of high electromotive force and low current strength into a current of lower electromotive force and higher current value. The use of transformers will be understood from the following illustration. Suppose that in a certain district 100,000 watts of electrical power are required in the form of a current of 500 ampères at 200 volts, and that this power has to be transmitted to the district from a central power station at a distance of one mile. If the required power were transmitted direct at the volts and ampères specified, the cost of conductors to carry the 500 ampère current would amount to nearly £5,000. If, however, the power were transmitted as a "high tension" current of 20 ampères at 5,000 volts, the cost of the conductors, apart from the question of insulation, would be under £10. The insulation of these conductors would however have to be unusually good and would greatly add to the cost of laying them. The power can not be used for ordinary purposes at the high voltage at which it is transmitted on account of the danger to life and also because of its unsuitability for working the ordinary types of lamps and motors. It is therefore desirable to transmit electric power from place to place as a *high tension* alternating current and to *transform* it, where required, to a *low tension* current at the required voltage.

A transformer for this purpose consists essentially of two coils of wire—a primary coil and a secondary coil—coiled round an endless core of soft iron. The primary coil carries the current to be transformed and consists of a large number of turns of highly insulated wire of the thickness necessary to carry the current transmitted to it. The secondary coil consists of fewer turns of insulated wire of thickness sufficient to carry the low tension current resulting from the transformation, and is connected directly with the system to which this current is to be supplied. The iron core is usually a core of soft iron wire or a system of thin plates of soft iron, arranged to give an endless magnetic circuit and to avoid loss of energy by Foucault eddy currents. The principle of action of the

transformer will be evident from its construction. The alternating current in the primary coil magnetises the iron core and sets up an alternating flow of induction in the magnetic circuit. This variation of the induction through the secondary coil gives an induced alternating current in the secondary coil. The period of this induced current is the same as that of the current in the primary coil but the two currents are not, in general, in the same phase.

When the turns of the secondary coil are fewer than those of the primary coil, the transformer transforms *down*, from high voltage to low voltage, and the ratio of the numbers of turns in the two coils gives approximately the ratio of transformation. Ruhmkorff's induction coil is an example of a transformer used to transform *up*, from low tension to high tension. It can be used to transform down by using the secondary coil as the primary. If, for example, the terminals of the secondary coil of a working induction coil be connected by thin insulated wires or air lines to the terminals of the "secondary" coil of another induction coil, the very high tension current transmitted from the one coil to the other will be transformed down to a low tension current in the "primary" coil, similar to the current in the primary coil of the working induction coil. In the first coil the intermittent current is said to be transformed up from a low to a very high tension current, and in the second coil the transformation is reversed, the current entering the coil being transformed down from the high tension to the low tension form.

The ratio of transformation depends mainly upon the ratio of the number of turns in the two coils. Thus to effect the transformation from 5000 volts and 20 ampères to 200 volts and 500 ampères, the number of turns in the primary coil of the transformer would have to be about 25 times as great as the number on the secondary coil.

If E_1, C_1, and E_2, C_2, denote respectively the electromotive forces and currents in the primary and secondary coils, then we have

$$E_1 C_1 = E_2 C_2$$

only if we assume that there is no loss of energy during

transformation. There are in practice two sources of loss of energy due to hysteresis and to eddy currents in the iron core. The losses during transformation are however small, the efficiency of a good transformer being well over ninety per cent.

When two coils are wound together in such a way that the flux of induction through one all passes through the other there is a simple relation between the coefficients of self-induction and the coefficient of mutual induction of the coils. For, if L_1, L_2 and M denote these coefficients, and n_1 and n_2 the number of turns in the coils, we have, for a current C in the coil to which L_1 and n_1 refer, the flow of induction through one turn measured by $L_1 C/n_1$, and the flow of induction through the other coil is $\frac{L_1 C n_2}{n_1}$. That is, $\frac{n_2}{n_1} L_1 = M$. Similarly, beginning with the other coil we get $\frac{n_1}{n_2} L_2 = M$ and we therefore have

$$L_1 L_2 = M^2 \text{ and } \frac{L_1}{L_2} = \left(\frac{n_1}{n_2}\right)^2.$$

In the coils of a transformer these relations hold only approximately, for there is always some magnetic leakage between the two coils, and the variation of the permeability of the iron core with the intensity of magnetisation complicates the result. If, however, we neglect the variation of permeability it may be assumed that $(M^2 - L_1 L_2)$ is a small quantity, and that $\frac{L_1}{L_2} = \left(\frac{n_1}{n_2}\right)^2$ is roughly correct. The latter relation shows that in a transformer L_2 is necessarily small, compared with L_1, for if the ratio of transformation n_2/n_1 is, say $\frac{1}{20}$, the value of L_2 is only the $\frac{1}{400}$th part of L_1.

If the currents in the primary and secondary coils at any instant be denoted by x and y respectively, and if we represent the harmonic electromotive force applied to the primary circuit by $E \sin \omega t$, the resistances of the primary and secondary coils by R_1 and R_2, and the coefficiency of

induction by L_1, L_2 and M, as above, we have the following relations for the two coils

$$L_1 \frac{dx}{dt} + M \frac{dy}{dt} + R_1 x = E \sin \omega t$$

$$M \frac{dx}{dt} + L_2 \frac{dy}{dt} + R_2 y = 0.$$

Taking the first of these relations for the primary coil, $L_1 \frac{dx}{dt}$ gives the back electromotive force due to self-induction, $M \frac{dy}{dt}$ the back electromotive force due to mutual induction, and $R_1 x$ the potential difference which determines the current x in the coil. The sum of these quantities must evidently be equal to the impressed electromotive force $E \sin \omega t$. Similarly for the second relation the sum of the corresponding terms is equal to zero, since there is no impressed electromotive force acting on the secondary coil.

From these equations the values of x and y are found to be

$$x = \frac{E}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t + \alpha);$$

$$y = \frac{M\omega}{\sqrt{(L_2\omega)^2 + R_2^2}} \cdot \frac{E}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t + \beta)$$

where

$$L = L_1 - \frac{(M\omega)^2 L_2}{(L_2\omega)^2 + R_2^2}$$

$$R = R_1 + \frac{(M\omega)^2 R_2}{(L_2\omega)^2 + R_2^2},$$

and $$\tan \alpha = \frac{L\omega}{R},$$

$$\tan(\alpha - \beta) = \frac{R_2}{L_2\omega}.$$

These results are readily verified by simple differentiations and substitutions.

They show that the currents in the two coils are periodic currents of the same period as the impressed electromotive

force in the primary coil, but differing in phase. Also from the form of the value for x, it is evident that the current in the primary coil may be taken as the current in a coil of self-induction L, and resistance R. That is, the apparent effect of the secondary coil is to *decrease* the self-inductance of the primary coil from L_1 to $(L_1 - a\,L_2)$ and to *increase* the resistance from R_1 to $(R_1 + a\,R_2)$, where

$$a = \frac{(M\omega)^2}{(L_2\,\omega)^2 + R_2{}^2}.$$

When the secondary coil of a transformer is open R_2 is infinite, and a therefore is zero, and L and R reduce to their real values L_1 and R_1. When the secondary coil is closed R_2 is usually small and may be negligibly small compared with $L_2\,\omega$, if ω is large. If this is so, the value of a reduces to $\left(\frac{M}{L_2}\right)^2$ and therefore

$$L = L_1 - \frac{M^2}{L_2}$$

and

$$R = R_1 + \left(\frac{M}{L_2}\right) R_2.$$

Also, since $\tan\,(\alpha - \beta) = \frac{R_2}{L_2\,\omega}$, we have, when R_2 is negligibly small compared with $L_2\,\omega$, $\alpha - \beta = 0$ or π. If we also assume that the transformer is so well designed that $M^2 - L_1\,L_2$ is negligibly small, then we may take $M^2 = L_1\,L_2$ as approximately true, and this gives us $L_1 - \frac{M^2}{L_2} = 0$ or $L = 0$, $R = R_1 + \frac{L_1}{L_2} R_2$, and $a = 0$. Hence, when the secondary coil is closed the apparent self-inductance of the primary coil is practically zero, the apparent resistance R is equal to $R_1 + \frac{L_1}{L_2} R_2$, where R_1, R_2, L_1 and L_2 are as defined above, and the current in the primary coil is in the same phase as the impressed electromotive force. Under these conditions, the current in the primary coil is $\frac{E \sin\,\omega\,t}{R}$, and power spent in the coil is

$\frac{1}{2}E^2/R$. This indicates that the power absorbed increases as R decreases, that is, as R_2 decreases, for $R = R_1 + \frac{L_1}{L_2} R_2$ and R_1, L_1, and L_2 are fixed. This is true, however, only if $M^2 = L_1 L_2$; when $M^2 - L_1 L_2$ is a small but appreciable quantity the power absorbed can be shown to increase as R_2 decreases to a critical value which depends upon the frequency of the primary current. Below this critical value of R_2 the power absorbed decreases.

If we apply the approximations here indicated to the values of x and y given above, we get

$$x = \frac{E \sin \omega t}{R},$$

where

$$R = R_1 + \frac{L_1}{L_2} R_2$$

and

$$y = \frac{M}{L_2} \cdot \frac{E \sin \omega t}{R} = \frac{M}{L_2} x.$$

We have seen that $M = \frac{n_1}{n_2} L_2$, and therefore $\frac{M}{L_2} = \frac{n_1}{n_2}$. Hence we have

$$y = \frac{n_1}{n_2} x,$$

where n_1 and n_2 are the number of turns in the primary and secondary coils respectively. This shows that $\frac{y}{x}$, the ratio of transformation for the current is practically equal to $\frac{n_1}{n_2}$, the ratio of the number of turns in primary coil to the number of turns in the secondary coil of the transformer.

The electromotive force in the secondary coil is

$$R_2 y \quad \text{or} \quad R_2 \cdot \frac{M}{L_2} \cdot \frac{E \sin \omega t}{R}$$

substituting

$$R_1 + \frac{L_1}{L_2} R_2$$

for R, this gives

$$\frac{R_2 M}{L_2 (R_1 + \frac{L_1}{L_2} R_2)} E \sin \omega t$$

and assuming R_1 to be small compared with $\frac{L_1}{L_2} R_2$ we get

$$\frac{L_2}{M} E \sin \omega t$$

as the electromotive force in the secondary circuit. Since $L_2/M = n_2/n_1$ we see that the electromotive force in the secondary coil is approximately n_2/n_1 of the electromotive force in the primary coil. The power absorbed in the primary coil is, as given above, $\frac{1}{2} \frac{E^2}{R}$. The power given out in the secondary coil is evidently given by

$$\frac{1}{2} \frac{M^2 E^2 R_2}{L_2{}^2 R^2}.$$

The efficiency of the transformer is therefore given by

$$\frac{M^2 R_2}{L_2{}^2 R} \text{ or } \frac{M^2}{L_2{}^2} \cdot \frac{R_2}{R_1 + \frac{L_1}{L_2} R_2}.$$

If R_1 be taken as small compared with $\frac{L_1}{L_2} R_2$, this result reduces to $\frac{M^2}{L_1 L_2}$. This is unity when $M_2 = L_1 L_2$, and is nearly unity in a well designed transformer.

CHAPTER XXX.

ELECTRIC WAVES.

190. Oscillatory discharge. In Art. 174 the process of discharge of a condenser has been described, but it is there assumed that the resistance is so adjusted that the discharge is a simple continuous one. When the resistance is not so adjusted the discharge is of an oscillatory character. The electrostatic energy of the condenser field is partly transformed into electromagnetic energy and partly dissipated as heat in the conductor. When the resistance exceeds a certain limit the proportion of energy dissipated may be considerable and the electromagnetic energy produced is at once dissipated as the heat effect of the self-induced extra current. When, however, the resistance is small the the amount of energy dissipated is small, and practically the first step in the discharge is a sudden transformation of the electrostatic energy into a nearly equal amount of electromagnetic energy. This energy is not at once dissipated, but is retransformed into electrostatic energy so as to recharge the plates of the condenser with charges of signs opposite to those of the initial charges. A small proportion of the energy is also, as before, dissipated as heat during the transformation. This process of transformation of energy from electrostatic to electromagnetic and from electromagnetic to electrostatic goes on time after time until the whole of the energy is finally dissipated during the transformations. This means that the condenser instead of being discharged by one steady continuous discharge accompanied by transformation and dissipation of energy is effected by a series of rapidly recurring successive discharges during which the condenser is successively charged in opposite senses, and at each discharge the

electrostatic energy of the condenser field is converted into electromagnetic energy accompanied by the dissipation of a small quantity of energy as heat in the conductor, and also by electric radiation as explained below. These successive discharges take place very rapidly, each oscillation occupying a time conveniently expressed in millionths of a second, and the whole process of successive charge and discharge constitutes what is called oscillatory discharge. The time of each oscillation is the same, and this time is known as the period of the oscillations.

Fig. 317 illustrates graphically the nature of this discharge. The abscissae represent time, and the ordinates

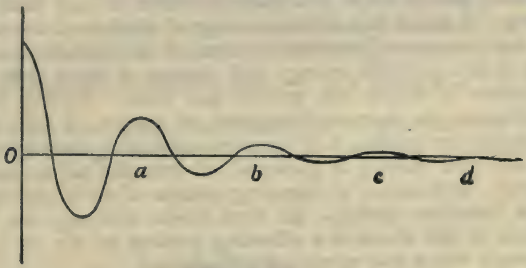


Fig. 317.

the charge of the condenser at any instant. The change in sign of the charge is clearly indicated by the ordinates of the curve, the period of the oscillation is represented by the length O *a*, *a b*, *b c*, etc., and it will be seen that the *amplitude* of the oscillation decreases with time, that is, on account of the energy dissipated, the maximum charge in the condenser decreases with each oscillation—and ultimately becomes zero.

As the result of the transformations of energy that go on in the medium during an oscillatory discharge, it is evident that the portion of the medium involved primarily in the transformations must undergo a definite periodic variation of state. This variation of state is propagated with finite velocity throughout the surrounding medium, that is, it is the origin of a set of electric waves which

travel outward in all directions into the medium with a definite finite velocity. By these waves energy is radiated out into the medium from the region of oscillation, and this energy is finally dissipated as heat.

The whole process of oscillatory discharge is closely analogous to the case of a vibrating tuning fork. Imagine the two prongs of a large fork to be pressed strongly towards each other, say, between the finger and thumb. They are now "charged" with a quantity of energy stored up as potential energy of strain in the material of the prongs. When the prongs are let go this energy is suddenly "discharged" by a rapid series of oscillations in each of which there is a transformation of the potential energy of strain into kinetic energy of motion of the mass of the prongs and retransformation of this kinetic energy into strain potential energy with change of sense as to the direction in which the prongs are bent. During each oscillation also there is dissipation of energy, as heat due to molecular friction in the prongs and during the vibration of the fork energy is radiated out into the air as sound waves. The case of a vibrating column of air in an organ pipe furnishes perhaps an even closer analogy, for the source of the waves is a vibrating portion of the medium through which the waves are propagated.

191. Period of Oscillation. A first approximation to the time of oscillation in any particular case is readily obtained if we consider a case in which the dissipation of energy at each oscillation is negligibly small. Let F denote the capacity of the condenser, L the self-inductance of the circuit, Q_0 the maximum charge, and C_0 the maximum current during discharge, then the energy in the medium is given by

$$E = \tfrac{1}{2}\frac{Q_0{}^2}{F} = \tfrac{1}{2} L C_0{}^2.$$

Hence, if the dissipation of energy be neglected and Q and C denote the charge and current at any instant, we have

$$\tfrac{1}{2}\frac{Q^2}{F} + \tfrac{1}{2} L C^2 = E.$$

where E is constant.

If this is differentiated with respect to time we get

$$\frac{Q}{F} \cdot \frac{dQ}{dt} + LC\frac{dC}{dt} = 0.$$

This result may be obtained without formal differentiation, for if a change dQ take place in Q in time dt, then the change in the energy of the condenser in the time dt is $dQ \cdot \frac{Q}{F}$ for $\frac{Q}{F}$ is the potential of the condenser, and the *rate* of change in the condenser energy is therefore $\frac{dQ}{dt} \cdot \frac{Q}{F}$. Also if dC be the corresponding change in C in the time dt, then $L \cdot dC$ is the change in the flow of magnetic induction through the circuit, and C being the current in the circuit $CL\,dC$ is the change in the energy of the magnetic field—and the *rate* of change in this energy is $LC\frac{dC}{dt}$. Since the total energy is assumed to be constant the total rate of change of energy must be zero, and therefore we have

$$\frac{Q}{F} \cdot \frac{dQ}{dt} + LC\frac{dC}{dt} = 0.$$

Now, $C = \frac{dQ}{dt}$, and therefore

$$\frac{dC}{dt} = \frac{d\left(\frac{dQ}{dt}\right)}{dt}$$

Hence we get

$$\frac{Q}{F} \cdot \frac{dQ}{dt} + L\frac{dQ}{dt} \cdot \frac{d\left(\frac{dQ}{dt}\right)}{dt} = 0.$$

and therefore

$$\frac{Q}{F} + L\frac{d\left(\frac{dQ}{dt}\right)}{dt} = 0.$$

That is

$$\frac{d\left(\frac{dQ}{dt}\right)}{dt} = \frac{1}{FL}Q.$$

This evidently means that Q is a quantity such that if we differentiate it with respect to time to get $\frac{dQ}{dt}$, and then differentiate $\frac{dQ}{dt}$ with respect to time we get the initial result multiplied by a constant.

This at once suggests that

$$Q = Q_0 \sin \omega t.$$

for

$$\frac{dQ}{at} = -Q_0 \omega \cos \omega t.$$

and

$$\frac{d\left(\frac{dQ}{dt}\right)}{dt} = -Q_0 \omega^2 \sin \omega t$$

or

$$\frac{d\left(\frac{dQ}{at}\right)}{dt} = -\omega^2 Q.$$

This result agrees with the one given above, the constant $\frac{1}{FL}$ being represented by ω^2.

Now the expression $Q = Q_0 \sin \omega t$, indicates that Q is a periodic quantity varying harmonically between the limits Q_0 and $-Q_0$, and the period of its variation is $\frac{2\pi}{\omega}$ for ωt changes from zero value to 2π in that time and goes through its full cycle of values within these limits. Now this period $\frac{2\pi}{\omega}$ is evidently the period of oscillation of the discharge, and since $\omega^2 = \frac{1}{FL}$ the period of oscillation is given by $t = 2\pi\sqrt{FL}$. Since $\sqrt{FL} = \sqrt{FR \cdot \frac{L}{R}}$, where FR is the time constant, c, of the condenser, and $\frac{L}{R}$ the the self-induction time constant c', of the circuit. We

may, therefore, write $\sqrt{\text{F L}} = \sqrt{c\,c'}$, and $t = 2\pi\sqrt{c\,c'}$ where c and c' are the two time constants.

This value of t is only a first approximation, for we have neglected the energy dissipated. If we include the energy dissipated by the Joule heating effect in the resistance, we must evidently change the equation given above by adding $\text{C}^2\,\text{R}$, the rate of dissipation of energy, to the two other rates of change of energy before equating their algebraic sum to zero. This gives, after substituting $\left(\frac{d\,\text{Q}}{d\,t}\right)^2\text{R}$ for $\text{C}^2\,\text{R}$, and simplifying,

$$\frac{\text{Q}}{\text{F}} + \text{L}\frac{d\left(\frac{d\,\text{Q}}{d\,t}\right)}{d\,t} + \text{R}\frac{d\,\text{Q}}{d\,t} = 0$$

or

$$\text{L}\frac{d\left(\frac{d\,\text{Q}}{d\,t}\right)}{d\,t} + \text{R}\frac{d\,\text{Q}}{d\,t} + \frac{\text{Q}}{\text{F}} = 0.$$

The expression for Q for which this relation is true depends upon whether the roots of the quadratic equation

$$\text{L}\,x^2 + \text{R}\,x + \frac{1}{\text{F}} = 0.$$

are real or imaginary.

If the roots are imaginary, that is if $\frac{4\,\text{L}}{\text{F}} > \text{R}^2$ the value of Q is given by

$$\text{Q} = \text{A}\,e^{-\frac{\text{R}\,t}{2\,\text{L}}}\cos\,(\omega\,t + a),$$

where

$$\omega^2 = \frac{1}{\text{L F}} - \frac{\text{R}^2}{4\,\text{L}^2}.$$

By this result Q is a periodic function of the time, its successive maximum values decreasing in geometrical progression as t increases in arithmetical progression, and, as above, the period of oscillation is

$$t = 2\,\pi\Big/\sqrt{\frac{1}{\text{L F}} - \frac{\text{R}^2}{4\,\text{L}^2}}.$$

When $R = 0$ this evidently reduces to the simpler result, $t = 2\pi\sqrt{LF}$, given above.

If the roots of the quadratic equation are real, that is if $R^2 > \frac{4L}{F}$, then the value for Q which satisfies the above relation is

$$Q = Ae^{\alpha t} + Be^{\beta t},$$

where α and β are the roots of the equation. From this equation it will be seen that as t increases Q never changes sign, but steadily decreases, and reaching zero value only after an infinite time, but becoming negligibly small in a very short time.

It follows from this that the relation $R^2 = \frac{4L}{F}$ or $R = \sqrt{\frac{4L}{F}}$ gives the limiting value of R for oscillatory discharge. If R exceeds this value the discharge is continuous, if less than this it is oscillatory.

The time of oscillation may be written in terms of the time constants c and c'.

$$t = \frac{2\pi\sqrt{cc'}}{\sqrt{1 - \frac{c}{4c'}}}.$$

This value of t is only a second approximation. The loss of energy by radiation, and the uncertainty as to the value of R for the rapidly alternating discharge current have not been considered.

In calculating the time of oscillation from the data of the circuit, the inconsistency of the electrostatic and electromagnetic units must be remembered. If K is taken equal to 1 for air, then, since $K\mu = \frac{1}{v^2}$, μ must, in a consistent system of units, be taken not as 1 but as $\frac{1}{v^2}$, where v is the velocity 3×10^{10} cms. per second. If the capacity of the system be 10^2 electrostatic units, the inductance ·00001 henry, and the resistance ·001 ohm. Then we

evidently have $F = 10^2$ C.G.S. units, $L = 10^4 \times \frac{1}{v^2}$ C.G.S. units and $R = 10^6 \times \frac{1}{v^2}$ C.G.S. units. The time constant of the circuit given by L/R is $\frac{1}{100}$ second, the time constant of the condenser, FR, is $\frac{1}{300}$ second, and the first approximation to the period of oscillation is given by

$$t = 2\pi \sqrt{FL} = 2\pi \sqrt{\frac{10^6}{v^2}} = \frac{2\pi \times 10^3}{v}.$$

That is

$$t = \frac{2\pi \times 10^3}{3 \times 10^{10}} = \frac{2\pi}{3 \times 10^7} = 2{\cdot}0944 \times 10^{-7}.$$

This gives the time of oscillation equal to 2·0944 ten-millionths of a second, that is, the frequency of the oscillations is about five million per second. The critical value of R given by $R^2 = \frac{4L}{F}$ is $R = \frac{2}{3} \,.\, 10^{-9}$ C.G.S. units.

If we take the more exact formula

$$t = 2\pi \Big/ \sqrt{\frac{1}{LF} - \frac{R^2}{4L^2}}$$

we get

$$t = 2\pi \Big/ \sqrt{9 \times 10^{14} - \frac{1}{400}} = \frac{2\pi}{3 \times 10^7} = 2{\cdot}0944 \times 10^{-7}$$

Here the value of $\frac{R^2}{4L^2} = \frac{1}{400}$, is negligibly small compared with that of $\frac{1}{LF}$, and therefore t, as given by this formula, is practically the same as that given by the simpler formula.

What has been said so far applies not only to a condenser, properly so called, but to any conductor. If the dumb-bell shaped conductor of Fig. 318 be charged in such a way that A is at a higher potential than B, then when the source of potential difference is removed the equalisation of potential between the

Fig. 318.

two ends will be accompanied by electrical oscillations such as have been described, and the time of oscillation is determined by the same considerations.

192. Hertz's experiments. Hertz investigated the phenomena of electric oscillations and waves by means of apparatus in which the *oscillator* or *vibrator* was a dumb-bell-shaped brass conductor similar to that referred to above. The rod connecting A and B was cut so as to give a small gap at $a\,b$ (Fig. 319) between the two small brass balls a and b. The portion Aa was then connected to one pole of an induction coil and Bb to the other pole as shown in the figure.

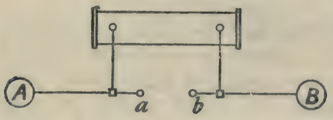


Fig. 319.

The system made up of Aa, Bb, and the air gap $a\,b$ must be considered as one conductor. When the induction coil is worked Aa, say, is charged positively and Bb negatively, then a spark passes at $a\,b$, and *the path of this spark* on account of its comparatively low resistance practically connects Aa and Bb as one conductor, and the difference of potential, established before the spark passed, is equalised by very rapid electric oscillations in this conductor. By working the coil continuously the vibrator becomes a steady source of electric waves passing out into the surrounding medium. The cycle of changes that go on in the state of the medium near the vibrator during one complete vibration is that associated with the transformation of electrostatic to electromagnetic energy, the re-transformation of electromagnetic to electrostatic energy, and, during these transformations, a gradual dissipation of energy as heat in the medium. This cycle of changes is transmitted from the vibrator out into the medium, and the sequence of states so transmitted constitute the electric waves. Hence it will be understood that when these waves pass a conductor they tend to reproduce in this conductor the electrical conditions which occur in the vibrator. that is, electrical oscillations are

induced in the conductor and differences of potential are set up between neighbouring conductors so that small sparks may be caused between conductors or pieces of metal placed close together.

Hertz took advantage of this to demonstrate the existence of the waves. One form of *detector* used by him was a piece of thick copper wire bent into a circle but with a small air-gap between the ends of the wire. One end was pointed and the other rounded, and the width of the gap could be adjusted either by hand pressure or by replacing the simple pointed end by a small pointed adjustable screw. As the waves passed through this circuit small sparks were observed at the gap, and it was found that the sparks were strongest and brighest when the dimensions of the detector were such that its period of oscillation as an oscillator or vibrator was the same as that of the vibrator originating the waves. This effect is an example of the general principle of *resonance*, and a circuit showing electric resonance may be called an electric resonator, and is specially effective in detecting the existence of electric waves. By arranging for the proper adjustment of inductance and capacity it is evidently possible to adjust the period of the detector within wide limits, and so *tune* it to resonance in any particular case.

Electric waves are found to pass through stonework, woodwork, and other similar substances, but suffer reflexion at a wall of good conducting material. Hence, if a large metal sheet be set up at some distance from an electric vibrator with its plane at right angles to the line of propagation of the waves, the incident waves are reflected back along their path of incidence, and the interference between the direct and reflected waves gives, in the usual way, a stationary wave between the vibrator and the reflector. The reflecting surface marks a node in the wave, and successive nodes or internodes are at a distance apart equal to one-half the wave length of the wave disturbance. By means of a detector the positions of the nodes and internodes are readily detected by its quiescence or activity, and the wave length may therefore be determined by direct measurement. Hertz adopted this method,

and succeeded in measuring the wave length in a number of cases.

If λ denote the wave length then the usual relation $v = n\lambda$ indicates that the velocity of the waves is given by the product of n, the frequency and λ the wave length. As we have seen, λ for electric waves can be measured directly, and n can be calculated from the data of the vibrator which determine t, its time of oscillation, for $n = 1/t$. From measurements made in this way Hertz found the value of v to be about 3×10^{10} cms. per second, that is the velocity of transmission of electric waves is the same as that of light waves. For a vibrator with a period of one-fifty-millionth of a second the wave length would evidently be about 6 metres and the distance between the nodes of a stationary wave 3 metres. Hertz, Lodge, Thomson, Fitzgerald, and others have shown by direct experiment that electric waves can be reflected, refracted, polarised and diffracted in accordance with the same laws that apply to the reflexion, refraction, polarisation, and diffraction of light. For experiments of this kind Hertz used a mirror of parabolic cross section with a simple vibrator fixed at its focal line as shown in Fig. 320. The waves originated at the vibrator are reflected from the mirror in a direction parallel to the axis of the cross section, giving a beam of parallel radiation suitable for experiment. If the beam is received on a parabolic mirror similar to that from which it started the waves are found to be focussed at the focal lines and well marked sparks are produced in the vibrator fixed at the focus. To exhibit refraction Hertz passed the beam through a prism of pitch using one parabolic mirror M (Fig. 321), fitted with a vibrator, to originate the waves, and another M′, also fitted with a vibrator, as a receiver of the refracted beam. The prism was about 150 cms. high, its angle about 30° and the width of its faces about 120 cms.

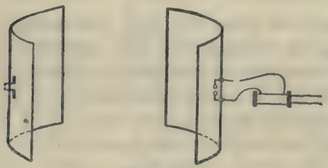


Fig. 320.

The beam of electric radiation from M was found to be refracted in the same way as a beam of light, the direction of the refracted beam being readily detected by the position of maximum spark activity at the vibrator

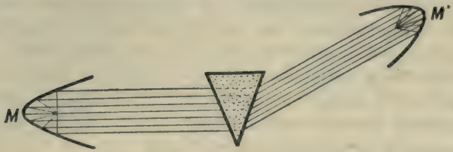


Fig. 321.

attached to M′. When the adjustment for minimum deviation was made the deviation was found, in one case, to be about 22°, and the refractive index of pitch for the incident waves was therefore about 1·7.

193. Wireless Telegraphy. An interesting effect of the incidence of electric waves is that involved in the action of Branley's *coherer*. If the circuit of a battery contains a bad contact, such as that caused by a needle point resting lightly on a metal plate or by a layer of metal filings between two metal plates, the incidence of the waves on this contact causes the points of contact to become more coherent, and the resistance of the contact may be reduced to such an extent that the current passing may be very greatly increased. A simple form of *coherer* is shown in Fig. 322. Two brass, copper, or silver discs *d*, *d*, each soldered to the end of a copper wire, *w*, are fitted into the glass tube *g*, *g* and a

Fig. 322.

thin layer of filings rests lightly between the discs. If this coherer is placed, as indicated in the figure, in the circuit of a battery and an electric bell the contacts made by the filings between the discs may be so adjusted that the current passing will not be strong enough to ring the bell.

If, however, waves from a distant vibrator fall upon the coherer, after this adjustment is made, the contact at once becomes good and the bell in the circuit rings. If the coherer is slightly tapped the contact again fails and the arrangement is again ready to detect the incidence of the waves. The tapping can easily be effected automatically by attaching a tapper to the bell, so that when the bell begins to ring the tube is tapped and the coherer at once returns to the sensitive condition. By suitably connecting the coherer circuit with a relay and a Morse telegraphic instrument it is evidently possible to receive and interpret signals sent out by the vibrator and transmitted to the receiver through the intervening medium. The arrangement here outlined is the starting point of *wireless telegraphy*. The vibrator or transmitter was first used by Hertz and Lodge, and the coherer as a receiver was invented by Branley and used by Lodge and others for the detection of electric waves before the recent commercial development of wireless telegraphy, which owes its origin to these experimenters, was originated.

The medium in which the electric waves are propagated is the ether, and there is satisfactory evidence that these waves differ from light and heat radiation only in wave length. The wave length of comparatively short electric waves may be several metres, whereas the wave length of light waves, for the middle of the spectrum, is about ·00005 cms., and the longest heat radiation wave measured is about ·003 cms.

194. Electric Elasticity and Density of the Ether. The velocity of wave transmission in any medium is usually expressed by the relation

$$v = \sqrt{\frac{e}{d}},$$

where e denotes the modulus of elasticity involved, and d the density of the medium.

In order to find the values of e and d, which determine the velocity of transmission of electric waves, it is perhaps simplest to compare the expressions for the energy per unit volume of a strained medium and the energy per unit volume of an electric field. As a simple case consider a

volume V of any medium to be compressed by hydrostatic pressure until the pressure per unit area increases from 0 to P, where P is very small, and let the change of volume be v. Then the *stress* is P and the strain is $s = v/V$. The work done in compression is $\frac{1}{2} P v$, and if this work is stored up as strain energy in the medium, the potential energy so stored up *per unit volume* of the medium is $\frac{1}{2} P\, v/V$ or $\frac{1}{2} P s$. The modulus of elasticity, the ratio of the stress to the strain, is given by $e = P/s$, so that the energy of strain per unit volume of the medium is given by

$$\tfrac{1}{2} P s \quad \text{or} \quad \tfrac{1}{2}\frac{P^2}{e} \quad \text{or} \quad \tfrac{1}{2} e s^2.$$

Now, in Art. 33, the energy per unit volume of an electric field is shown to be expressed by $\frac{F^2 K}{8\pi}$, and by Art. 46 the value of F is given by $F = \frac{4\pi\sigma}{K}$. Hence, combining these two expressions we may express the energy per unit volume as

$$\tfrac{1}{2} F \sigma \quad \text{or} \quad \tfrac{1}{2} F^2 \frac{4\pi}{K} \quad \text{or} \quad \tfrac{1}{2} \cdot \frac{4\pi}{K} \cdot \sigma^2.$$

If now F be taken to represent the stress in the electric field, and σ to represent the strain, then the three expressions just given correspond exactly with the three similar ones given higher up, and it will be seen that the electric elasticity is represented by $4\pi/K$. The energy of an electric field has in this way been associated with potential energy of strain and the analogue of elasticity deduced. In the same way if the energy of a magnetic field be associated with the kinetic energy of a moving mass, the analogue of mass may be determined. In the case of a mass m moving with a velocity u, the kinetic energy $\frac{1}{2} m u^2$. By Art. 106 the energy per unit volume in a magnetic field may be expressed by $\frac{F^2 \mu}{8\pi}$, and if we take the case of the field inside an endless uniformly wound coil of n turns per unit length carrying a current C, the value of F is $4\pi n C$, and the energy per unit volume is $2\pi (n C)^2 \mu$ or

$\frac{1}{2} . 4 \pi \mu (n C)^2$. Now $n C$ is the rate of displacement of electricity round unit length of the coil and represents electrical velocity in the same way as σ represents electric strain or displacement. Hence, comparing $\frac{1}{2} . 4 \pi \mu (n C)^2$ with $\frac{1}{2} m v^2$, the quantity $4 \pi \mu$ evidently corresponds to m and measures the electric mass or inertia per unit volume. That is $4 \pi \mu$ is the electric density of the medium.

The velocity of electric waves in a medium for which $4 \pi/K$ is the electric elasticity, and $4 \pi \mu$, the electric density is evidently given by

$$v = \sqrt{\frac{4 \pi}{K} / 4 \pi u} \text{ or } v = \sqrt{\frac{1}{K \mu}}.$$

Now in Art. 198 the quantity $\sqrt{\frac{1}{K \mu}}$ is shown to be a velocity of 3×10^{10} cms. per second. This, therefore, is the velocity of electric waves and is identical with the velocity of light. This result confirms the assumption that electric waves and light waves are propagated through the same medium, the ether, and differ only in wave length.

195. The Relation between the Index of Refraction and the Specific Inductive Capacity of a medium. If v_1 and v_2 are the velocities of electric waves in two media, then

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{K_2}{K_1} \cdot \frac{\mu_2}{\mu_1}.$$

Now for all transparent media μ_1 and μ_2 are practically equal and $v_1/v_2 = \rho$, the index of refraction from the first medium into the second. Hence for transparent media we have

$$\rho^2 = \frac{K_2}{K_1}.$$

That is, the square of the index of refraction from one medium to a second is the ratio of the specific inductive capacity of the second to that of the first.

If the second medium is air, for which $K_1 = 1$, then

$$\rho^2 = K_2.$$

That is, the specific inductive capacity of any medium (relative to air as unity) is equal to the square of the index

of refraction of that medium. The index of refraction here involved is the index of refraction for electric waves of long wave length. This constant is not known accurately for any transparent medium, but if we take the optical index of refraction as probably not very different from the required index the truth of this result can be verified for a number of media. The table given below shows the extent of the agreement with theory. In the last two cases the discrepancy is very marked.

	K.	ρ^2.
Paraffin	2·29	2·02
Petroleum oil.	2·07	2·07
Carbon bisulphide	2·67	2·67
Flint Glass	10·12	2·92
Water	76·	1·78.

The value of ρ for very long waves has been calculated from the value for light by a formula, due to Cauchy, which gives—

$$\rho = A + \frac{B}{\lambda} + \frac{C}{\lambda_4},$$

where A, B, and C are constants.

This formula is correct within the limits of the spectrum and the values of A, B, and C within these limits are known, but it cannot be expected with the same values of the constants to give trustworthy results for wave lengths several million times those for which the values of A, B, and C were determined.

CHAPTER XXXI.

ELECTRICAL UNITS.

196. Dimensions of Electrostatic Units. The dimensions of any quantity express by a formula the extent to which the fundamental units of mass, length, and time are involved in the unit selected to measure the given quantity. Thus, in the usual mechanical units the unit of velocity involves the unit of length directly and the unit of time inversely. The unit of accleration involves the unit of length directly and the unit of time inversely to the second power. The unit of force involves the unit of mass directly, the unit of length directly, and the unit of time inversely to the second power. These results are expressed more concisely and conveniently by the dimensional formula LT^{-1}, LT^{-2}, MLT^{-2} for the dimensions of velocity, acceleration, and force, the dimensions of the fundamental units of mass, length, and time being denoted by M, L, and T. To determine the dimensions of any quantity the simplest process is to take a relation involving that quantity, form from it a dimensional relation and deduce from this the required dimension. Thus to obtain the dimensions of work or energy we have from the usual relation of mechanics

$$\text{Work} = \text{force} \times \text{distance}$$

or

$$W = Fs.$$

Hence we get the dimensional relation

$$[W] = [F]\,[s],$$

which is merely a short way of expressing

The dimensions of work = dimensions of force × dimensions of distance.

This gives

$$[W] = MLT^{-2} \times L$$

for the unit of length is the unit in which the distance s is measured. Hence

$$[W] = M L^2 T^{-2}.$$

That is the unit of work involves the unit of mass directly, the unit of length directly to the second power, and the unit of time inversely to the second power.

Similarly from the relation

$$\text{Kinetic energy} = \tfrac{1}{2} m v^2$$

or
$$E = \tfrac{1}{2} m v^2$$

we get
$$[E] = [m] [v^2],$$

or
$$[E] = M \times (L T^{-1})^2$$

$$[E] = M L^2 T^{-2}.$$

It will be noticed that in forming the dimensional equations, numbers which have no dimensions are omitted.

If the dimensions of a quantity are known the change in the unit of that quantity due to given changes in the fundamental units can readily be found. Thus, if in one system of units the unit of work is w, then in another system in which the units of mass, length, and time are respectively a, b, and c times these units in the first system, the unit of work will evidently be from the dimensions just found $\frac{a b^2}{c^2} w$. Comparing, for example, the English F.P.S. system of units and the C.G.S. system, the units of mass, length, and time in the first system are respectively 453·6, 30·48, and 1 times the corresponding units in the second system. The *foot-poundal* or unit of work in the first system will therefore be $\frac{453 \cdot 6 \times (30 \cdot 48)^2}{1^2}$ *ergs*, the *erg* being the unit of work in the second system.

Taking now the relation

$$f = \frac{q q'}{d^2}$$

it will be possible to obtain the dimensions of the unit of quantity of electricity defined in Art. 6.

Taking $q' = q$ we get

$$f = \frac{q^2}{d^2},$$

or $$q^2 = d^2 f,$$

that is $$q = d\sqrt{f}.$$

Hence $$[q] = [d] \cdot [f]^{\frac{1}{2}}$$

$$[q] = \mathrm{L} \,.\, (\mathrm{M\,L\,T^{-2}})^{\frac{1}{2}}$$

or $$[q] = \mathrm{L} \,.\, \mathrm{M^{\frac{1}{2}}\,L^{\frac{1}{2}}\,T^{-1}}.$$

That is $$[q] = \mathrm{M^{\frac{1}{2}}\,L^{\frac{3}{2}}\,T^{-1}}.$$

Again, the potential at any point in the electric field is the potential energy *per unit quantity of electricity.*

That is

$$[\mathrm{V}] = \frac{[\text{energy}]}{[\text{quantity of electricity}]}$$

that is $$[\mathrm{V}] = \frac{\mathrm{M\,L^2\,T^{-2}}}{\mathrm{M^{\frac{1}{2}}\,L^{\frac{3}{2}}\,T^{-1}}}$$

or $$[\mathrm{V}] = \mathrm{M^{\frac{1}{2}}\,L^{\frac{1}{2}}\,T^{-1}}.$$

Similarly for capacity we have

$$\mathrm{Q} = \mathrm{VC} \quad \text{or} \quad \mathrm{C} = \frac{\mathrm{Q}}{\mathrm{V}}.$$

and therefore $$[\mathrm{C}] = \frac{[\mathrm{Q}]}{[\mathrm{V}]},$$

that is $$[\mathrm{C}] = \frac{\mathrm{M^{\frac{1}{2}}\,L^{\frac{3}{2}}\,T^{-1}}}{\mathrm{M^{\frac{1}{2}}\,L^{\frac{1}{2}}\,T^{-1}}}$$

or $$[\mathrm{C}] = \mathrm{L}.$$

It will be noticed here that the dimensions of capacity on this system of units are the same as those of length. This does not mean, as it should do in a logically derived system, that capacity and length are similar quantities. It is a purely accidental result of the fact that the system of electrostatic units we are here dealing with, is a conventional one based upon the definition of unit quantity of electricity given in Art. 6. The conditions of this definition involve the suppression of the dimensions of the quantity K for air. This question will be dealt with later on.

By an extension of the method indicated above it is easy to obtain the dimensions of all the electrostatic

quantities. The more important of these are given below.

Quantity	[Q]	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$
Electric force	[F]	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$
Potential	[V]	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
Capacity	[C]	L
Electric Polarisation	[P]	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$.

197. Electromagnetic units. The system of units for magnetic quantities is based upon the definition of unit pole given in Art. 74. Taking the relation

$$= \frac{m\, m'}{d^2}$$

and putting $m' = m$

we get $f = \dfrac{m^2}{d^2}$.

That is $m^2 = f d^2$

or $m = d \sqrt{f}$.

Proceeding as in the case of electrostatic units we get—

$$[m] = L\,.\,M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$$

or $$[m] = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

From this result as a starting point we readily obtain the dimensions of the magnetic quantities tabulated below.

Strength of pole	[m]	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$.
Magnetic force	[H]	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$.
Magnetic potential	[P]	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$.
Flow of force	[F]	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$.
Reluctance	[ρ]	L^{-1}.
Magnetic moment	[M]	$M^{\frac{1}{2}} L^{\frac{5}{2}} T^{-1}$.
Intensity of magnetisation	[I]	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$.
Magnetic induction	[B]	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$.

It should here be noticed that as the dimensions of K are suppressed in the electrostatic system so the dimensions of μ are suppressed in the system of magnetic and electrostatic units.

To obtain the dimensions of the electromagnetic units we may start with the definition of the electromagnetic unit of current given in Art. 129. From this we get the relation

$$I = \frac{2\pi n C}{r},$$

where I denotes the strength of the magnetic field at the centre of the coil. This gives the dimensional relation

$$[I] = \frac{[C]}{[r]},$$

or
$$[C] = [I]\,[r].$$

That is
$$[C] = M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} . L$$

or
$$[C] = M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$$

The method of measuring the electromotive force induced in a circuit by displacement in a magnetic field (Art. 165), may also be taken as a starting point. The induced electromotive force is measured by the rate of change in the flow of induction through the circuit, hence

$$[E] = \frac{[F]}{T}.$$

That is
$$[E] = \frac{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}}{T} = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}.$$

From these results it is easy to build up the dimensions of the electromagnetic units given below

Current	[C].	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
Quantity	[C *t*]	$M^{\frac{1}{2}} L^{\frac{1}{2}}$
Electromotive force	[E]	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$
Resistance	[R]	LT^{-1}
Capacity	[C]	$L^{-1} T^{2}$
Inductance	[I]	L

198. Irrationality of Electromagnetic and Electrostatic units. It will be seen by reference to the tables given in the foregoing article that the dimensions of the *same* quantity are not the same in the two sets of units. For example, the dimensions of Quantity of Electricity are $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$ in the electrostatic system, and $M^{\frac{1}{2}} L^{\frac{1}{2}}$ in electromagnetic units; this results, as already explained, from the suppression of the dimensions of K and μ. If we include these quantities in determining the dimensions of any quantity in the two sets of units it will be possible to determine a condition that the dimensions of K and μ must satisfy in order that the dimensions of the given quantity shall be the same for the two sets of units. Taking quantity of electricity for example, we have for the electrostatic units

$$f = \frac{1}{K} \frac{q^2}{d^2}$$

or
$$q^2 = K f d^2,$$

or
$$q = \sqrt{K} \sqrt{f} d.$$

That is
$$[q] = [\sqrt{K}] [\sqrt{f}] [d]$$

or
$$[q] = [\sqrt{K}] M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

Similarly for the electromagnetic units

$$[Q] = [Ct] = [C] T.$$

Now to introduce μ into [C] we have

$$f = \frac{1}{\mu} \cdot \frac{m^2}{d^2}$$

or
$$m = \sqrt{\mu} \cdot \sqrt{f} \cdot d.$$

That is,
$$[m] = [\sqrt{\mu}] M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1},$$

then the dimensions of magnetic potential, V, are those of work per unit pole or

$$[V] = [\sqrt{\mu}] \frac{M L^2 T^{-2}}{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}} = \frac{1}{[\sqrt{\mu}]} \cdot M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$$

Also magnetic force or strength of field H is measured by rate of change of potential with distance, and therefore

$$[H] = \frac{[V]}{L} = \frac{1}{[\sqrt{\mu}]} M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$$

And, as in Art. 197,

$$[C] = [H]\,L = \frac{1}{[\sqrt{\mu}]} \cdot M^{\frac{1}{2}}\,L^{\frac{1}{2}}\,T^{-1}$$

and therefore

$$[Q] = \frac{1}{[\sqrt{\mu}]}\,M^{\frac{1}{2}}\,L^{\frac{1}{2}}.$$

If the two sets of units are consistent the two dimensions for Q should be the same, that is

$$[\sqrt{K}]\,M^{\frac{1}{2}}\,L^{\frac{3}{2}}\,T^{-1}$$

and

$$\frac{1}{[\sqrt{\mu}]} \cdot M^{\frac{1}{2}}\,L^{\frac{1}{2}}.$$

should be identical. This evidently involves

$$[\sqrt{K}]\,[\sqrt{\mu}] = \frac{M^{\frac{1}{2}}\,L^{\frac{1}{2}}}{M^{\frac{1}{2}}\,L^{\frac{3}{2}}\,T^{-1}}$$

or

$$[\sqrt{K\mu}] = L^{-1}T.$$

That is

$$\left[\frac{1}{\sqrt{K\mu}}\right] = L\,T^{-1}$$

on the dimensions of $\frac{1}{\sqrt{K\mu}}$ are those of a velocity. If we work out the result for any other quantity whose dimensions can be expressed in both sets of units we always get the same result. We are therefore not able to give the dimensions of K or μ separately, but only to state that

$$\left[\frac{1}{\sqrt{K\mu}}\right] = L\,T^{-1} = [v].$$

If we extend the method indicated in this article it is possible to express the dimensions of the electrical quantities in terms of M, L, T, and K for the electrostatic system and in terms of M, L, T, and μ for the electromagnetic system. If in these dimensions we suppress K or μ we get the usual electrostatic or electromagnetic dimensions. The table given below gives these dimensions for the more important quantities.

Quantity.	Electrostatic (M, L, T, K) Dimensions.	Electromagnetic (M, L, T, μ) Dimensions.	Quantity.	Electromagnetic (M, L, T, μ) Dimensions.	Electrostatic (M, L, T, K) Dimensions.
Specific Inductive Capacity	K	$L^{-2} T^{2} \mu^{-1}$	Permeability ..	μ	$L^{-2} T^{2} K^{-1}$
Quantity of Electricity	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}$	Quantity of Magnetism	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \mu^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} K^{-\frac{1}{2}}$
Electromotive force Potential Difference	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} \mu^{\frac{1}{2}}$	Magnetic Potential	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{2} K^{\frac{1}{2}}$
Electric force ..	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}$	Magnetic force ..	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} K^{\frac{1}{2}}$
Surface Density Electric Polarisation or Displacement	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{3}{2}} \mu^{-\frac{1}{2}}$	Intensity of Magnetisation Magnetic Induction	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{-\frac{3}{2}} K^{-\frac{1}{2}}$
Capacity	$L K$	$L^{-1} T^{2} \mu^{-1}$	Inductance ..	$L \mu$	$L^{-1} T^{2} K^{1}$
*Conductance ..	$L T^{-1} K$	$L^{-1} T \mu^{-1}$	*Resistance ..	$L T^{-1} \mu$	$L^{-1} T K^{-1}$
Current	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	Current	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}$

* Conductance and Resistance are reciprocals not analogues. The apparent analogy here results from the fact that as $[K \mu]$ are identical with $[1/v^2]$ the dimensions of the reciprocal of a quantity of dimensions $L T^{-1} K$, that is $v k$ must be $L T^{-1} \mu$ or $v \mu$.

199. Practical Units. The practical units adopted for the measurement of the more important electrical quantities have already been defined. For convenience they are given below and their magnitude in terms of electromagnetic and electrostatic C.G.S. units specified. The quantity $1/\sqrt{K\mu}$ is indicated by v, and its measure in air may be taken as 3×10^{10}.

Quantity.	Practical Unit.	Electro-magnetic C.G.S. units.	Electro-static C.G.S. units.
Electromotive Force	Volt	10^8	$10^8/v$
Resistance	Ohm	10^9	$10^9/v^2$
Current	Ampere	10^{-1}	$10^{-1}v$
Quantity of Electricity	Coulomb	10^{-1}	$10^{-1}v$
Capacity	Farad	10^{-9}	$10^{-9}v^2$
	Micro-farad	10^{-15}	$10^{-15}v^2$
Inductance	Henry	10^9	$10^9/v^2$

200. Ratio of Electrostatic and Electromagnetic units. If we take the ratio of the dimensions of any electrical quantity in the ordinary electrostatic and electromagnetic units we find that the result is of the dimensions of a velocity or a power of a velocity. Thus, for quantity of electricity, the ratio of the electrostatic to the electromagnetic *dimensions* is

$$\frac{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}}{M^{\frac{1}{2}} L^{\frac{1}{2}}} \text{ or } LT^{-1} \text{ or } [v].$$

Similarly for electromotive force or difference of potential we get

$$\frac{M^{\frac{1}{2}} L^{\frac{1}{2}} T^{1}}{M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}} \text{ or } \frac{1}{L\,T^{-1}} \text{ or } \left[\frac{1}{v}\right]$$

and for capacity we get

$$\frac{L}{L^{-1}\,T^{2}} \text{ or } L^2 T^{-2}. \quad \text{That is } (L\,T^{-1})^2 \text{ or } [v^2],$$

and so on for the other quantities.

Now if the *same* quantity of electricity be measured in electrostatic and electromagnetic units, and if s and m be the measures of the quantity of electricity in the two sets of units, we have

$$s \text{ electrostatic units} = m \text{ electromagnetic units}$$

and therefore
$$\frac{1 \text{ electromagnetic unit}}{1 \text{ electrostatic unit}} = \frac{s}{m}.$$

Now if the numerator of the left hand side of this relation be multiplied by a quantity of the dimensions of a velocity it becomes of the same dimensions as the denominator, for LT^{-1} [electromagnetic unit] = [electrostatic unit], and if the measure of this velocity be s/m units the relations becomes true numerically, for we then have

$$\frac{s/m \text{ [electrostatic units]}}{1 \text{ [electrostatic unit]}} = \frac{s}{m}.$$

Hence the measure of the velocity v is given by s/m, where s and m are the measures of the same quantity of electricity in electrostatic and electromagnetic units. The ratio s/m is also the ratio of the electromagnetic to the electrostatic unit and this ratio is sometimes given as the measure of v.

The result just obtained is more easily and clearly deduced from the complete dimensions of quantity of electricity in the two sets of units. From the table of Art. 198 we get

$$s[M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}] = m[M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}].$$

From this we get

$$\frac{1}{\sqrt{K\mu}} = \frac{s}{m}[LT^{-1}].$$

As in Art. 198 the quantity $1/\sqrt{K\mu}$ is seen to be of the dimensions of a velocity and its measure is here shown to be s/m, where s and m denote as before the electrostatic and electromagnetic measures of the same quantity of electricity.

In the same way, if the same capacity be measured in electromagnetic and electrostatic units and m and s be the measures, we have as before

$$\frac{1 \text{ electromagnetic unit}}{1 \text{ electrostatic unit}} = \frac{s}{m}.$$

And as

$$(\mathrm{L}\,\mathrm{T}^{-1})^2\,[\text{electromagnetic unit}] = [\text{electrostatic unit}]$$

we have, in order that the relation may be numerically correct,

$$\frac{(\sqrt{s/m}\,\mathrm{L}\,\mathrm{T}^{-1})^2\,[\text{electromagnetic units}]}{1\text{ electrostatic unit}} = \frac{s/m\text{ electrostatic units}}{1\text{ electrostatic unit}} = \frac{s}{m}.$$

That is, the measure of v in this case is $\sqrt{s/m}$, where s and m are the electrostatic and electromagnetic measures of the same *capacity*.

From what has been said it will be clear that, in order to determine practically the value of v, all that is necessary is to measure the same magnitude of some convenient electrical quantity in electrostatic and in electromagnetic units. The value of v can then be calculated from the values of the two measures so obtained. The two electrical quantities most conveniently and accurately measurable in both electrostatic and electromagnetic units are electromotive force or difference of potential and capacity.

In the case of *difference of potential* the necessary measurements could be made as follows. Let the difference of potential to be measured be that between two points on a conductor carrying a current C, and let R denote the resistance between the points, then for the *electromagnetic measure* of the difference of potential we have

$$m = \mathrm{C}\mathrm{R},$$

where C and R are expressed in C.G.S. electromagnetic units.

Then, if an absolute electrometer be connected to the two points so as to measure the difference of potential in absolute electrostatic units we get s, the required *electrostatic measure.* A consideration of the dimensions of difference of potential in electrostatic and electromagnetic units by the method indicated above for quantity of electricity and capacity will show that the value of v will be found from the relation

$$v = \frac{m}{s}.$$

In order to determine v, from measurement of the *capacity* of a condenser, the following method may be adopted.

A very accurately made condenser of definite geometrical form — plane, spherical, or cylindrical — is carefully measured, and its capacity calculated from its dimensions by the appropriate formula (Chap. VII.). This gives s, the *electrostatic measure* of the capacity in C.G.S. electrostatic units.

To determine the *electromagnetic measure* of its capacity the condenser is charged to a known difference of potential V and then discharged through a ballistic galvanometer. If δ denotes the angular throw of the galvanometer needle, corrected for damping, we have

$$Q = \frac{HT}{\pi G} \frac{\delta}{2}$$

But $Q = mV$, where m is the electromagnetic measure of the capacity and V is expressed in C.G.S. electromagnetic units.

Hence we get
$$mV = \frac{HT}{\pi G} \frac{\delta}{2}$$

or
$$m = \frac{HT}{\pi GV} \frac{\delta}{2}$$

The measures s and m being thus found, the value of v is given, as shown above by the relation

$$v = \sqrt{\frac{s}{m}}$$

The values obtained for v by methods of this kind all approximate very closely to 3×10^{10} cms. per sec. This is the same as the velocity of light, and it is shown in Art. 194, the velocity v really is the velocity of transverse waves in the ether, and is therefore identical with the velocity of light.

Maxwell assumed that an infinitely long conductor carrying a fixed static charge q per unit length, and moving with a velocity x in the direction of its length, is equivalent to a current of $q\,x$ electrostatic units and $q\,x/v$ electromagnetic units. It differs from a conductor carrying a current in that a conductor carrying a current has no statical charge.

If two infinitely long parallel straight wires, each charged with q units of electricity per unit length, move in the direction of their lengths with a velocity x there will be attraction between them as two parallel currents and repulsion as two statical linear charges. The attraction per unit length for both conductors is by (Art. 136) equal to $2\left(\frac{q\,x}{v}\right)^2\Big/d$ or $\frac{2\,q^2\,x^2}{v^2\,d}$ and the repulsion per unit length is $2\,q^2/d$. Hence if the repulsion equal the attraction we must have $x = v$, that is if each conductor move with the velocity 'v' then there is no action between the two conductors. Recent experiments by Professor Rowland confirm Maxwell's assumption.

201. Absolute Measurement of Units. Most of the measurements dealt with so far have been comparative and not absolute. Thus in measuring resistance the practice is to *compare* the unknown resistance with a known standard resistance. Similarly, by the method usually adopted, electromotive forces are compared with each other and not measured absolutely. In order to obtain our standards for comparative measurements and to realise the practical units, it is, however, necessary to make certain absolute measurements based only on measurements in the fundamental units of mass, length, and time. That is, to measure any electrical quantity in absolute units no units are to be assumed known, but the necessary fundamental units of mass, length, and time.

For example, in order to measure current in absolute C.G.S. electromagnetic units, as defined in Art. 129, it would be necessary to proceed as follows. The current to be measured is passed through a standard tangent galvanometer set in earth's field, and from the theory of the galvanometer we have

$$\mathrm{C} = \frac{r\,\mathrm{H}}{2\,\pi\,n}\,\tan\delta.$$

But this relation involves H, the horizontal component of the earth's field, the value of which must not be assumed but determined absolutely. This can, however,

be done by the deflection and oscillation method of Art. 94. This gives

$$H = \frac{2\pi}{t}\sqrt{\frac{2K}{d^3 \tan\alpha}}.$$

Substituting this value of H in the expression for C, we get

$$C = \sqrt{\frac{2K}{d^3 \tan\alpha}} \cdot \frac{r}{tn} \tan\delta.$$

In this expression for C there is no quantity which cannot be measured directly in one or more of the fundamental units of mass, length, and time—and so C can be determined absolutely by measurement made with the balance, the scale of length, and a seconds clock. To make an accurate absolute measurement of current it would be necessary to adopt less approximate relations than those given above, and to secure the utmost accuracy possible in making the fundamental measurements, but the outline just given sufficiently indicates the method.

Again *electromotive force* might be measured absolutely by applying the rule for determining the electromotive force induced in a circuit by moving it in a magnetic field. If a current of simple geometrical form and easily measured dimensions is moved in some definite way in an uniform magnetic field the induced electromotive force, defined as the rate of change of the flow of induction through the circuit, is readily measured. Thus if a thin circular conductor be made to rotate in a uniform field about an axis at right angles to the direction of the field with a uniform velocity ω then, neglecting self-induction, the electromotive force induced in the conductor at any instant is, as in Art. 165, given by

$$e = H A \omega \sin \omega t,$$

The maximum value of e obtains when the plane of the conductor is parallel to the direction of the field, so that if the circular conductor be cut, and the ends made to connect automatically with the terminals of a voltaic cell every time the conductor passes through this position, it would be possible to arrange that the E. M. F. of the cell should be opposed to the induced E. M. F. in the conductor, and to adjust the speed of rotation until the two opposed

electromotive forces should exactly balance. We should then have

$$E = H a \omega \sin \omega t,$$

where H can be determined absolutely as before and the other quantities involved are fundamental. The electromotive force of the cell can thus be determined in absolute units. It will be noticed that as, by this method of adjustment, no current is produced in the rotating ring the co-efficient of self-induction does not enter into the calculation and no error arises from its being neglected.

Instead of balancing the induced electromotive force in the rotating coil against the fixed E. M. F. of a cell by adjusting the speed of rotation it would be more convenient to balance it against the difference of potential between two points on a conductor carrying a current C. This difference of potential is adjustable, as in the Potentiometer method of Art. 149, by adjusting the resistance between the points, and this does away with the necessity for adjusting the speed of rotation. We should then have

$$CR = H a \omega \sin \omega t,$$

where C is the current in the conductor and R the resistance between the points when the adjustment for no current is exact. Now C can be measured absolutely by the method outlined above, which gives

$$C = \frac{r H}{2 \pi n} \tan \delta.$$

And if H in this relation be the same as in the one preceding it, that is if the magnetic field for the standard galvanometer is the same as for the rotating coil, then from the two relations we get—

$$R = \frac{2 \pi n a \omega}{r} \sin \omega t.$$

That is, we get an *absolute measure of the resistance* R without having to measure H at all.

202. The Determination of the Ohm. In order to realise the practical electrical units and to set up a concrete standard for comparative measurements it is evidently best to select the standard unit of resistance as a starting point. We cannot practically realise a standard unit of current or of electromotive force although

instruments can be *standardised* to give absolute measures of these quantities, but in the case of resistance it is easily possible to construct a durable invariable standard unit. For these reasons experimental work on the absolute measurement of the unit has centred round the *determination of the Ohm*, that is, the exact specification of the resistance which is equal to 10^9 C.G.S. electromagnetic units. It has been found desirable to specify this resistance in terms of the specific resistance of pure mercury, and the Ohm is now said to be the resistance to a steady current of "a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of constant cross-sectional area, and of a length of 106·3 centimetres." This specification is practically the same as that which states the ohm to be the resistance of a column of mercury at 0°C., 106·3 cms. long, and 1 square mm. in cross section.

The experimental determination of the ohm has been the object of a number of very careful researches. An outline only of the general principles of the more important methods can be given here.

1. *The British Association Rotating Coil method.* A thin circular coil is rotated rapidly and uniformly about a vertical axis in the earth's field. As a result currents are induced in the coil, and if a magnetic needle be suspended exactly at the centre of the coil it will be deflected by the action of the induced currents in the direction of the rotation of the coil. From the constant of the coil, the speed of rotation, and the deflection of the needle it is possible to obtain an absolute measure of the resistance of the coil.

Let H denote the horizontal component of the earth's field, A the area of the coil, n the number of turns, and α the angle between the plane of the coil at any instant and the plane at right angles to the magnetic meridian. Then F, the flow of force through the coil, is given by $F = n\,H\,A \cos\alpha$ or, since $\alpha = \omega t$, by $F = n\,H\,A\cos\omega t$ and the induced electromotive force is $n\,H\,A\,\omega \sin\omega t$.

This gives

$$C = \frac{n\,H\,A\,\omega}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - \phi).$$

But $$\sin(\omega t - \phi) = \sin\omega t\cos\phi - \cos\omega t\sin\phi.$$

Also $\cos\phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$ and $\sin\phi = \dfrac{L\omega}{\sqrt{R^2 + L^2\omega^2}}$.

Therefore we may write

$$C = \frac{n\,H\,A\,\omega}{R^2 + L^2\omega^2}\,[R\sin\omega t - L\omega\cos\omega t].$$

Now, if G be the constant of the coil, the field at the centre of the coil at right angles to its plane will be G C, and the components of this field in and at right angles to the meridian are G C $\cos\omega t$ and G C $\sin\omega t$. Substituting the value of C we get

$$\frac{G\,n\,H\,A\,\omega}{R^2 + L^2\omega^2}\,[R\sin\omega t\cos\omega t - L\omega\cos^2\omega t],$$

or $$\frac{G\,n\,H\,A\,\omega}{R^2 + L^2\omega^2}\cdot\frac{1}{2}\left[R\sin 2\,\omega t\;\frac{L\omega(1 + \cos 2\,\omega t)}{4}\right],$$

or $$-\frac{G\,n\,H\,A\,L\,\omega^2}{2(R^2 + L^2\omega^2)} + \frac{G\,n\,H\,A\,\omega}{2(R^2 + L^2\omega^2)}\,[R\sin 2\,\omega t - L\omega\cos 2\,\omega t],$$

or $$-\frac{G\,n\,H\,A\,L\,\omega^2}{2(R^2 + L^2\omega^2)} + \frac{G\,n\,H\,A\,\omega}{2\sqrt{R^2 + L^2\omega^2}}\sin(2\,\omega t - \phi)$$

for the component parallel to the meridian, and

$$\frac{G\,n\,H\,A\,\omega}{R^2 + L^2\omega^2}\,[R\sin^2\omega t - L\omega\cos\omega t\sin\omega t],$$

or $$\frac{G\,n\,H\,A\,\omega}{R^2 + L^2\omega^2}\,\tfrac{1}{2}\,[R(1 - \cos 2\,\omega t) - L\omega\sin 2\,\omega t],$$

or $$\frac{G\,n\,H\,A\,R\,\omega}{2(R^2 + L^2\omega^2)} - \frac{G\,n\,H\,A\,\omega}{2(R^2 + L^2\omega^2)}\,[R\cos 2\,\omega t + L\omega\sin 2\,\omega t],$$

or $$\frac{G\,n\,H\,A\,R\,\omega}{2\,R^2 + L^2\omega^2} - \frac{G\,n\,H\,A\,\omega}{\sqrt{R^2 + L^2\omega^2}}\cos(2\,\omega t - \phi)$$

for the component at right angles to the meridian.

These expressions for the components of the field due to the induced currents each consist of two parts, a constant part and a periodic part of period double that of the rotation of the coil, and if the needle suspended at the centre be assumed to have a sufficiently large moment of inertia the effect of the periodic parts of the components

will have a negligible effect in determining its position, and we may write the components effectively determining the position of the needle as

$$H - \frac{G n H A L \omega^2}{2(R^2 + L^2\omega^2)}$$

parallel to the meridian and

$$\frac{G n H A R \omega}{2(R^2 + L^2\omega^2)}$$

at right angles to the meridian.

If therefore δ be the observed deflection of the needle from the meridian, we have

$$\tan\delta = \frac{\dfrac{G n H A R \omega}{2(R^2 + L^2\omega^2)}}{H - \dfrac{G n H A L \omega^2}{2(R^2 + L^2\omega^2)}}$$

or

$$\tan\delta = \frac{G n A R \omega}{2(R^2 + L^2\omega^2) - G n A L \omega^2}.$$

If $L\omega$ is very small then this reduces to

$$\tan\delta = \frac{G n A \omega}{2R}.$$

From these relations R can be determined absolutely, for all the other quantities can be calculated from the dimensions of the coil and the speed of rotation. The resistance of the coil circuit thus determined in electromagnetic C. G. S. units can then be compared with the specific resistance of pure mercury and the resistance equivalent to the ohm, or 10^9 C. G. S. electromagnetic units deduced from the results. In this way the ohm can be specified in terms of the specific resistance of pure mercury.

This method was suggested by Lord Kelvin and was carried out by a Committee of the British Association. The ohm determined from the results of these experiments was called the B.A. unit, and its value is about ·9863 of the ohm as now specified. That is, the results of the experiment were not correct, and gave a unit which was about $1\frac{1}{2}$ per cent. below the real equivalent of 10^9 C. G. S. electromagnetic units.

2. *Lorenz's method.* When a disc of conducting material is rotated in its own plane in a magnetic field about its geometrical axis a difference of potential is set up, as already explained, between the axis and the circumference of the disc. As in Art. 165, if H denote the component of the field at right angles to the plane of the disc, and if the field be assumed uniform, we have

$$e = \frac{\pi r^2 H}{T},$$

where r denotes the radius of the disc, T the time of rotation, and e the difference of potential set up between the axis and the circumference of the disc. If, then, a disc be set up in the interior of a long carefully-wound solenoid with its axis parallel to the axis of the coil (which should be at right angles to the meridian in order to eliminate the earth's field) and made to rotate uniformly we shall have

$$e = \frac{\pi r^2 . 4 \pi n C}{T} = \frac{4 \pi^2 r^2 n C}{T},$$

where C is the current in the coil, for the plane of the disc is at right angles to the field $4 \pi n C$ inside the coil due to the current C passing through the coil. It is easily possible by the usual arrangement to balance the difference of potential e against the difference of potential between two points on a conductor forming part of the circuit of the coil and carrying therefore the *same* current. If R be resistance between these two points when the balance is exact we evidently have

$$\frac{4 \pi^2 r^2 n C}{T} = C R.$$

or

$$\frac{4 \pi^2 r^2 n}{T} = R.$$

In this way C is eliminated and the resistance R is determined in absolute units, for r is a length, n the number of turns *per unit length*, and T a time.

The arrangement of connections for this method is shown diagrammatically (Fig. 323). The elementary

theory of the method is simple, but there are a number of troublesome corrections. This method is due to Lorenz

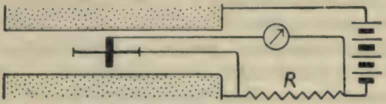


Fig. 323.

and careful determinations made by Lord Rayleigh and Mrs. Sidgwick and by the late Principal Viriamu Jones show that it is capable of giving very good results.

3. *Lippmann's method.* If in Lorenz's method a rotating coil be used instead of the disc, and made to rotate uniformly round a vertical axis in the field inside the coil, a periodic electromotive force will be set up in the coil. But if the terminals of the coil make connection only for an instant, once in each revolution, with the two points on the resistance R, it will be possible to balance the electromotive force at that instant against the difference of potential C R. If the connection is made automatically at the instant when the plane of the coil is parallel to the axis of the solenoid coil the electromotive force will have its maximum value

$$e = \frac{\pi r^2 \mathrm{N} \,.\, 4\pi n \mathrm{C} \,.\, 2\pi}{\mathrm{T}}.$$

That is,
$$e = \frac{8\pi^3 \mathrm{N} n r^2 \mathrm{C}}{\mathrm{T}},$$

where N is the number of turns in the coil, r its radius, and T its time of revolution.

This gives for R the relation

$$\mathrm{C\,R} = \frac{8\pi^3 \mathrm{N} n r^2 \mathrm{C}}{\mathrm{T}}$$

or
$$\mathrm{R} = \frac{8\pi^3 \mathrm{N} n r^2 \mathrm{C}}{\mathrm{T\,C}},$$

that is,
$$\mathrm{R} = \frac{8\pi^3 \mathrm{N} n r^2}{\mathrm{T}}.$$

This method is in one respect more workable than that of Lorenz; by taking a coil with a large number of turns a comparatively large electromotive force is induced in the coil, and the adjustment for exact balance can therefore be more accurately made.

4. *Mutual Inductance Method.* This method has been used by Kirchhoff and by Glazebrook. Two coils, for which the coefficient of mutual induction is M, are connected one with a battery giving a steady current C, the other with a ballistic galvanometer.

When the steady current in one is suddenly reversed an induced effect takes place in the other, causing a *quantity* of electricity q to pass round the circuit and through the ballistic galvanometer.

But by Art. 165 we have

$$q = \frac{2\,M\,C}{R},$$

where R is the resistance of the coil and ballistic galvanometer circuit.

Also if δ be the angular deflection of the galvanometer needle corrected for damping we have

$$q = \frac{H\,T}{\pi\,G} \cdot \frac{\delta}{2}.$$

Hence we have $$\frac{2\,M\,C}{R} = \frac{H\,T}{\pi\,G} \cdot \frac{\delta}{2}$$

and therefore $$R = \frac{4\,\pi\,M\,G\,C}{\pi\,T\,\delta}.$$

The current C is measured by making it pass through a tangent galvanometer, and for C we may write

$$C = \frac{H}{G'} \tan \delta',$$

where G′ is the constant of the galvanometer coil, and δ' the deflection caused by the current. Substituting this value of C we get

$$R = \frac{2\,\pi\,M}{T} \cdot \frac{G}{G'} \cdot \frac{\tan \delta'}{\delta},$$

The ratio G/G' may be obtained experimentally by passing the same current through the two galvanometers in series, and the other quantities involved can be directly determined in fundamental units.

203. Current. The absolute measurement of current can be made, as already indicated, by the standard tangent galvanometer. It can also be made with considerable accuracy by measuring the attraction between two coils carrying the same current and placed with their planes parallel, at right angles to the line joining their centres. If C be current in the coils and M the coefficient of mutual induction, the force of attraction is given by $C^2 (dM/dx)$, where dM/dx denotes the *rate* of change of M with x, the distance between the centres of the coils. This rate of change of M can be calculated from the dimensions of the coil system, and the force of attraction between the coils can be determined in dynes directly. One coil is attached to one pan of an accurate balance and the other is fixed in the specified position under it. When the current passes the force of attraction between the coils can be exactly balanced by placing weights in the other pan of the balance, and from these weights the measure of the force is at once obtained in dynes.

From absolute measurements of this kind it is found that 1 C.G.S. unit of current deposits in one second from an aqueous solution of silver nitrate 0·01118 gramme of silver, and from this we get the specification of the *ampère*, which is one-tenth of a C.G.S. unit, as that current which deposits in one second from a solution of silver nitrate 0·001118 gramme of silver.

204. Electromotive Force. When the units of current and resistance are definitely specified and realised it is a comparatively simple matter to determine electromotive force absolutely. The E.M.F. of a Latimer Clark cell, for example, has been determined absolutely by Lord Rayleigh by an elaboration of the potentiometer method of Art. 149. The usual adjustment of this method gives

$$E = CR.$$

The current C is measured absolutely by the amount of silver deposited in a given time from a solution of silver

nitrate, and R is determined by comparison with a standard resistance. The value found is given below.

205. Practical Units. For convenience of reference the most important practical units are again briefly specified below.

The practical unit of *resistance*, the *ohm*, is theoretically 10^9 C.G.S. electromagnetic units, and is specified as the resistance of a column of mercury at 0° C., 106·3 cms. long, of uniform cross-section and of mass 14·4521 grammes.

The practical unit of *current*, the *ampère*, is theoretically 10^{-1} C.G.S. electromagnetic units, and is specified as that current which in one second deposits from a solution of silver nitrate 0·001118 gramme of silver.

The practical unit of *quantity*, the *coulomb*, is theoretically 10^{-1} C.G.S. electromagnetic units, and is specified as the quantity of electricity conveyed by one ampère in one second.

The practical unit of *electromotive force* or *difference of potential*, the *volt*, is theoretically 10^8 C.G.S. electromagnetic units and is that difference of potential which gives a current of one ampère through a resistance of one ohm. For a practical standard the E.M.F. of a Latimer Clark cell has been determined as 1·434 volts at 15° C. and 1·434 $[1 - \cdot 00078\ (t - 15)]$ volts at $t°$ C.

The practical unit of *capacity*, the *farad*, is 10^{-9} of a C.G.S. electromagnetic unit, and is specified as the capacity of a condenser in which a charge of one coulomb gives a difference of potential of one volt.

The *microfarad* is 10^{-6} of a *farad* and is therefore 10^{-15} of a C.G.S. electromagnetic unit of capacity.

The practical unit of *inductance*, the *henry*, is theoretically 10^9 C.G.S. electromagnetic units, and is specified as the inductance of a circuit in which an induced electromotive force of one volt is set up by the variation of the inducing current at the rate of one ampère per second.

The English *legal* ampère, volt, and ohm are defined by the standard current balance (Art. 203), standard 100-volt multi-cellular electrostatic voltmeter, and standard ohm (platinum silver), kept in the Board of Trade Electrical Standards Laboratory, Whitehall, S.W.

CHAPTER XXXII.

THERMO-ELECTRICITY.

206. Introductory. It is found that if a circuit be made up of two wires of different metals, and one of the junctions be heated, a current is set up in the circuit. Currents so produced are called *thermo-electric currents*, and the E. M. F. to which they are due is called *thermo-electromotive force.* For example, if a piece of iron wire and a piece of copper wire of about equal length have their loose ends soldered or twisted together a thermo-electric current may be established in the circuit by heating either of the junctions, or rather by establishing a difference of temperature between the two junctions, for a current may also be produced by cooling either of the junctions. The existence of this current may be shown by including a galvanometer in the circuit; this is readily done by cutting either of the wires and joining the terminals of the galvanometer to the loose ends of the cut wire, or by taking about equal lengths of copper and iron wire, twisting one pair of ends together and joining the other pair to the terminals of the galvanometer,—the ends twisted together form one junction which can be heated or cooled, and the coil of the galvanometer itself forms the other junction which always remains at the temperature of the room, and is therefore practically constant. It should here be noted that the junction between two wires may be effected in any way without influence on the result, so long as the temperature of the extremities of whatever forms the junction is the same: for example, in the case of the galvanometer just mentioned, the wire of the coil forms one junction; and if the extremities of the coil, that is, the terminals of the galvanometer, are of the same temperature, then

the result will be the same as if the ends of the wires were directly twisted together, and the junction so formed maintained at the temperature of the terminals of the galvanometer. The point here mentioned may be more generally put in this way:—in a compound circuit made up of several conductors of different materials, any difference of temperature of the several junctions produces a current in the circuit, *but the thermo-electric effect of any element of the circuit is nil if the extremities of that element have the same temperature*—that is, such an element may be considered as a mere junction between the two adjacent elements, and the temperature of the junction is the common temperature of the extremities of the element. Further, in such a compound circuit the total electromotive force in the circuit is the sum of all the thermo-electromotive forces at the several junctions in the circuit.

The production of thermo-electric currents by heating junctions of different metals was first discovered by **Seebeck** in 1822.

207. A Simple Thermo-Electric Couple Let the extremities of two pieces of wire of different metals, say copper and iron,† be twisted together and a low resistance reflecting galvanometer included in the circuit. Let one junction be slowly heated by a Bunsen flame while the other remains at the temperature of the room. It will be noted that as the temperature of the junction rises the deflection of the galvanometer at first increases, indicating an increase of the thermo-electric current in the circuit. This increase, however, becomes less and less rapid as the heating goes on, before long it ceases altogether, and the current attains a maximum. If the heating of the junction is still continued the current now begins to *decrease*, and the decrease goes on continuously until the current is first reduced to zero and then *reversed* in the circuit. The important features of this experiment are: first, the current first increases, attains

† Copper and iron are chosen simply because the couple they form conveniently exhibits the phenomena to be studied; any other pair of metals could be made to give the same results, but in most cases the necessary range of temperature would not be easily obtained.

a maximum and then decreases; and second, the reversal of the current in the circuit after it has been reduced to zero.

If the experiment be so conducted that the temperature of the heated junction can be noted during the experiment—for example, if the junction be inserted in a long glass tube closed at one end and dipping into an oil bath—the following facts will be noticed. First, the temperature at which the current attains its maximum value is always the same, no matter what the temperature of the other junction may be. Second, the temperature at which reversal of the current takes place is as much above the temperature of maximum current as that temperature is above the temperature of the cool junction, that is the temperature of maximum current is the mean of the temperature of reversal, and the temperature of the cool junction. For example, in the case of the copper and iron couple here considered, supposing the cold junction to be at 20° C., the temperature of maximum current will be about 260° C. † and the temperature of reversal about 500° C. Or, if the cold junction be kept at 0° C. then the temperature of maximum current will be the same as before—about 260° C.—but the temperature of reversal will be about 520° C.

The temperature of maximum current for any couple is known as the **neutral point** for that couple, and it is always the mean of the junction temperatures when reversal takes place. It will be evident from what has been said that the neutral point is a constant for a given couple, but that the temperature of reversal depends upon the temperature of the cold junction.

208. The Peltier Effect. In 1834 it was discovered by Peltier that if a current was sent round the circuit of a thermo-electric couple heat was evolved at one junction and absorbed at the other—that is, one junction was heated and the other cooled. Further, it was found that the junction which was cooled was the one which, if heated, would give a thermo-electric current in the same direction round the circuit as the current which causes the cooling

† The temperature varies for different specimens of iron and copper.

effect. For example, in the copper-iron couple considered above, for temperatures below the neutral point, the *thermo-electric* current flows round the circuit, so as to pass from the copper to the iron at the hot junction; hence, if a current be sent round the circuit of this couple, the junction at which it passes from copper to iron will be cooled, and the other will be heated. From this it will at once be seen that, when the thermo-electric current passes round the circuit of the couple, it absorbs heat at the hot junction and evolves heat at the cold junction. This result indicates the source of energy in a thermo-electric couple,—the heat absorbed at the hot junction is greater than that evolved at the cold junction, and the difference between the two quantities of heat is the source of energy to which the current in the circuit is due. In fact, a thermo-electric couple acts as a small heat-engine, taking in heat at the hot junction as its source, and giving out heat at the cold junction as its condenser.

209. Thermo-Electric Power. If the temperature of one of the junctions of a thermo-electric couple be changed, there will be a corresponding change in the thermo-electromotive force of the couple; and if these changes be supposed to be very small, the thermo-electric power of the junction may be defined as the *ratio* of the change of thermo-electromotive force to the change of temperature to which it is due. This ratio varies with the temperature at which the changes are supposed to take place. The meaning of the term *power* used in this relation is rather ambiguous; the probable reason of its use will be understood by considering the following case. Suppose a couple to have both its junctions at the *same* temperature—say that of the room; then, if the temperature of one of them is slightly changed, a thermo-electromotive force is set up in the circuit, and the ratio of this E. M. F. to the small change of temperature to which it is due gives an idea of the thermo-electric *power* of the couple at the temperature of the room. The thermo-electric power of a couple is evidently least in the neighbourhood of the neutral point, and is zero at that temperature, for as the current approaches its maximum value its rate of change with temperature gradually

diminishes, and finally changes sign (that is, changes from a rate of *increase* to a rate of *decrease*) after passing through the value zero.

210. The Thomson Effect. From theoretical considerations of the action of thermo-electric couples, Sir William Thomson was led to assume that the Peltier effect is zero when the thermo-electric power is zero: that is, if the temperature of a junction is at the neutral point for the couple, then at that junction there is no Peltier effect. If, however, this junction is the hot junction, then we meet with a difficulty in explaining where the energy of the current is derived from; for if the Peltier effect at the hot junction is zero no heat is absorbed there, and at the cold junction heat is *given out*, hence heat must be absorbed in a thermo-electric circuit at other points than at the junctions. This result was first pointed out by Sir William Thomson, and he showed that heat must be absorbed in one or both of the wires in virtue of the difference of temperature between the ends, or that heat may be absorbed in one wire and generated in the other, but that the quantity of heat absorbed must, in the case considered, be greater than that given out. This effect is known as the *Thomson effect*. Just as the Peltier effect may be considered as the reverse of the Seebeck effect, so also there is a reverse to the Thomson effect. The Thomson effect states that the passage of a current along a conductor whose ends are at different temperatures is attended with an absorption or evolution of heat according to the material of the conductor; and the reverse of this evidently is that if the ends of a conductor are maintained at different temperatures, then a current will be produced in the conductor. The Thomson effect will of course vary with the direction of the current in the conductor—that is, if when the current flows from the hot to the cold end, and heat is absorbed, then heat will be evolved when it flows from the cold to the hot end. Thus, when a current flows in a copper conductor from the cold to the hot end heat is absorbed, and when it flows from hot to cold heat is evolved. In iron the effects are opposite. Lead is the only metal in which there is no appreciable Thomson effect.

The existence of the Thomson effect indicates that in a

conductor difference of temperature between any two points is associated with difference of potential between the points. Thus, if a copper bar L M (Fig. 324) be heated at the point N to a constant temperature, and the two ends be maintained at a constant lower temperature, the final steady distribution of temperature will be such that temperature rises continuously along L N and falls along N M. In a metal such as copper, in which heat is absorbed when electricity travels from cold to hot and is evolved when it travels from hot to cold, the rise of temperature along L N must be associated with a rise of potential, and similarly the fall of temperature along N M is attended with a corresponding fall of potential. For, if this is so, when electricity flows from L to M it gains electric energy on account of the rise of potential, indicated diagrammatically in the figure by the increase in the ordinate L l at L to N n at N. This energy is absorbed as heat from the portion L N of the conductor. Similarly, when electricity passes along from N to M it loses electric energy and the energy so evolved appears as heat in the portion N M of the bar. Hence, when a current is sent along the bar from L to M, heat is absorbed from the portion L N and evolved in the portion N M, that is, there is an apparent transfer of heat in the direction of the current. In metals such as iron, however, rise of temperature must be associated with fall of potential and fall of temperature with rise of potential. That is, in the case of iron and similar metals, the potential at N is lower than at L and M as indicated in Fig. 325. In this case, electricity in passing from L to N loses or gives out electric energy and consequently heat is developed in the portion L N. Similarly, in the part N M, where the potential rises the electricity gains energy which is absorbed as heat from

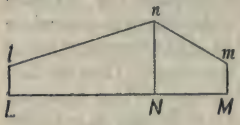


Fig. 324.

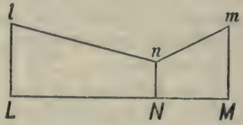


Fig. 325.

the part N M. There is therefore an apparent transfer of heat in a direction opposed to that of the current. Of ordinary metals platinum, bismuth, mercury, cobalt, nickel, palladium and german silver behave like iron, while silver, zinc, cadmium, tin, antimony, aluminium, brass, behave like copper. In the latter group of metals the effect is said to be positive and in the former group negative.

The Thomson effect in any metal may be exhibited experimentally by means of a bar, such as L M (Fig. 326), heated at its middle point to a fixed constant temperature and having its ends maintained at a lower fixed temperature. If the distribution of temperature along the bar is perfectly symmetrical on each side of the middle point, so that points equidistant from the middle point on opposite sides of it are at the same temperature, it will be found on passing a strong current along the bar that the symmetry of the distribution is disturbed. In metals for which the Thomson effect is positive the temperature at points in the first half falls, and rises at points in the second half, so that a difference of temperature is established between corresponding points such as a, b or opposite sides of the middle point, and the temperature at a is lower than at b.

Fig. 326.

In metals for which the effect is negative the symmetry of the distribution of temperature is similarly disturbed by the passage of a current, but the difference of temperature set up between corresponding points is such that the temperature at a is higher than at b.

211. Coefficients of the Peltier and Thomson effects. In order to deal quantitatively with the Peltier and Thomson effects it is necessary to specify how these effects can be definitely measured. When a current flows round a thermo-electric circuit, energy in the form of heat is absorbed at one junction and evolved at the other. This indicates that there is a difference of potential at these junctions, and the energy absorbed or evolved when *unit quantity* of electricity crosses a junction is the measure of

the difference of potential existing at the junction. If this difference of potential be denoted by Π, the energy absorbed or evolved at a junction when a current C passes through it for a time t is evidently measured by $C\,t\,\Pi$, and Π is the coefficient of the Peltier effect. This coefficient is not a constant but depends upon the temperature of the junction. In the case of the Thomson effect the difference of potential between two points is determined by difference of temperature, and the coefficient of the effect must be associated with difference of temperature. In any conductor consider two points very close together, at temperatures T and $T + d\,T$, and having therefore a difference of temperature of $d\,T$. The difference of potential between these two points may be expressed as $\sigma \,.\, d\,T$, where σ is the coefficient of the Thomson effect at temperature T. The energy absorbed between these two points when a current C passes along the conductor for a time t in the direction of increase of temperature is $C\,t \,.\, \sigma \,.\, d\,T$. The coefficient σ is not a constant, but a function of the temperature. It has been called the *specific heat of electricity*, the term being derived from the following analogy. Imagine L M in Fig. 324 to represent a copper tube heated in the same way as the bar, and that a liquid flows through the tube under the condition that it takes the temperature at each point of the tube as it flows along. Under this condition of flow it is evident that the liquid will absorb heat in flowing from L to N and give out heat from N to M. Also, if s denote the specific heat of the liquid, the heat absorbed or evolved in passing from one point to another through a difference of temperature $d\,T$ is $s \,.\, d\,T$. That is the thermal effect, when a current flows along a conductor for which the Thomson effect is positive, is analogous to this case of liquid flow along a tube, and σ the coefficient of the Thomson effect corresponds to s the specific heat of the liquid. For metals in which the Thomson effect is positive, σ is positive, and for metals with a negative Thomson effect, σ is negative. Hence, in the former class of metals the specific heat of electricity is said to be positive, and in the latter class it is said to be negative. That is, in the positive class electricity is

supposed to behave like a real liquid in the absorption and evolution of heat, but in the negative class it behaves like a hypothetical liquid which absorbs heat in cooling and gives out heat in heating. To determine the energy absorbed by unit quantity of electricity in passing along a conductor from a point where the temperature is T_1 to one where the temperature is T_2 it is evident that, since σ is not a constant but varies with the temperature, the difference of temperature $T_2 - T_1$ must be divided into infinitely small steps each denoted by $d\,T$, then the energy gained at each step is $\sigma\,.\,d\,T$, where σ has its proper value for each step, and $\Sigma_{T_1}^{T_2}\,\sigma\,d\,T$ or rather $\int_{T_1}^{T_2} \sigma\,d\,T$ is the energy gained by unit quantity of electricity in passing from a point at temperature T_1 to one at temperature T_2.

212. Experimental Laws. There are two simple laws applying to thermo-electric circuits which have been established by experiment. They are the law of successive temperatures and the law of successive contacts. The law of successive temperatures states that, for a given thermo-electric couple, the electromotive force for any specified range of temperature is the sum of the electromotive forces for any number of successive steps into which the given range of temperature may be divided. That is,

$$E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + \dots . E_{t_{n-1}}^{t_n}$$

where t_1 t_2 t_2 . . . . are successive temperatures intermediate between t_1 and t_n.

The law of successive contacts states that if, at a given temperature, a number of metals are in successive contact so as to form a chain of elements connected in series, the electromotive force between the extreme elements, if placed in direct contact, is the sum of the electromotive forces between successive adjacent elements. That is, for metals A, B, C, D. . . . N in successive contact

$$E_A^N = E_A^B + E_B^C + E_C^D + \dots . E_M^N$$

provided all *the junctions* in the series are at the same

temperature. The temperature at points other than the junctions does not affect the law. This law may evidently be stated in direct relation to a thermo-electric circuit. It evidently states that for given fixed junction temperatures, if E_A^B, E_B^C, E_C^D, etc., denote the electromotive forces for circuits with the metals A and B, B and C, C and D, etc., then the electromotive force for metal A and N is given by

$$E_A^N = E_A^B + E_B^C + E_C^D + \dots . E_M^N .$$

The law given in Art. 206 is evidently a simple case of this general law. By the law the junction at any temperature between any two metals may be made either by direct contact, or by means of a simple element of any metal with its *ends* at the temperature of the junction, or by means of a chain of elements of different metals with *its ends and all the junctions it includes* at the given temperature.

213. The Electromotive Force in a Thermo-electric Circuit. By considering the energy absorbed and evolved in a thermo-electric current it is possible to find an expression for the electromotive force in the circuit. Take the case of a circuit with junctions at temperature T_1 and T_2, the latter temperature being the higher. Let Π_1 and Π_2 denote the Peltier coefficients at T_1 and T_2 and σ_A and σ_B, the Thomson effects for the metals A and B of the circuit. Then, assuming the current to pass from A to B at the hot junction, the energy *gained* by unit quantity of electricity in passing round the circuit is

Π_2 for the Peltier effect at the junction at temperature T_2.

$-\Pi_1$ for the Peltier effect at the junction at temperature T_1

$\int_{T_1}^{T_2} \sigma_A \, dT$ for the Thomson effect in metal A.

$\int_{T_1}^{T_2} \sigma_B \, dT$* for the Thomson effect in metal B.

*This quantity is evidently positive if σ_B happens to be negative, in the case of iron.

Hence, the total gain for the complete circuit is

$$\Pi_2 - \Pi_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT,$$

and since this total must be numerically equal to the total electromotive force in the circuit, we have

$$E = \Pi_2 - \Pi_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT.$$

If we consider a circuit with junctions at temperatures T and $T + dT$, where dT is infinitely small, the result is more simply expressed, for the Peltier coefficients are Π and $\Pi + d\Pi$, where $d\Pi$ is the increment or difference in Π corresponding to the difference in temperature dT, and σ_A and σ_B are the values of these coefficients for the temperature T. Hence the four gains of energy given above are respectively $\Pi + d\Pi$, $-\Pi$, $\sigma_A\, dT$, and $-\sigma_B\, dT$, and we therefore have for dE, the infinitely small electromotive force in the circuit

$$dE = d\Pi + (\sigma_A - \sigma_B)\, dT.$$

If the principles of thermo-dynamics be applied to this thermo-electric circuit the value for dE may be expressed in another form. In a thermo-electric circuit the operations are reversible if we neglect the Joule heating effect in the conductors, and when the electromotive force in the circuit is infinitely small the current is infinitely small, and therefore the Joule heating effect, being proportional to the square of the current is negligible. The Peltier and Thomson effects being directly proportional to the current are reversible in the thermo-dynamic sense. Hence, in the thermo-electric circuit just considered, with its junctions at temperatures T and $T + dT$, since quantities of energy $\Pi + d\Pi$, $-\Pi$, $\sigma_A\, dT$, and $-\sigma_B\, dT$ are absorbed at temperatures $T + dT$, T, T, and T it follows thermodynamically that

$$\frac{\Pi + d\Pi}{T + dT} - \frac{\Pi}{T} + \frac{(\sigma_A - \sigma_B)}{T}\, dT = 0.$$

or

$$d\Pi - \frac{\Pi}{T}\, dT + (\sigma_A - \sigma_B)\, dT = 0.$$

This gives $$(\sigma_A - \sigma_B)\, d\mathrm{T} = \frac{\Pi}{\mathrm{T}} d\mathrm{T} - d\Pi,$$

and substituting this value in the expression given above for $d\,\mathrm{E}$, we get

$$d\mathrm{E} = \frac{\Pi}{\mathrm{T}}\, d\mathrm{T}.$$

From the relation

$$(\sigma_A - \sigma_B)\, d\mathrm{T} = \frac{\Pi}{\mathrm{T}}\, d\mathrm{T} - d\Pi$$

it follows that if for metal A the value of σ be zero, as is probably the case for lead, then

$$-\sigma_B = \frac{\Pi}{\mathrm{T}} - \frac{d\Pi}{d\mathrm{T}}.$$

This result expresses the coefficient of the Thomson effect at temperature T for the metal B in terms of the Peltier coefficient for B and a metal in which the Thomson effect is zero.

Similarly from the relation

$$d\,\mathrm{E} = \frac{\Pi}{\mathrm{T}}\, d\,\mathrm{T},$$

we get $$\Pi = \mathrm{T}\frac{d\,\mathrm{E}}{d\,\mathrm{T}}.$$

This relation indicates the only practicable method of measuring Π. By plotting a curve giving the relation between E and T in the circuit, the value of $d\,\mathrm{E}/d\mathrm{T}$ at any temperature T can be determined and the corresponding value of Π obtained.

From the value of Π just given, the value of σ in terms of $d\,\mathrm{E}/d\,t$ may readily be obtained. For we have

$$\sigma_A - \sigma_B = \frac{\Pi}{\mathrm{T}} - \frac{d\Pi}{dt}.$$

Now $$\Pi = \mathrm{T}\frac{d\,\mathrm{E}}{d\,\mathrm{T}},$$

and, by differentiating, $$\frac{d\Pi}{d\,\mathrm{T}} = \mathrm{T}\frac{d^2\,\mathrm{E}}{d\,\mathrm{T}^2} + \frac{d\,\mathrm{E}}{d\,\mathrm{T}}.$$

Hence $$\sigma_A - \sigma_B = \frac{dE}{dT} - \left(T\frac{d^2E}{dT^2} + \frac{dE}{dT}\right),$$

$$\sigma_A - \sigma_B = -T\frac{d^2E}{dT^2}.$$

That is, if A is a metal for which σ is zero

$$\sigma_B = T\frac{d^2E}{dT^2}.$$

The quantity dE/dT already defined as the thermo-electric power of the couple will be more fully explained further on, but if dE/dT be denoted by y, then

$$\Pi = Ty$$

and $$\sigma_A - \sigma_B = -T\frac{dy}{dT},$$

or when $$\sigma_A = 0,$$

then $$\sigma_B = T\frac{dy}{dT}.$$

214. Thermo-electric curves. In any thermo-electric circuit the electromotive force in the circuit under given conditions of temperature may generally be measured by a suitable modification of the potentiometer method. As

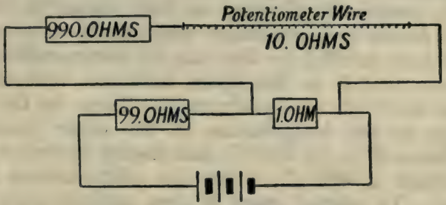


Fig. 327.

the electromotive force in the circuit, usually expressed in *microvolts*, or millionths of a volt, is very small, the difference of potential along the potentiometer must be small and so subdivided that a difference of one-millionth of a volt may easily be read. One method of arranging this is indicated in Fig. 327. If the wire be divided into

1,000 divisions the difference of potential for each division will be about $1/10^7$ of the E. M. F. of the battery, and may therefore be small enough to admit of sufficiently accurate measurement. If one junction of a couple with the metals A and B is maintained at a constant temperature, say the temperature of melting ice, and the electromotive force in the circuit measured when the other junction is adjusted to a number of known successive temperatures it will be possible to plot a curve showing the relation between the electromotive force in the couple and the difference of the temperatures at the junctions. The form of this curve in practically all cases is that shown in Fig. 328, and it is found to approximate very closely to a parabola with its axis parallel to the axis of electromotive force. The curve O A B in the figure indicates that with one junction at 0° C. the electromotive force in the circuit increases as the temperature of the other junction is raised from 0° C. to 200° C., and thence decreases till its temperature is 400° C., where the electromotive force is reversed. In normal cases it goes on increasing in the reversed direction indefinitely, the curve beyond B being a geometrical continuation of the parabolic segment O A B. If, with this couple, the junction of constant temperature be maintained at 100° C. instead of 0° C., then the curve of electromotive force is obtained from O A B simply by changing the origin O to O′ and taking O′ B′ instead of O B as the axis of temperature. This is evidently in accordance with the law of successive temperatures, for the

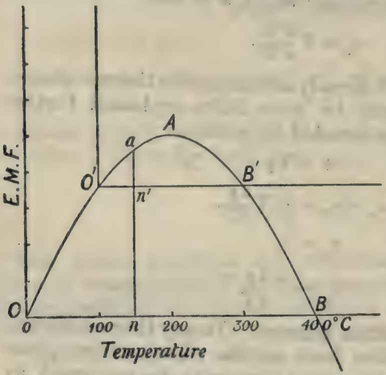


Fig. 328.

relation $E_0^{150} = E_0^{100} + E_{100}^{150}$ is correctly represented in the figure by the geometrical relation $n\,a = n\,n' + n'a$, where $n\,a$ represents E_0^{150}, $n\,n' = E_0^{100}$, and $n'a = E_{100}^{150}$.

So far we have considered the case of a couple made with the metals A and B, and O A B in Fig. 328 is the curve for the couple. If now, in the same way, we obtain the curves for other couples made with the metals A and C

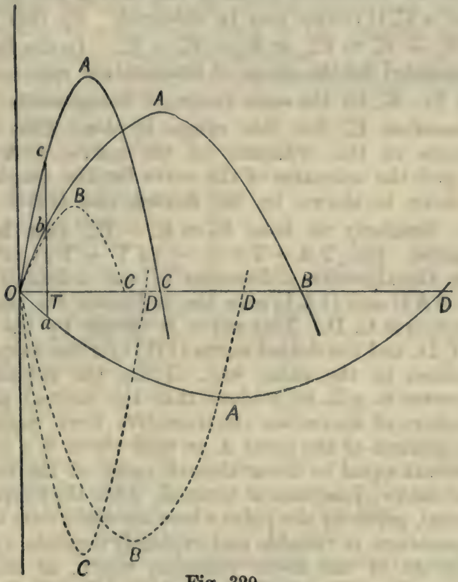


Fig. 329.

A and D, A and E, and so on, we shall have a set of curves similar to the three shown in Fig. 329, the curve O A B being for the metals A and B, O A C for the metals A and C, and O A D for the metals A and D. The curve

O A D is drawn with its ordinates negative to indicate that for the metals A and D the direction of the current, under given conditions, is opposite to that in the A, B and A, C circuits under the same conditions. If, for example, the B conductor in the A, B couple be replaced by the D conductor the direction of the current in the circuit is reversed. When the current passes in the standard metal, A, from cold to hot, the direction of the current is to be taken as positive. From these curves by application of the law of successive contacts the curves for a B, C, a B, D, or a C, D couple may be obtained. By this law we have $E_A^B + E_B^C = E_A^C$, or $E_B^C = E_A^C - E_A^B$. In the figure E_A^C is represented for the range of temperature represented by O T by T*c*, E_A^B for the same range of temperature by T *b*, and therefore E_B^C for this range by *b c*. That is, the differences of the ordinates of the curves O A B and O A C give the ordinates of the curve for the couple B, C. This curve is shown by the dotted line O B C in the figure. Similarly we have $E_C^D = E_A^D - E_A^C$, that is E_C^D is represented by $\mathrm{T}\,d - \mathrm{T}\,c = -(d\,\mathrm{T} + \mathrm{T}\,c) = -d\,c$, That is, the algebraic differences of the ordinates of the curves O A C and O A D give the ordinates for the curve for the couple C, D. This curve is shown by the dotted line O C D, and the dotted curve O B D for the couple B, D is obtained in the same way. From the symmetry of these curves it will be evident that the neutral point, or temperature of maximum electromotive force represented by the abscissa of the point A, in each curve is a constant temperature equal to the arithmetic mean of the temperatures of the two junctions at reversal. Also, the temperature of reversal, given by the point where the curve cuts the axis of temperature, is variable and evidently depends upon the temperature of the lower junction, which, as shown in Fig. 328, fixes the position of the origin of the axes of temperature and electromotive force.

215. Thermo-electric power. The electromotive force curves dealt with in the last Article are very approximately segments of parabolas with their axes at right angles to the axis of temperature. If for each of these curves a

differential curve be drawn (by any graphic method), showing the relation between the thermo-electric power, dE/dT, and the temperature T for each couple, it will be found that this curve is practically a straight line. For example, if for each degree of temperature the *difference* of electromotive force be plotted as the approximate value of dE/dT at the middle of the degree, it will be found that the curve obtained approximates very closely to a straight line. On account of the simplification of geometrical relations attending the use of straight line curves these lines, giving the relation between thermo-electric power and temperature, are almost exclusively used in the graphic methods of exhibiting thermo-electric relations.

In Fig. 330, let O A B be the electromotive force curve for an A, B couple. Then between the points *a b*, on the curve, dE is represented by *c b* and dT by *a c* and dE/dT therefore by *c b*/*a c*, that is by tan *b a c*, where *b a c* is the direction that the curve at *a b* makes with the axis O B. Inspection of the curve will show that the tangent of this angle is positive at O and decreases from O to A where it becomes zero, then, changing sign, it again increases from A to B. For the curve O A B, therefore, the thermo-electric power line, L N M, starts with a positive ordinate, O L, at O, crosses the axis of temperature at N, the foot of the perpendicular from A on O B, and continues from N onwards with negative ordinates. The point N, where the line crosses the axis of temperature and where the thermo-electric power is zero, evidently indicates the neutral point for the given couple.

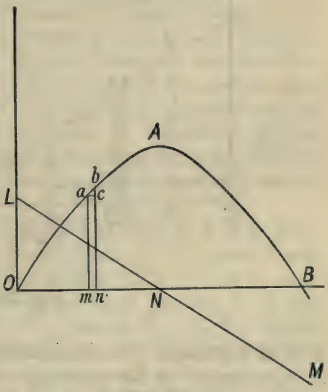


Fig. 330.

The electromotive force in a circuit for which the thermo-electric power line is known is readily found. Let L N M, Fig. 331, be the thermo-electric power line for a given couple. At any temperature represented by O A the thermo-electric power, that is, the value of dE/dt, is represented by the ordinate A a. For a very small increment of temperature, dt, at this temperature the increment in the electromotive force of the couple is given by $dE/dt \,.\, dt$, the product of the *rate* of change of thermo-electric power with temperature, and the change of temperature. Hence,

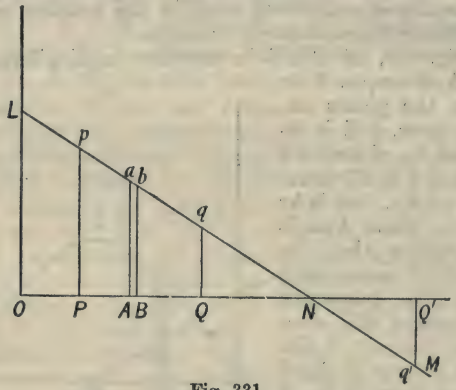


Fig. 331.

if A B represent dt, the increment of electromotive force is represented by the rectangle determined by A a and A B, and if dt be infinitely small this rectangle is practically equal to the area of the strip A B b a standing on A B. It follows from this that, if one junction of a couple be maintained at a temperature represented by O P while the temperature of the other is raised from this temperature to a higher one represented by O Q, the electromotive force set up in the couple will be represented by the area P Q q p. For, if the interval of temperature represented by P Q be divided into a number of very small steps such as that

represented by A B, the increment of electromotive force corresponding to each step will be represented, as for the step A B, by the area of the strip similar to A B $b\,a$ standing on the very short length representing the step of temperature, and the sum of these increments giving the electromotive force in the circuit must therefore be represented by the area P Q $q\,p$, which is the sum of the strip areas representing the increments.

If the temperatures of the junctions of the couple be represented by O P and O Q′ then the electromotive force in the circuit is, as before, represented by the area bounded by P p, Q′ q', L N M, and the axis of temperature; but, of this area, P N p. to the left of N, is positive and Q′ N q' to the right of N is negative, therefore the electromotive force is represented by the *difference* between the two areas P N p and Q′ $n\,q'$. Hence, if these two areas were equal, that is if N were midway between P and Q′, the electromotive force would be zero. That is, when the temperature of the neutral point represented by O N is the arithmetic mean of the temperatures of the junctions represented by O P and O Q′ the point of reversal is reached. When P N p is the greater area the electromotive force is positive, and when it is the smaller area the electromotive force is negative.

If we take the curves O A B, O A C, and O A D in Fig. 329, and draw the corresponding thermo-electric power lines we get lines such as are shown in Fig. 332, A B, A C, and A D being the thermo-electric power lines for the A, B the A, C and the A, D couples. The points N_1, N_2, N_3, where these lines cross the axis of temperature, are the neutral points for these couples.

The electromotive force for an A, C couple for junction temperatures represented by O P and O Q is represented by the area P Q $q\,p$, and the electromotive force for the A, B couple for the same junction temperatures is represented by P Q $s\,r$, therefore, by the law of successive temperatures the electromotive force in the B, C couple is represented by the area $r\,s\,q\,p$. It follows from this that the differences of corresponding ordinates of the A, C line and the A, B line give the ordinates for the thermo-electric

power line of the B, C couple. Similarly, from the differences of the ordinates of the lines A B, A C, and A D the ordinates of the thermo-electric power lines for the B, D and C, D couples may be obtained. From these differences separate thermo-electric power lines might be plotted from the axis O T for the B, C, the B, D, and the C, D couples, giving lines corresponding to the dotted electromotive force curves, O B C, O B D, and O C D in Fig. 329. It would, however, only needlessly complicate the diagram to do this. In representing the relative thermo-electric powers for a

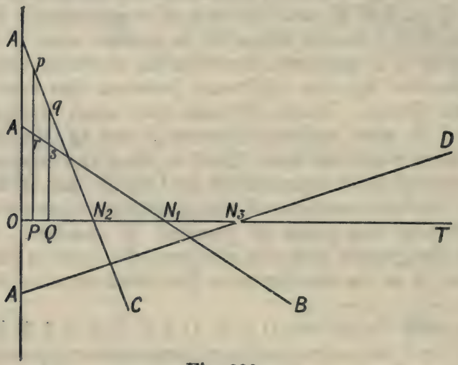


Fig. 332.

number of metals A, B, C, D, etc., it is most convenient to take one of the metals, say A, as a standard and plot the thermo-electric power lines for the couples A and B, A and C, A and D, and so on. Then, for any two metals P and Q, the relative thermo-electric power at any temperature may be found, as explained above, from the difference of the ordinates of the A, P and the A, Q lines at that temperature, and the electromotive force for a P, Q couple between any two temperatures may also be found, as already described, from these lines. It will also be evident that the point of intersection of the A, P and the A, Q lines gives the neutral point of the P, Q couple.

If, therefore, we have a diagram with thermo-electric power lines for the couples B and A, C and A, D and A, etc., the axis of temperature may be associated with the metal A, and lines may be called the thermo-electric power lines for the metals B, C, D, etc., and for a couple made up of the metals P and Q the P and Q lines serve to determine the constants of the couple. A diagram drawn in this way and giving the thermo-electric powers of the metals B, C, D, etc., supplies, with proper conventions as to sign and regard to the position of the origin, all the necessary thermo-electric data for a couple of any two given metals included in the diagram. Such a diagram is called a *thermo-electric diagram.*

216. The Thermo-electric diagram. For theoretical purposes it is most convenient to take the origin of the axes of temperature and thermo-electric power at the absolute zero of temperature and to express temperature on the absolute scale. The necessary sign conventions of the diagram are most simply understood by considering the case of a particular couple. Take the case of a couple made with the metals A and B, and suppose both junctions to be at the same temperature T. If at the junctions the metal B is at a higher potential than the metal A, then B is said to be positive to A. If this difference of potential is denoted by Π, then we have at each junction a seat of electromotive force Π tending to send a current from B to A, *not* across the junction, but round through the circuit of the couple. These two electromotive forces are equal and opposite in the same circuit, and therefore balance each other. Similarly, since the two ends of each element of the couple are at the same temperature T, the algebraic sum of the electromotive forces associated with the Thomson effects in the A and B conductors is zero in each conductor. Hence, when both junctions of this couple are at the same temperature the total electromotive force in the circuit is zero, although at each junction the potential of B is higher than that of A. Let one of the junctions be now raised in temperature from T to $T + dT$. The electromotive forces set up at the junctions are no longer equal, for the increment of temperature dT at

one junction causes an increment $d\,\Pi$ of the electromotive force at that junction, and their difference is therefore $d\,\Pi$. At the same time the electromotive forces associated with the Thomson effect cease to balance, for there is now a difference of temperature $d\,T$ between the ends of each conductor, and this gives a difference of potential $\sigma_A \,.\, d\,T$ between the ends of the A conductor tending to send a current through B, and a difference of potential $\sigma_B \,.\, d\,T$ between the ends of the B conductor tending to send a current through A. If σ_A and σ_B are assumed to be positive these differences of potential are opposed to each other, and as in Art. 213 the algebraic sum of the electromotive forces in the circuit is given by

$$d\,E = d\,\Pi + (\sigma_A - \sigma_B)\, d\,T,$$

and is seen to be made up of $d\,\Pi$, the increment of Π at the junction, and $(\sigma_A - \sigma_B)\, d\,T$, the increment due to the Thomson effect in the A and B conductors. This electromotive force is for diagram purposes taken as positive or negative, according as the current passes in the standard metal of the diagram from cold to hot or hot to cold. As the current always passes (except in the case illustrated by Fig. 334) at the hot junction from the negative to the positive metals, this convention involves that $d\,E$ is positive when the metal coupled with the standard metal is positive to the standard, and negative when this metal is negative to the standard.

From what has been said we can now deduce the sign conventions that apply to thermo-electric power lines. When, at any temperature T, the increment of electromotive force $d\,E$ for a positive increment of temperature $d\,T$ is positive, then the thermo-electric power $d\,E/d\,T$ is positive. If the line A B in Fig. 332, giving the thermo-electric power line for the A, B couple, be taken as the line for the metal B and the axis of temperature as the line for the metal A taken as a standard, then the diagram is evidently so drawn that at any temperature for any two metals the thermo-electric power of the positive metal of the two is the greater. Thus in Fig. 333, the thermo-electric power lines B B and

C C for the metals B and C indicate that up to the neutral point, represented by O *n*, the metal C is positive

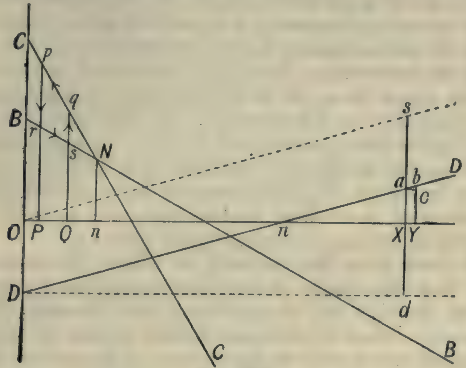


Fig. 333.

to B, but for temperatures above the neutral point B is positive to C. Also, if the temperatures of the junctions of a B, C couple be represented by O P and O Q, then the direction of the current at the hot junction will, in accordance with what has been said above, be from B to C. This may be indicated on the diagram by an arrow from B to C, and, in the same way, the direction of the current in each element of the couple and at the cold junction may be indicated by arrows round the area *s q p r*, representing, as explained in Art. 215, the electromotive force in the circuit.

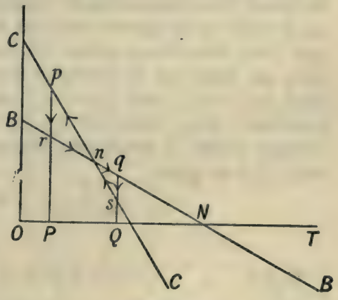


Fig 334.

In Fig. 334, if the temperatures of junctions are

represented by O P and O Q and arrows be drawn, as above, to indicate the direction of the current in the circuit, we get the area $s\,q\,n\,r\,p\,n\,s$ to represent the electromotive force, and, of this area, the part $n\,p\,r$ is positive and $n\,q\,s$ negative, so that the electromotive force is represented by the difference of the two areas. It should be noticed in this figure that the arrow in $s\,q$ is in the direction opposite to that given above for the hot junction. In this case, where the neutral point lies between the junction temperatures the current passes at the hot junction in a direction opposed to that in which the electromotive force at the junction tends to send it so long as the area $p\,n\,r$ is greater than $n\,q\,s$, that is so long as $r\,p$ is greater than $s\,q$.

It follows from the method in which the diagram is drawn that the slope of the thermo-electric power lines indicates the sign and magnitude of the Thomson effect in the metals. Since the thermo-electric power of the more positive of two metals is the greater, it is evident that on the diagram a path with increasing ordinates is also a path of increase of potential. Hence, in going along the B and C lines, Fig. 333, from cold to hot, the potential falls, and therefore, by Art. 210, σ, the coefficient of the Thomson effect, is negative. In the case of the D line, however, the potential rises from cold to hot, and therefore σ is positive. It must be remembered, however, that these lines are supposed to be drawn relative to metal A as a standard, and that, therefore, *unless the metal* A *is one for which* σ *is zero*, all that can be deduced from the above argument is that $(\sigma_B - \sigma_A)$ and $(\sigma_C - \sigma_A)$ are negative and $(\sigma_D - \sigma_A)$ positive. The value of σ_P, or more correctly $(\sigma_P - \sigma_A)$, for any metal P is also represented on the diagram. From Art. 213 we get

$$\sigma_A - \sigma_B = -T\frac{d^2 E}{d T^2}.$$

That is $$\sigma_A - \sigma_B = -T\,\frac{d\left(\frac{d\,E}{d\,T}\right)}{d\,T}.$$

This gives $$\sigma_A - \sigma_B = -T\frac{dy}{dt} \quad \text{or} \quad \sigma_B - \sigma_A = T\frac{dy}{d\,T},$$

where y denotes $d\,E/d\,T$, the thermo-electric power. Now,

taking the A, D line for which $(\sigma_D - \sigma_A)$ is positive, the value of y at temperature T, represented by O X, is represented by X a, and for a small increment of temperature, d T, represented by X Y, the increment dy is represented by $b\,c$. Hence dy/dT is represented by $b\,c/a\,c$, the tangent of the angle which the A, D line makes with the positive direction of the axis of temperature.

Hence, if through the origin O, *taken at the absolute zero of temperature*, a line O s is drawn parallel to D D, then the value of $(\sigma_D - \sigma_A)$ at any temperature T, represented by O X is given by the ordinate X s, for

$$\frac{\mathrm{X}\,s}{\mathrm{O\,X}} = \tan \mathrm{X\,O}\,s,$$

and therefore $\mathrm{X}\,s = \mathrm{O\,X}\,.\,\tan \mathrm{X\,O}\,\sigma = \mathrm{T}\,.\,\tan a\,n\,\mathrm{X} = \mathrm{T}\dfrac{dy}{d\,\mathrm{T}}.$

By an evident alternative construction the line $d\,a$ may also be taken to represent $\sigma_D - \sigma_A$. It will be seen here that when $d\,y/d\,t$ is positive, that is, when the thermo-electric power line slopes upwards like D D, the value of $(\sigma_P - \sigma^A)$ is positive.

Experiment has shown that for lead the value of σ is zero or negligibly small, so that by taking lead as the standard metal A the value of σ_A may be taken as zero, and $(\sigma_B - \sigma_A)$, $(\sigma_C - \sigma_A)$, etc., become simply σ_B, σ_C, σ_D, etc., the Thomson coefficients for the metals for which the lines are drawn. It must be remembered that the magnitude of σ for any metal can be obtained from the diagram in the way described, only if the origin of the axes is taken at the absolute zero of temperature.

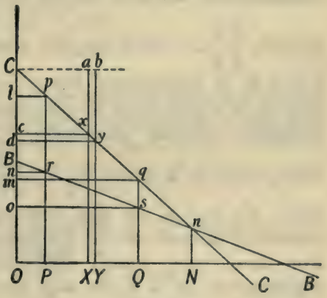


Fig. 335.

The absorption and evolution of energy associated with the Peltier and Thomson

effects in a thermo-electric circuit can also be represented in the diagram.

Take a B C couple with junctions at temperatures T_1 and T_2, represented in Fig. 335 by O P and O Q. As before explained the diagram is drawn so that a path with increasing ordinates is one of increasing potential, and therefore one of *absorption* of energy. Similarly a path with decreasing ordinates is one of evolution of energy. If we suppose unit quantity of electricity to travel round the circuit then the energy absorbed or evolved at a junction is measured by Π, the Peltier coefficient for the temperature of the junction and the energy absorbed or evolved in each of the conductors is given by $\int_{T_1}^{T_2} \sigma \, d\,T$, where σ is the Thomson coefficient for the metal of the conductor. From the diagram of Fig. 333 it will now be clear that for the B, C couple with junctions at temperatures T_1 and T_2 energy is absorbed at the hot junction and in conductor C, and evolved at the cold junction and in conductor B.

The energy absorbed at the hot junction is, for unit quantity of electricity measured by Π_2, the Peltier effect at temperature T_2, and as, by Art. 213, $\Pi = T \frac{d\,E}{d\,T}$ we have $\Pi_2 = T_2 \frac{d\,E}{d\,T}$. Now in the diagram, at temperature T_2, $d\,E/d\,T$ is represented by $s\,q$ and T_2 by O Q or $o\,s$, therefore Π_2 is represented by the area of $o\,s \times s\,q$, that is by the rectangle $o\,s\,q\,m$. Similarly Π_1, the Peltier effect at the cold junction is given by $T_1 \frac{d\,E}{d\,T}$, and is represented by the area $n\,r \times r\,p$ or the rectangle $n\,r\,p\,l$. The energy absorbed in each of the conductors can also be represented. For any very small difference of temperature, $d\,T$, represented by X Y, in say the C conductor, the energy absorbed for unit quantity of electricity is given by $-\sigma_c\,d\,T$. Now σ_c is negative, so that $-\sigma_c\,d\,T$ is a positive quantity, and it is represented by $a\,x$ and X Y

which represents $d\mathrm{T}$, is equal to $a\,b$, therefore $\sigma_c\,d\mathrm{T}$ is represented by the area $a\,x \times a\,b$, that is, by the strip $a\,x\,y\,b$, when $d\mathrm{T}$ is infinitely small. From the geometry of the diagram the strip $c\,x\,y\,d$ is equal to $a\,x\,y\,b$, and may therefore be taken to represent the energy absorbed in the element $x\,y$ of the C conductor. It follows at once from this that the total energy absorbed in the C conductor must be represented by the area $m\,q\,p\,l$. Similarly the energy evolved in the B conductor is represented by the area $n\,r\,s\,o$. Hence the total energy absorbed in the circuit is represented by $o\,s\,q\,m + m\,q\,p\,l$, that is, by the area $o\,s\,q\,p\,l$, and the total energy evolved is represented by $l\,p\,r\,n + n\,r\,s\,o$, that is, by the area $l\,p\,r\,s\,o$. The energy dissipated in the circuit is consequently represented by $o\,s\,q\,p\,l - l\,p\,r\,s\,o$, that is, by the area $r\,s\,q\,p$. This area therefore represents the energy spent in the circuit for each unit quantity of electricity travelling round it. It therefore represents the electromotive force in the circuit, in accordance with the result given above.

The magnitude of the electromotive force in the circuit can also be determined from the diagram by finding an expression for the measure of the area $p\,q\,s\,r$. This area is measured by the product of one half the sum of the parallel sides and the perpendicular distance between them, that is by

$$\mathrm{P\,Q} \times \tfrac{1}{2}(p\,r + s\,q).$$

If the diagram is drawn to scale this value is readily determined.

It can also be reduced to a formula, for P Q represents $\mathrm{T}_2 - \mathrm{T}_1$ and, from the figure, $s\,q/r\,p = \mathrm{N\,Q}/\mathrm{N\,P}$, where O N represents T_n, the neutral point of the B, C couple. That is

$$\frac{s\,q}{r\,p} = \frac{\mathrm{T}_n - \mathrm{T}_2}{\mathrm{T}_n - \mathrm{T}_1} \quad \text{or} \quad s\,q = k\,(\mathrm{T}_n - \mathrm{T}_2) \quad \text{and} \quad r\,p = k\,(\mathrm{T}_n - \mathrm{T}_1),$$

where k is a constant. This gives for E, the electromotive force in the circuit, the expression

$$\mathrm{E} = (\mathrm{T}_2 - \mathrm{T}_1)\,k\left(\mathrm{T}_n - \frac{\mathrm{T}_1 + \mathrm{T}_2}{2}\right)$$

or

$$\mathrm{E} = k\,(\mathrm{T}_2 - \mathrm{T}_1)\left(\mathrm{T}_n - \frac{\mathrm{T}_1 + \mathrm{T}_2}{2}\right).$$

This expression shows that E is zero when $T_2 = T_1$, that is, when both junctions are at the same temperature, and also when $T_n = (T_1 + T_2)/2$, that is, at the point of reversal when the neutral point is the arithmetic mean of the junction temperatures.

Fig. 336 gives a thermo-electric diagram for a number of metals.

The diagram is drawn from recent experimental results by Noll and by Dewar and Fleming. The better possibilities of obtaining pure material, the improvements of electrical methods and apparatus, and the advance in low temperature research have made these results more concordant and reliable than at the time of Tait's researches in 1873.

The right-hand diagram shows a portion of the left-hand one drawn on a larger scale.

217. Formulæ.—The theory of the thermo-electric couple may be given more concisely in the analytical form than by the geometrical method explained above.

The thermo-electric power curves, being practically parabolas, may, when the origin is at the absolute zero, be expressed by the equation

$$E = aT + bT^2,$$

where E denotes the electromotive force in a couple with one junction at absolute zero and the other at temperature T on the absolute scale, and a and b are constants depending upon the metals of the couple.

It follows from this that the electromotive force for a couple with junctions at temperatures T_1 and T_2 is given by

$$E_{T_1}^{T_2} = a(T_2 - T_1) + b(T_2^2 - T_1^2).$$

Further, from the equation

$$E = aT + bT^2,$$

we get, by differentiating,

$$\frac{dE}{dT} = a + 2bT.$$

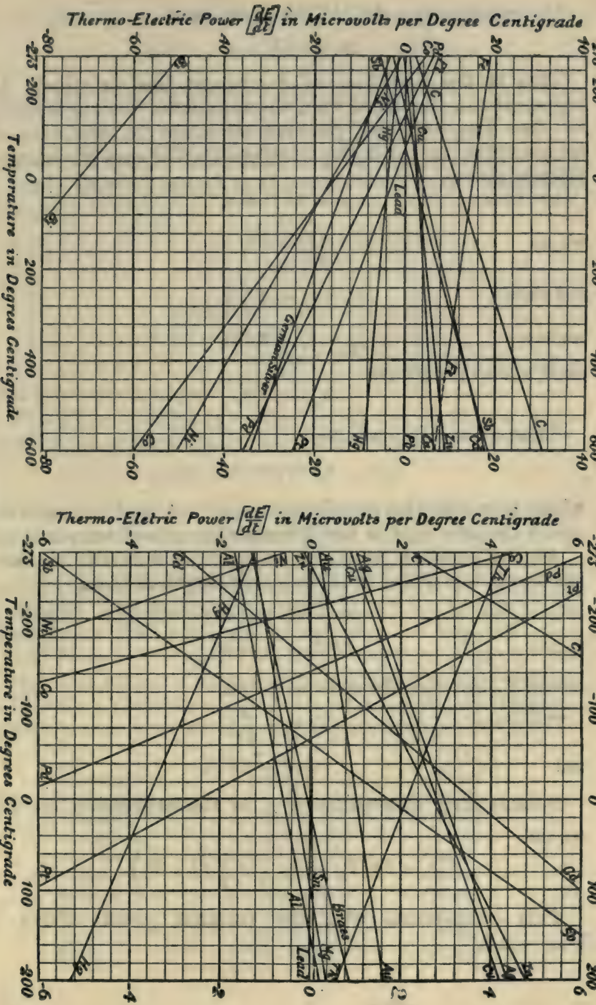

Thermo-Electric Power $\left[\frac{dE}{dt}\right]$ in Microvolts per Degree Centigrade
Temperature in Degrees Centigrade
Common Silver
Lead
Thermo-Eletric Power $\left[\frac{dE}{dt}\right]$ in Microvolts per Degree Centigrade
Temperature in Degrees Centigrade
Brass
Lead

Fig. 336.

Or, representing $d\,E/d\,T$ by y, we have

$$y = 2\,b\,T + a.$$

This is the equation of a straight line in which $2\,b$ represents, in the usual way, the tangent of the angle made by the line and the positive direction of the axis of temperature, and a is the intercept on the axis of thermo-electric power.

Also from
$$y = 2\,b\,T + a,$$
we get
$$\frac{d\,y}{d\,T} = 2\,b,$$
and since
$$\sigma_B - \sigma_A = T\frac{d\,y}{d\,T},$$
we have
$$\sigma_B - \sigma_A = 2\,b\,T,$$
or if $\sigma_A = 0$, we have
$$\sigma_B = 2\,b\,T.$$

Thus at any temperature

$$\sigma_B = y - a,$$

where y is the thermo-electric power at that temperature and a the intercept of the B line on the axis of temperature.

At the neutral point for any two metals the thermo-electric power $d\,E/d\,T$, is zero, and therefore from

$$y = 2\,b\,T + a,$$
we get
$$2\,b\,T_n + a = 0,$$
where T_n denotes the neutral point.

This gives
$$T_n = -\frac{a}{2\,b},$$
and
$$a = -2\,b\,T_n.$$

Substituting this value of a in the expression for $E_{T_1}^{T_2}$ we get

$$E_{T_1}^{T_2} = 2\,b\,(T_2 - T_1)\left(\frac{T_2 + T_1}{2} - T_n\right).$$

This corresponds with the formula given above if $k = -2\,b$.

For the value of Π we have, from the relation,

$$\Pi = T\frac{dE}{dT},$$

by substituting for dE/dT the equation

$$\Pi = aT + 2bT^2.$$

Also in a couple with junctions at temperature T_1 and T_2, the energy absorbed in the circuit per unit quantity of electricity is given by

$$\Pi_2 = aT_2 + 2bT_2^2 \text{ at the } T_2 \text{ junction,}$$

$$-\Pi_1 = -(aT_1 + 2bT_1^2) \text{ at the } T_1 \text{ junction,}$$

$$\int_1^2 \sigma_A\, dT - \int_{T_1}^{T_2} \sigma_B\, dT = \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT$$

$$= -2b\int_{T_1}^{T_2} T\, dT = -2b(T_2^2 - T_1^2)$$

in the two conductors

Hence the total energy absorbed in the circuit is given by the sum of these quantities, and is equal to

$$a(T_2 - T_1) + b(T_2^2 - T_1^2).$$

This it will be seen is, as it should be, the measure of the electromotive force in the circuit.

If the thermo-electric power at 0° C. be denoted by a_0, the thermo-electric power at t° C. is evidently given by $a_0 + 2bt$. The values of a_0 and $2b$ for the metals whose lines appear in Fig. 336 are given below. They are taken mainly from Dewar and Flemings' results, and apply, primarily, to a range of temperature extending from — 200° C. to 100° C. The values are such as to give the thermo-electric power in microvolts per degree centigrade.

	a_0	$2b$
Aluminium	−0·593	·00398
Antimony	1·800	·02817
Bismuth	−72·630	−·08480
Cadmium	3·610	·02365
Carbon	11·170	·03251
Copper	2·815	·00683
Cobalt	−15·582	−·07340
Gold	1·004	·00315
Iron	15·087	−·01330
Steel	10·146	−·01092

	a_0	$2b$
Magnesium	−0·302	·00353
Mercury	−4·460	−·00860
Nickel	−16·050	−·05639
Palladium	−6·743	−·04714
Platinum	−2·493	−·03708
Silver	2·960	·00714
Thallium	2·140	−·00770
Tin	0·047	·00021
Zinc	2·713	·01040

218. Thermo-electric Piles. The thermo-electric current produced by a single couple is usually extremely small—for example, in an iron-copper couple, with its junctions at 0° C. and 100° C. respectively, its E. M. F. is only about 1143 micro-volts*—and the current produced will of course depend, in accordance with Ohm's law, on the resistance of the circuit. Other pairs of metals may, however, be chosen so as to make more effective couples. A couple of bismuth and antimony gives perhaps the greatest E. M. F. for a given difference of temperature between its junctions: for example, at ordinary temperatures for 1° C. difference of temperature such a couple gives an E. M. F. of about 117 micro-volts. The E. M. F. in a thermo-electric circuit may, however, be increased by using, in the same circuit, a large number of couples of small resistance, all arranged so as to give a current in the same direction, just as the cells of a battery are arranged in series. Thus, in a single bismuth-antimony couple the current flows in the circuit so as to cross the hot junction from bismuth to antimony: if, then, a number of couples be arranged as in Fig. 337, with *alternate* junctions *a*, *a*, *a*, and *b*, *b*, *b*, brought near together, then a current may be set up in the circuit

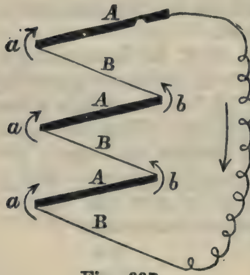


Fig. 337.

* A micro-volt is the millionth part of a volt—that is, about the millionth part of the E. M. F. of a Daniell's cell.

by keeping one set (a, a, a), say, hot, and the other set (b, b, b) cold. For example, if the figure represent an arrangement of bismuth-antimony couples of which the junctions $a, a, a,$ are hot and $b, b,$ cold, then the direction of the current in the circuit is shown by the arrows. The intensity of this current will depend upon the difference of temperature between the two sets of junctions; and when this difference is small, the intensity of the current may be said to be directly proportional to it. This fact suggested the use of an arrangement of thermo couples for the measurement of difference of temperature. If a large number of couples be arranged as in Fig. 337, and small slips of mica or other insulating material be placed between the bars of each couple, then the whole arrangement may be folded up into a bundle and mounted as shown in Fig. 338. The instrument here shown is known as a thermopile, and is used with a low-resistance reflecting galvanometer as a very delicate differential galvanometer.

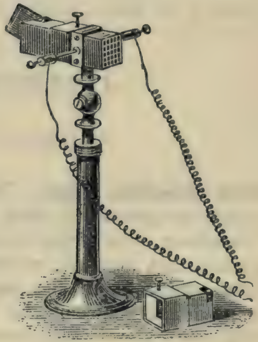


Fig. 338.

CHAPTER XXXIII.

PRACTICAL APPLICATIONS.

219. Electric Bells. These are simple applications of electro-magnetism; they are of two typical kinds, viz., *Tremblers*, which ring continuously as long as the current passes, and *Strikers*, which give only a single sound each time contact is made; the former are commonly used in houses and offices, the latter for signalling purposes. Fig. 339 shows diagrammatically the construction of a trembler. On a wooden base (not drawn) is mounted a small horse-shoe electro-magnet L M N R. The end L of the wire which is wound on it is permanently attached to one of the terminals T of the instrument, and the other to a nut K. Fixed to K is a steel spring S carrying a soft-iron armature I, to the other end of which is fitted an arm terminating in the clapper C. The spring S is so adjusted that when no current passes, the armature rests lightly against the point of a brass screw E, which for purposes of adjustment is made to work in a bearing H, and from this bearing, which is also of brass, a wire proceeds and is permanently attached to the other terminal T′. In the external circuit are a battery B, and bell-push or contact-maker P.

When contact is made a current passes by the route B P T′ H I S K R N M L T. This magnetises the core of the electro-magnet, draws I towards its poles, and makes the clapper C strike the gong G. Contact being thus broken between the point of the screw E and the armature, the core becomes demagnetised, and the armature, by reason

of the elasticity of the spring S, falls back against the point. This re-establishes contact, the core is again magnetised and the armature attracted again immediately to fall back, and so on, the result being a continuous titinnabulation. Two small brass projections are attached to the poles of the electro-magnet in order to prevent

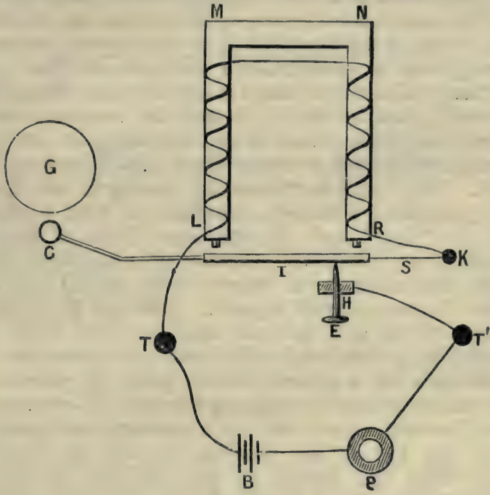


Fig. 339.

actual contact between them and the armature and consequent magnetic retention, which would prevent the armature falling back when the current stopped. It is usual also to tip the screw E with platino-iridium and to attach a minute plate of the same alloy to the armature for this tip to play against; in this way on account of the hardness of the alloy wear and tear are diminished and good contact is maintained.

In *strikers* the screw E and bearing H are absent, the terminal T′ being connected directly to the nut K. Thus, as soon as contact is made by the push P, the armature is

attracted and the clapper gives one stroke, but as there is now nothing to break the circuit automatically it does not fall back until the finger is taken off the push.

220. Telegraphy. To deal anything like thoroughly with this subject would require a book of several hundred pages; all, therefore, we shall attempt is to give an outline of its general principles together with a brief account of one of the most commonly employed systems, viz. the Morse.

The object of the telegraph is to enable two persons at distant places to communicate with one another. The place from which the message is sent is called the *sending* station and that to which it is sent the *receiving* station. Now call the places A and B, and first suppose that it was only required to send messages from A to B and not from B to A. This could be effected by placing A and B in a circuit containing a battery, providing A with a contact-maker or "key" and B with some form of receiving instrument or "recorder" capable of responding to the current, the message of course being spelled out by a system of preconcerted signals. But as it is always necessary to be able to send messages *both* ways there must also be in the circuit a key at B and a recorder at A, and this calls for a special device, partly to prevent the sender transmitting the messages through his own recorder (which, however, would not much matter), and partly to prevent the key at the receiving station breaking the circuit when the person at the sending station was endeavouring to transmit. Fig. 340 shows the arrangement commonly adopted; it should be noted that in this as in all telegraphic systems only *one* wire (called the *line*) is used to connect the stations, the circuit being completed through the earth.

Each of the stations A and B is provided with a battery (zc, $z'c'$) as well as with a key (K, K′) and recorder (R, R′), The zinc plates of both batteries are in good electrical communication with the earth by means of earth-plates E E′ while their copper plates are capable of contact with the front ends of the keys. The recorders are placed in branch circuits as shown, and when a key is *not* depressed by the sender (as at K′) its back end is kept, by means of

a spring, in contact with the recorder while it is lifted from the battery. On the other hand, when it *is* so depressed (as at K) its fuse-end makes contact with the battery while it is lifted from the recorder. The result is that each station is always in readiness to receive messages from the other and that the sender does not transmit through his own recorder: the figure shows A transmitting to B

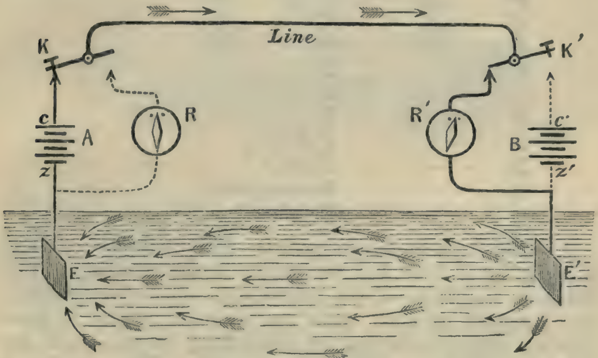


Fig. 340.

In the Morse system above referred to, the recorder consists essentially of an electro-magnet and armature similarly arranged to those of the "striker" form of electric bell (see preceding section), the armature thus remains drawn towards the magnet just so long as the key is pressed down. In the most modern form of recorder, known as the *ink-writer*, the armature is attached to a small wheel covered with printers' ink and under this is drawn by means of clockwork a long narrow strip of paper; when the armature is not attracted the wheel remains clear of the paper, but when attraction occurs it comes in contact therewith and prints a dot thus . or a dash thus — according as the duration of the current is "short" or "long." The letters of the alphabet, stops,

etc., are represented by combinations of dots and dashes according to the annexed code:—

A	. —	P	. — — .
B	— . . .	Q	— — . —
C	— . — .	R	. — .
D	— . .	S	. . .
E	.	T	—
F	. . — .	U	. . —
G	— — .	V	. . . —
H	. . . .	W	. — —
I	. .	X	— . . —
J	. — — —	Y	— . — —
K	— . —	Z	— — . .
L	. — . .	Full stop	. . . . . .
M	— —	Comma	. — . — . —
N	— .	Interrogation	. . — — . .
O	— — —	Hyphen	— . . . . —

Another form of Morse recorder is the *sounder*. In this no printing is done, but the person at the receiving station listen to the clicks of the instrument and notices whether the intervals between them are short or long. A short interval corresponds to a dot and a long one to a dash, the code of course being the same as above. The sounder is much more difficult to work with than the ink-writer, as the ear can interpret the signals only after careful training.

It frequently happens, especially over long distances, that the current from the sending station is too weak to work the recorder. To remedy this a delicate auxiliary electro-magnet, called a *relay*, is made use of, it and the aforesaid current being made to pass through this *and not through the recorder at all*. Each time the current passes the relay armature is attracted, and matters are so arranged that this completes a local circuit through the recorder and a battery at the receiving station. It is thus this latter battery which is the *direct* producer of the signals. Fig. 343 still shows the arrangement of the line, etc., *only that R, R′ now represent the relays*, the recorders and local batteries not being depicted.

The key (K or K′, Fig. 340) employed in the Morse system is of special construction and is shown in detail in Fig. 341. Its centre A works on a pivot connected with the "line"

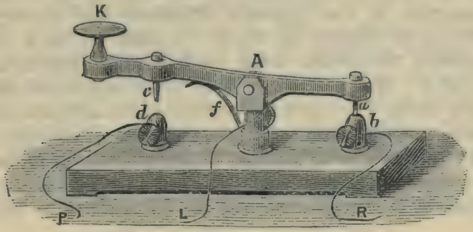


Fig. 341.

(*cf.* K, K′, Fig. 340); when not pressed down its back-end *a* is kept by the spring *f* in contact with the terminal *b*, which runs to the relay at its own station (K′, Fig. 340), while when pressed down its fuse-end *c* makes contact with the terminal *d* running to the battery (K, Fig. 340).

221. Duplex Telegraphy. In order that two stations may transmit messages to each other simultaneously two arrangements are adopted in practice. The principle of the differential method may be gathered from the following considerations. Suppose the coil of an electromagnet is wound (Fig. 342) with two equal wires in opposite directions, and that these two wires are joined to two different batteries. Then, if the currents supplied by the two batteries are equal they will neutralise each other, and the core of the magnet will remain unmagnetised. Moreover, if the two currents are slightly different the magnetisation produced will depend on the resultant of the two currents. Fig. 343 shows how this idea is used in duplex telegraphy, the details of keys and relays, etc., being left out for simplicity's sake. The line wire *l* connects the two stations

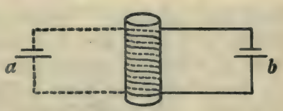


Fig. 342.

a and *b*. The circuit, after leaving the key *k*, divides itself into two, one wire going round the electromagnet in one direction and through the resistance R, which is equal to the line resistance, back to the battery; the other going round the electromagnet in the opposite direction and through the line wire *l* to station *b*, where it joins on to a similar arrangement, the return being made through the earth connection at *b*. Now let us consider what happens in the two following cases: (1) when one station alone is transmitting; (2) when both stations are working simultaneously. In the first case, suppose that *a* alone is sending messages; then, because the resistances of R and *l* are equal, the electromagnet *m* is not affected, for the currents in its coils are

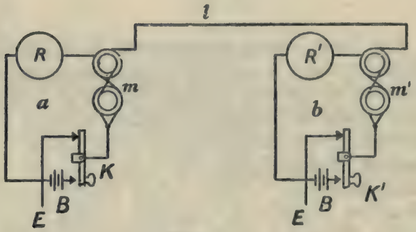


Fig. 343.

equal and opposite. Since only one of the electromagnet circuits at *b* is closed it is evident that the receiving instrument at *b* will be influenced by the current sent out from *a*. (2) Next suppose that both stations are working simultaneously. When K is depressed and K′ makes contact at its back end the effect produced will be the same as that traced above. When both keys are depressed, then, owing to the equality of the resistance of the two systems, there will be no current in the wire, and so the receiver at each station will be worked by its *compensating* current, as the current through the resistance R, R′ is called. And even in the case when the line current is not exactly zero, owing to leakages, the effective current at *a* or *b* will be the algebraic

sum of the currents which flow round the respective electromagnets. It is found that this effective current, when both keys are depressed, is almost equal to the line current when only one key is depressed—so that there is practically no difference in the signals recorded.

The other system which is used in duplex telegraphy depends on the principle of the Wheatstone bridge. From the figure (Fig 344), supposing station *a* alone is working, it is clear that, if the resistances in *c d* and *c e* are equal, or bear the same proportion as the resistances in the line and in R, the potentials at *d* and *e* will be equal, and those at *d'*

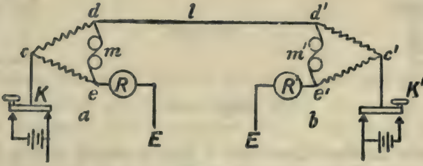


Fig. 344.

and *e'* unequal. Hence *m* will be unaffected, and *m'* will be actuated by the current. If proper balancing is made at both ends, then the equality of potentials between *d* and *e*, say, will be disturbed, due to the current from *b*, when both stations are working. Similarly, the balance of potentials at *d'* and *e'* will be disturbed only by the current sent from *a*. In this way it is seen that both stations can transmit messages without disturbing the receiver at the sending end by the current sent out from that end.

222. Submarine Cable Telegraphy. A submarine cable is really a large condenser, one plate of which is the metallic core and the other the protecting sheath and the surrounding water. If one of the systems above described were employed to transmit messages along such a cable it is apparent that much of the energy of the batteries would be used in charging the cable and so reducing the speed of transmission. Consequently, it is found more advantageous to use another system in cable telegraphy, the principle of

which is illustrated by the diagram (Fig. 345). When the key K is depressed the first plate of the condenser at *a* is charged. It will be seen from the figure that this will induce a charge of an opposite sign on the condenser attached to the line, and this in its turn will induce a charge of the same sign as the original one on the first plate of the condenser at *b*. Thus a pulse will be sent through the receiving

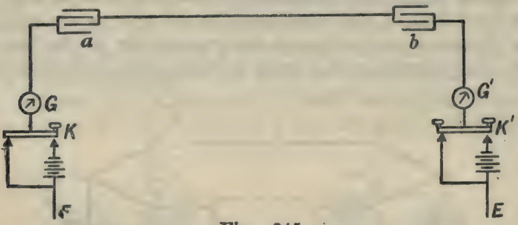


Fig. 345.

instrument at the other end. The receiver is a mirror galvanometer, and the messages are read from the movements of the spot of light on the scale. An automatic recording instrument, called a *syphon recorder*, invented by Lord Kelvin, is now used for receiving the messages. It is practically a recording form of the moving coil galvanometer. By suitable arrangements of condensers placed in the arm of a Wheatstone's bridge, cables are now worked on the duplex bridge system.

223. The Telephone. This instrument, which was invented by Prof. Graham Bell in 1876, depends for its sending power upon the fundamental fact of "magneto-electricity," and for its receiving power upon that of "electro-magnetism"; the former is that if a coil of wire be placed in a magnetic field and if anything be done to alter the strength of the part of the field where the coil is situated, then so long as that alteration is in progress a current is induced in the coil, which goes one or the other way round according as the said part of the field is being strengthened or weakened; and the latter is that if a current circulates round a magnet it strengthens or weakens the

latter according to whether it goes one way round or the other.

Before describing the instrument, let us consider the simple device of Fig. 346. N S is a bar magnet and I a piece of soft iron in front of one of its poles N. When I is introduced into the field it produces a re-arrangement of the lines of force, and any motion of I towards or away from N produces a further distortion of the lines of force,

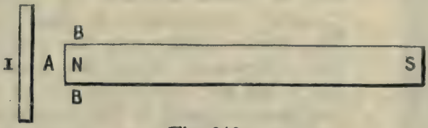


Fig. 346.

causing them to open out or close in near the end of the magnet according as I is moved away from or towards N. Now suppose N surrounded by a bobbin coiled with insulated wire, then, when I moves, lines of force will cut the turns of the coil transversely, and therefore an electromotive force will be set up which will be proportional to the rate at which the lines cut the turns of the coil. Hence, if I vibrates to and fro in front of N, alternating currents will be generated in the coil if its circuit is closed. Moreover, if the ends of the latter be connected with those of another on a distant bobbin similarly surrounding one of the poles of another bar-magnet, in front of which is another piece of iron, these varying currents will alternately strengthen and weaken that pole and cause the iron (if suitably mounted) to vibrate in unison with the first piece,* though with much less vigour because by no means all the energy of the first is conveyed to the second, a great deal being dissipated as heat, etc., in the wires.

The above is the essence of the telephone; Fig. 347 shows the actuality in section. A A is a cylindrical

* Approach of the first piece may of course produce either approach or recession of the second according to the polarity of the magnets and the way the bobbins are wound.

bar-magnet fixed in a wooden case, and C C a bobbin surrounding one of its poles, and wound with fine insulated copper wire, the ends of which are connected to the terminals F, F. In front of the pole which the bobbin encircles, and supported at its edges by the wooden case is a very thin diaphragm of soft iron D, and in front of this a mouthpiece. At the distant station is a precisely similar contrivance, and the two are in communication by "line," and "earth" connections to their terminals. The sender speaks into the mouthpiece of his instrument while the receiver holds his to his ear; the vibrations of the voice are communicated to the diaphragm of the former and these, as explained in the preamble, to that of the latter. Hence the receiver hears the words of the speaker, though with diminished strength and a somewhat "tinny" clang.

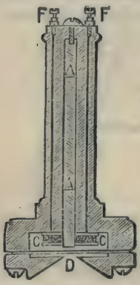


Fig. 347.

It is now usual for both sender and receiver to be supplied with *three* instruments, viz., one for each ear and a larger one for the mouth. The inconvenience of shifting is thus avoided and better effects obtained.

224. Microphone and Carbon Transmitter. The Graham-Bell transmitter has now been superseded by a much more efficient arrangement due to an ingenious contrivance invented by the late Professor Hughes. He found that if a telephone receiver be placed in series with a battery and a bad contact resistance made of three loosely fitting pieces of carbon, any motion of one of the pieces of carbon produced a sound in the telephone. The device he adopted for showing their effect was termed a microphone. Two pieces of carbon A, B (Fig. 348), are fixed to a

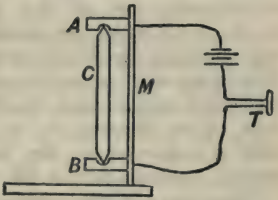


Fig. 348.

vertical rod of wood M and a carbon rod C fits loosely between them. The whole is connected in series with a battery and a telephone receiver T. Any motion of C varies the resistance of the circuit, and therefore causes a sudden change in the current which is detected by the telephone. It is found that this apparatus is exceedingly sensitive, and if a person speaks in front of it the sound is readily communicated to the telephone.

The transmitter now in general use is an application of the principle of the microphone. The vibrating plate is a thin plate of carbon fixed round its edges, between it and another piece of carbon is a loose packing of granular carbon. This forms the bad contact, and the whole arrangement is enclosed in a suitable case, and the carbon plate protected on the outside by a piece of wire gauze.

225. Dynamos. From what has been said above it is known that if a closed circuit be moved in a magnetic field in such a way as to vary the magnetic flux through the circuit then a current is induced in the circuit. In the case of the earth inductor, described in Art. 164, it is shown how, by rotation of a coil of wire round an axis at right angles to the direction of the magnetic field, a continuous variation in the magnetic flux through the coil may be obtained. It has also been explained that during each complete revolution of the coil the direction of the induced current changes as the coil passes through the position at right angles to the direction of the lines of force, but that by the action of a split ring commutator and spring brushes resting on the ring a continuous current in a fixed direction may be obtained. Such an arrangement constitutes a simple form of dynamo. A dynamo is thus a machine for the production of a current by means of magneto-electric induction. Theoretically, the essentials of such a machine are a magnetic field and the coil in which the induced current is produced. The magnetic field usually employed is the field between the poles of a suitably constructed magnet. In a very few cases a permanent steel magnet is employed, but in all

modern dynamos an electro-magnet is used. Fig. 349 shows diagrammatically the arrangement of the iron circuit of *field magnets* of a dynamo. The iron cores C, C, are fixed in the bed-plate B, and carry the pole pieces *p*, *p*, which are so shaped as to give a fairly uniform field in the suitably shaped space F between the poles. The field magnet coils are wound round the cores C C, the number of turns being adjusted to suit the strength of current they will carry and the magnetic quality of the iron. The iron circuit, or as it is sometimes called the *magnetic circuit*, of the dynamo may be cast all in one piece or in separate pieces, but when separate the pieces must be closely and accurately fitted together in order to ensure the efficient action of the electro-magnet. The current through the field coils may be obtained from a suitable battery, in which case the dynamo is said to be *separately excited.* It is evident, however, that if the field magnets have once been excited there will, even when no current passes through the field coils, be a weak magnetic field in the space F. If a coil is rotated in this field a weak current is induced in the coil, and if the field coils be connected either in series or in parallel with this coil, then this weak current increases the magnetism of the field magnets and therefore strengthens the field at F. This increase in the intensity of the field causes an increase in the induced current, and this again results in a further increase in the field intensity. This action may go on until the field magnets approach saturation, when an increase in the current gives little or no increase in the intensity of the field at F. By properly adjusting the windings of the coil rotating between the poles and the field

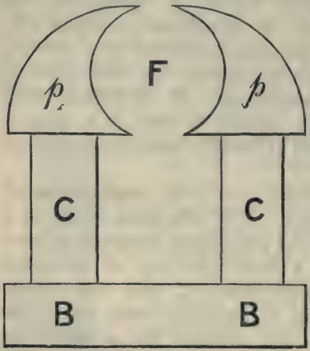


Fig. 349.

coils it is thus possible for a dynamo to self-excite its field magnets. Dynamos which work in this way are said to be *self-excited*—most modern dynamos are of this type. When the rotating coil and the field coils are in series the machine is said to be *series wound*, when they are in parallel or when the field coils are connected as a shunt on the rotating coil the machine is *shunt wound*. When the winding combines the series and shunt methods the machine is said to be *compound wound*.

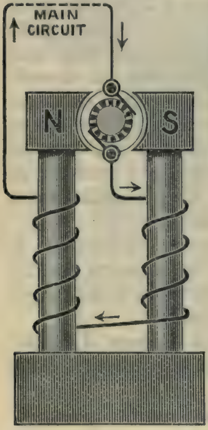


Fig. 350.

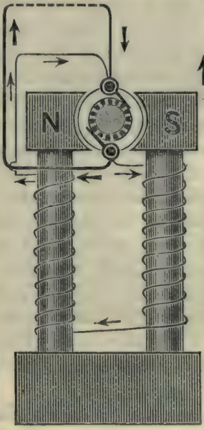


Fig. 351.

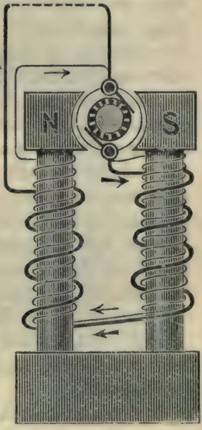


Fig. 352.

Figs. 350, 351, 352, show diagrammatically the connections of the coils in a series wound, shunt wound, and compound wound dynamo.

The coil which rotates in the field F is usually wound on a properly constructed frame containing soft laminated iron, the object of the soft iron being to increase by its permeability the magnetic flux through the coil. This frame with the coil constitutes the *armature* of the dynamo. The armature is mounted by the spindle, fitted through its frame

on bearings, and may be rotated from an engine by a belt fitted round the fly wheel of the engine and on a pulley on one end of the armature spindle, or, as in some cases, the engine may be directly coupled to the armature without the intervention of a belt. The mechanical work done in driving the armature against the resistance due to the field at F is spent in producing the electrical energy developed in the circuit of the dynamo.

The coils of the armarture are not closed circuits. In the simplest case the armature coil may be one continuous wire, one end of which is connected to one half of the split ring commutator and the other end to the other half. Brushes or springs rest upon the commutator ring and they are connected to the terminals of the external circuit, so that the closed circuit in which the current is induced consists of the armature coil as internal circuit and the external circuit connected to the brushes. In the case of most dynamos the armature coil is wound in a number of separate sections, and the commutator ring is split into a corresponding number of sections or bars.

The commutator ring, with each section insulated, is fitted on the armature spindle and the brushes are usually carried by a brush holder, which allows them to be raised off or pressed on the ring at the proper points.

Fig. 353 shows a simple form of the modern dynamo. The student should be able to make out the various parts for himself.

A dynamo, such as has been described above, gives, by the action of the commutator, a *continuous* or one-direction current in the external circuit, and is called a *continuous current* dynamo. If, instead of the commutator, we have two insulated rings or collars fitted on the spindle, one ring connected to one end of the armature coil and the other to the other end, then by means of brushes, resting one on each of these collars, we can obtain a current which changes direction in the circuit twice during each revolution of the armature. Such a current is an instance of what is called an alternating current, and a dynamo supplying an alternating current is called an *alternator*. The number of times a current *alternates* during a complete revolution of

the armature depends upon the arrangement of the field magnets and the coils of the armature.

226. Electromotors. In a dynamo mechanical energy is expended in causing the rotation of the armature in the magnetic field due to field-magnets in the space between the pole pieces—and by its action mechanical energy is

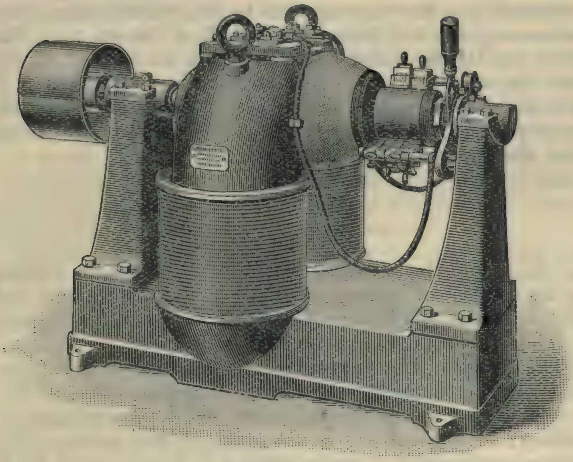


Fig. 353.

expended in producing electrical energy. In an electromotor the reverse action takes place. A current from a battery, or another dynamo, is passed through the coils of the armature, and the field magnets (either in shunt or in series), and the couple acting on the armature, due to the force exerted on its coils carrying current in a magnetic field, causes its rotation. If the armature is properly connected to suitable mechanism the torque causing rotation may be made to do work. In construction a motor is essentially the same as a dynamo—in fact any well constructed dynamo will run as a motor, but there

are numerous differences in design necessitated by the very different uses to which dynamos and motors are put.

When a motor is running, and the rotating armature is made to do work, then the motor is a sink of energy in the circuit, and is therefore a seat of back electromotive force. This is also evident from the fact that as the armature rotates the machine acts as a dynamo, and tends to produce a current in a direction opposed to that which causes it to run as a motor, and thus a back E. M. F. is set up. The more rapidly the motor runs the higher the back E. M. F., and the smaller the current in the circuit. Thus if R denote the resistance of the circuit, E the E. M. F. of the battery or dynamo, and e the back E. M. F. of the motor, then the current in the circuit is given by

$$C = \frac{E - e}{R}.$$

Also, under the same conditions, E C denotes the *activity* of the dynamo or battery, e C the activity of the motor, and $EC - eC$ or $C(E - e)$ the loss of power due to the heating of the circuit by the current.

The electrical efficiency of this dynamo-motor or battery-motor arrangement is evidently given by the ratio eC/EC or e/E, for, of the power E C given out by the dynamo or battery, an amount e C is absorbed by the motor. This ratio approximates to unity when e is nearly equal to E. Hence the more rapidly a motor runs, that is the lighter the load at which it works, the greater the efficiency of the arrangement and the less the loss of power in the circuit.

The activity, e C, of the motor refers to the power it absorbs in the circuit as an equivalent of the mechanical work done by the motor. If we include the energy dissipated as heat in the coils of the motor then the total power absorbed from the circuit is $eC + C^2 r$, where r denotes the resistance of the coils of the motor. Or, if e' denote the difference of potential between the terminals of the motor, then $e'C$ gives the total power absorbed by the motor, that is, $e'C = eC + C^2 r$. The ratio of the power absorbed as the equivalent of mechanical work done by

the motor to the total power absorbed by the motor, that is, the ratio $eC/e'C$ or e/e' is known as the *electrical efficiency* of the motor. The mechanical work done by the motor as the equivalent of eC is not all useful external work; part of it is spent in the motor doing internal work against the friction of the bearings, air resistance, and the inertia of the moving parts of the machine. If P denote the power of the motor to do external work, as measured say by a suitable brake, then the ratio P/eC gives what is called the *mechanical efficiency* of the motor.

When the connection is first made between a motor and the battery or dynamo which is to run it there is no back E. M. F. in the circuit, and an extra resistance has to be put in to prevent the production of an excessively high current, then, when the motor has got up speed, the resistance may be taken out. For this purpose all simple motors are provided with a starting switch.

Motors may be series wound, shunt wound, or compound wound, just as dynamos are. Most small toy motors are series wound, owing to the simplicity of the winding, but series wound motors of a large size are not much used. They are much more variable in speed than shunt wound machines, which, within certain small limits, automatically regulate their speed according to the load. Motors with compound winding may be adjusted to have practically a constant speed at all loads.

227. Ammeters and Voltmeters. In engineering practice it is often necessary to measure the current in a circuit in ampères, or to determine the difference of potential in volts between two parts of a circuit; and in lighting and power stations it is always necessary to know the conditions of the circuits fed by the machines, and to be able to tell at a glance the voltage at which any machine is working and the current generated by it. With a suitably graduated scale it is evident that a galvanometer could be used for each of these purposes. When the galvanometer is placed in the circuit, the readings on its scale may be taken to be proportional to the current if it is in series, and proportional to the potential difference between its terminals if it is in parallel. Now, it is essential that the resistance of a

current measurer should be small, so that the introduction of the instrument into the circuit does not alter the current if, as is sometimes the case, it is not necessary to keep the current measurer always in the circuit. On the other hand, the resistance of a potential difference measurer should be high, in order that it may not disturb the distribution of current in the circuit when it is switched on. The galvanometer, however, is too delicate, too sensitive, and too cumbersome to be used for these purposes by engineers, and special modifications of it have been devised, where scales indicate current directly in ampères and potential differences in volts. Generally speaking, then, an ammeter is a special type of a low resistance galvanometer, and a voltmeter is a particular form of a high resistance galvanometer. The general principles of the construction of these instruments are, then, precisely the same, except that, as will be pointed out further on, there is a distinct kind of voltmeter which is an electrostatic instrument. The voltmeters and ammeters in common use may be roughly classified as electromagnetic and hot wire instruments. Of the electromagnetic instruments we shall describe briefly two types—the moving coil and the moving magnet instruments. The description, of course, applies to both voltmeters and ammeters.

228. Moving Coil Ammeters and Voltmeters. The moving coil type of instrument (Fig. 354) is essentially a form of D'Arsonval galvanometer. The coil, instead of being suspended by fine wire, is mounted on a pivot working in jewels and fitted with hair-spring controls. Inside the coil there is mounted, co-axial with it, a fixed soft iron cylinder. The coil and soft iron cylinder are placed between the poles of a permanent horse-shoe steel magnet. The arrangement of the magnet and the fixed iron core ensures a uniform magnetic field for the region through which the coil rotates, and therefore the index attached to the coil moves over equal distances for equal increments of current. Further, the action of the instrument is perfectly dead beat, for the coil is wound on a light copper frame which, therefore, itself forms a complete electrical circuit in which a feeble current is generated when the system is moving. The direction of this current is such that the

magnetic field produced by it is opposite to the field of the permanent magnet, and therefore tends to stop the motion of the coil. Instruments of this type can only be used for continuous currents. Of course, in these and other instruments, it is not necessary to send the whole of the current through the coil. The greater part of it, in the case of

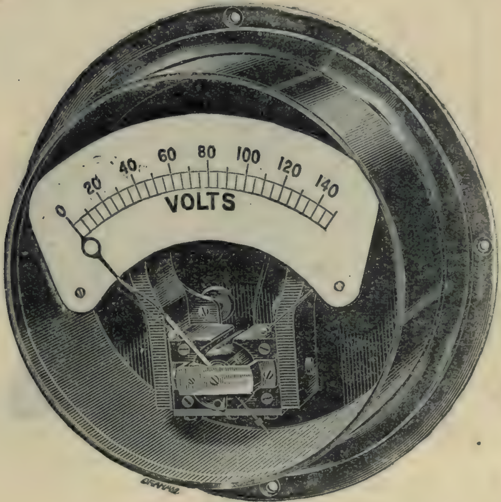


Fig. 354.

ammeters, is usually sent through a low resistance shunt, and only a definite fraction passes through the moving coil.

229. Moving Magnet Ammeters and Voltmeters. If a piece of iron is introduced into a non-uniform magnetic field it has a tendency to move into the stronger parts of the field. The amount of motion thus produced depends on the strength of the field, which in its turn, if produced by a current in a coil, depends on the strength of that

current. A thin piece of iron is easily magnetised to saturation, and therefore, when placed in a sufficiently strong field, even though that field may vary, it will behave as a magnet of practically constant strength. The force on the piece of iron under those conditions will then be

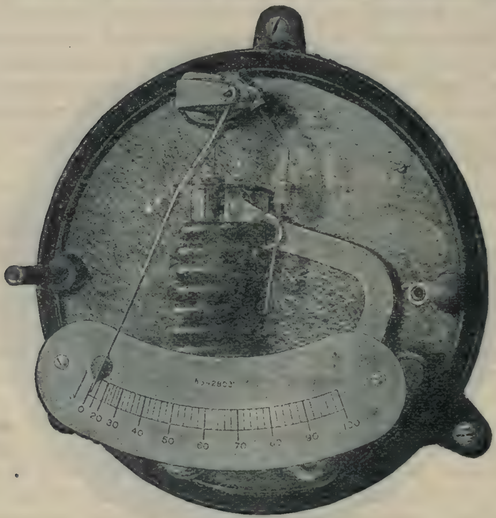


Fig. 355.

proportional to the intensity of the current producing the field. Hence the displacement of the iron may be taken as a measure of the current—whether it be direct or alternating. Several instruments acting on this principle have been devised by different inventors. One form is shown in Fig. 355.

230. Hot Wire Voltmeters and Ammeters. The next class of instruments are called hot wire ammeters and voltmeters. When dealing with the heating effect of a current, it was pointed out that the heat produced in a

wire through which a current C is flowing is proportional to C^2. It is, therefore, independent of the direction of the current, so that any measuring instrument depending on this principle can be used for measuring direct and alternating currents. The effect of the heat on the wire is to cause it to elongate, and the extension produced may be employed to indicate the strength of the current producing it. Fig. 356 illustrates the principle of the working of these instruments. A B is a fine platinum wire stretched between A and B. Another wire is attached at D and fixed at the other end C. This wire is thus kept taut. A piece of silk fibre is attached to C D and passes over a pulley P to which the index, I, is fixed. The other end of the fibre is fastened on to a weak spring S. It will be evident from the figure that any elongation of A B produces a sag in it, which, being transferred to the pulley by C D, causes the index to move over the scale.

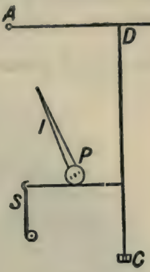


Fig. 356.

The chief drawback in instruments of this class is that the pointer does not indicate immediately the proper value of the current, because the wire takes some time to attain its final temperature.

This temperature is reached when the rate of loss of heat by radiation from the surface of the wire is equal to the rate of generation of heat by the current. The relation between the elongation and the strength of the current is easily obtained.

Let $C =$ current in ampères,

$r_0 =$ radius of the wire at temperature $t_0°$ Centigrade, the temperature of the air,

$r =$ radius of the wire at temperature $t_0°$, the temperature of the wire,

$l_0 =$ length of the wire at temperature $t_0°$,

$l =$ length of the wire at temperature $t°$,

$s =$ specific resistance of the wire at $t°$.

ϵ = emissivity of the wire at $t°$,

a = coefficient of linear expansion of the wire.

Then, when the steady state has been reached, the rate of loss of heat by radiation

$$= 2\pi r l \epsilon (t - t_0) \text{ calories.}$$

The rate of generation of heat by the current

$$= \frac{s l}{\pi r^2} C^2 \times \cdot 24 \text{ calories,}$$

$$\therefore\ 2\pi r l \epsilon (t - t_0) = \frac{s l}{\pi r^2} C^2 \times \cdot 24,$$

or

$$(t - t_0) = \frac{s}{\epsilon} \cdot \frac{C^2}{\pi^2 r^3} \times \cdot 12.$$

Now, if x = the elongation produced in the wire,

$$x = l_0 a . (t - t_0) \text{ approximately,}$$

$$\therefore\ x = l_0 a . \frac{s}{\epsilon} \cdot \frac{C^2}{\pi^2 r^3} \times \cdot 12,$$

or, since

$$r = r_0\{1 + a (t - t_0)\} \text{ approximately,}$$

$$x = l_0 a . \frac{s}{\epsilon} \cdot \frac{C^2 \times \cdot 12}{\pi^2 r_0^3 \{1 + a (t - t_0)\}^3}.$$

Since the wire is very fine, r will not differ very much from r_0, and hence we might employ the first formula to calculate x. Both s and ϵ vary with temperature, and ϵ is found to depend also on the dimensions of the wire. But for the same wire, for a *small* excess of temperature above the ordinary temperature of the air, experiment shows that the ratio of s to ϵ is fairly constant, generally, however, this ratio varies with the temperature of the wire. We could then write $c = \kappa \sqrt{x}$, where κ is a function of the temperature of the wire, and might be termed the controlling factor of the instrument. It must be clearly understood, however, that this is a mere theoretical result and that it is always necessary to calibrate and standardise the scales of ammeters and voltmeters constructed on this principle. A convenient form of this type of instrument is shown in Fig. 357.

An arrangement for measuring very feeble alternating currents, such as those generated in a telephone circuit, is described in the *Philosophical Magazine* for May 1904. The currents are made to pass through a fine wire, which is placed immediately opposite the radiation receiver of a

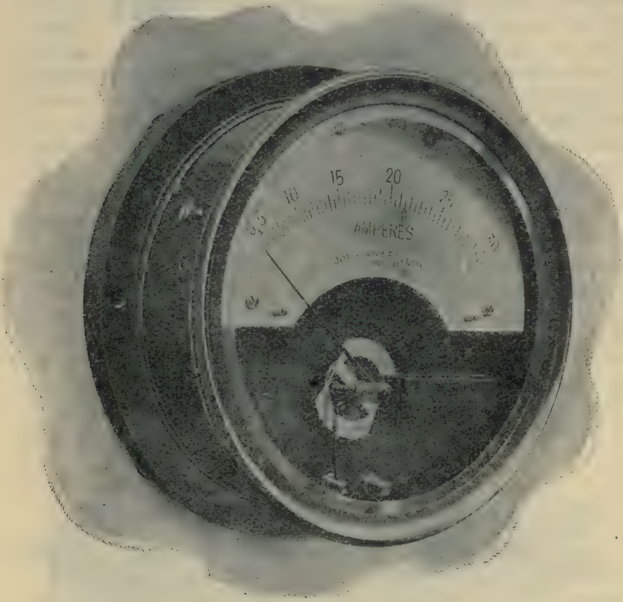


Fig. 357.

radio-micrometer. The heat generated in the wire is radiated away and measured by the deflection of the mirror of the radio-micrometer.

231. Electrostatic Voltmeters. Since it is absolutely necessary that a voltmeter should absorb little or no

current from the circuit, it seems obvious that an excellent type of instrument would be one depending on the electrostatic attraction of two conductors, and it is shown, in the

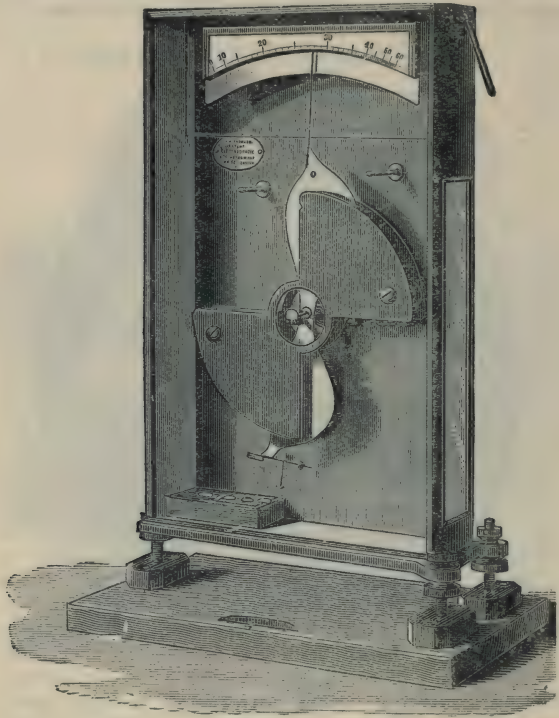


Fig. 358.

chapter dealing with electrometers, how this principle is adopted in the construction of the quadrant electrometer. If the needle were pivoted instead of being suspended by a

fibre, and a pointer was attached to the needle, then, after calibration of the scale, the instrument could be used as a voltmeter. It was also pointed out in the chapter referred to, that under certain conditions the reading on the electrometer scale was proportional to the square of the potential difference between its terminals. A voltmeter constructed on this pattern could, then, be used for measuring potential difference in alternating as well as continuous current circuits. It would have the additional advantage of being entirely unaffected by a magnetic field, of having no self-induction, and of being entirely free from temperature errors. Such an instrument is shown in Fig. 358. It will be seen that it consists essentially of an air condenser of variable capacity, one plate of which is pivoted, and it is the motion of this movable plate or needle which causes the change in the capacity of the instrument. A pointer attached to the needle indicates volts on a properly calibrated scale.

232. Fuses. In order to prevent too large a current from accidently passing through an ammeter or voltmeter, these instruments are often supplied with fuses, which are made of a wire of such material and diameter that a current slightly less than the maximum for the case in question will melt the wire. Fuses are also fitted to all switches which tap the lighting mains in a building. The diameter d, in inches, of a wire which will fuse with any given current, C ampères, is given approximately by the relation

$$d = \left(\frac{C}{a}\right)^{\frac{2}{3}}$$

where a is a constant depending on the material.

233. Electroplating and Electrotyping. The term *electroplating*, which was originally applied to the process of coating a baser metal with silver, is now extended to include the coating of any object with a coherent layer of any metal; in practice, however, the metals employed as the coat are usually either silver, gold, nickel, or copper, while the object may be either another metal, *e.g.* iron, or some non-conducting material such as wood or plaster-of-Paris.

The process is an application of the principle of electrolysis. The object to be coated is attached to the negative pole of a battery or dynamo, and immersed in a suitable solution or "bath" of some salt of the metal it is desired to deposit thereon; a plate of the last-named metal is also immersed in the solution and is attached to the positive pole. If the object be a non-conductor its surface must first be rendered conducting by covering it with a thin layer of graphite or sulphide of silver. When the current passes, the object (the *kathode*) receives the desired deposit from the "bath," while the other plate (the *anode*) becomes dissolved, thus maintaining the solution of approximately constant strength.

When copper is the metal to be deposited it is not unusual to do without the external battery and simply to make the object the positive or high-potential plate of a Daniell cell.

Great care has to be taken to employ solutions of appropriate composition and strength, and also properly to regulate the "density," as it is called, of the current, *i.e.* its strength per unit area of the object; otherwise the deposit is not adherent.

The most important electroplating industry is *electro-silvering*. In this case the salt is a double cyanide of silver and potassium, the bath being prepared by dissolving one part (by weight) of silver cyanide and two of potassium cyanide in 40 of distilled water. The things to be silvered have to be first freed from grease by means of a solution of potassic hydrate, then washed in weak sulphuric acid, then "scoured," *i.e.* washed in a mixture of sulphuric and nitric acid and salt, and finally amalgamated by immersion in a dilute solution of mercuric sulphate. The most appropriate current "density" is from four to five ampères per square foot, and this is maintained until the deposit amounts to about one ounce per square foot, which corresponds to a thickness of about $\frac{1}{800}$ inch. On removal from the bath the silvered articles are immersed in a solution of potassium cyanide, washed in boiling water and dried in sawdust.

Electrogilding is the art of coating objects with gold.

The bath is a solution of the double cyanide of gold and potassium. The details are similar to those for electrosilvering except that the current employed is weaker (about one ampère per sq. ft.) and the deposit thinner.

Electronickeling is the art of coating objects (usually of steel, as the handle-bars of bicycles) with nickel. The bath is a slightly acid solution of the double sulphate of nickel and ammonium. Prior to its immersion in the bath the object is cleaned and polished; the current is made fairly strong to begin with and then slackened: a good coat is of about $\frac{1}{1000}$ inch in thickness.

Electrocoppering is the easiest of all the plating processes. The bath is simply a strong and very slightly acid solution of copper sulphate; it is best to have the current at first very weak and subsequently increase it to a density of from 2 to 20 ampères per square foot. When *iron* is the metal to be coppered the simpler copper sulphate bath is unsuitable because it would be rapidly decomposed by the iron; in this case the bath consists of an alkaline solution of the double tartrate of copper and sodium obtained by adding excess of sodic hydrate to a solution of copper sulphate and tartaric acid.

Electrotyping is a particular application of electrocoppering, and is used for the production of pictures and diagrams in books. The author first makes a drawing on paper, and a left-handed copy of this is made on wood by a draughtsman, when, the wood being cut away by the engraver, the lines in the picture show up in relief. The wood, as thus prepared, constitutes the "block" and could be employed directly for printing off *a few* copies, but not many, because on account of its softness it would soon wear away and the lines become indistinct. Accordingly, instead of so using it, a mould of it is made in gutta percha, and this is electrocoppered, as above explained, when we get a very thin relief impression (left-handed) of the original drawing, which is strengthened by filling up the hollows with melted type-metal. This is called the "cliché" or "electro," and is capable of yielding some 80,000 impressions.

CHAPTER XXXIV.

THE ELECTRON THEORY OF MATTER AND RADIO-ACTIVITY.

This chapter contains only an elementary, yet, it is hoped, a sufficiently accurate account of the Electron Theory of Matter and Radio-activity. For convenience of study the chapter is divided into four sections, but the phenomena with which the different sections deal are so intimately related that the division line between the sections cannot be decisively drawn. The Sections are as follows :—

SECTION I.—THE CATHODE RAYS AND SIMILAR PHENOMENA.

SECTION II.—ELECTRICITY AND THE ETHER. THE STRUCTURE OF THE ATOM.

SECTION III.—CONDUCTION OF ELECTRICITY THROUGH GASES.

SECTION IV.—RADIUM AND RADIO-ACTIVITY.

Those who wish to pursue the subject further should read, for Sections I., II., III., J. J. Thomson's *Conduction of Electricity through Gases* and his *Electricity and Matter*, and, for Section IV., Rutherford's *Radio-activity*.

SECTION I.

THE CATHODE RAYS AND SIMILAR PHENOMENA

234. The Effect of the Motion of an Electrostatically Charged Body is the same as that of a Current. In Art. 200, p. 593, it is mentioned that Maxwell proved theoretically that a charge of q electrostatic units moving with a velocity x is equivalent to a current of $q\,x$ electrostatic units or $\frac{q\,x}{v}$ electromagnetic units. This was experimentally verified by Rowland as follows:—An insulated charged metal disc is caused to rotate about a vertical axis between two earthed gilt glass discs, and just above the upper disc was suspended a magnetic needle. (The upper glass disc prevented any motion of the needle due to air currents caused by the metal disc.) When the rotating disc was positively charged the needle moved as if deflected by a current in the plane of the disc in the direction of motion. When the disc was negatively charged the needle moved as if deflected by a current in the plane of the disc in the opposite direction to the motion of the disc. The values of the deflection observed confirmed Maxwell's calculations.

235. Deflection of a stream of Electrified Particles shot across a Magnetic Field. Let n be the number of particles in unit length, m the mass of each particle, e its charge, v the velocity of propagation, and H the strength of the magnetic field supposed uniform and perpendicular to the direction of motion. Now by the last section the equivalent current due to the particles $= n\,e\,v$. The force exerted on particles in unit length by the field is, by Art. 127, equal to $n\,e\,v\,\mathrm{H}$, and is at right angles to both the direction of motion and the field. The force on each particle is therefore $e\,v\,\mathrm{H}$. The particle will therefore be deflected from its straight-line path, and will move along an arc of a circle in a plane perpendicular to the lines of

magnetic force. Let R be the radius of this circle, then, since $e v$ H is the centripetal force on a particle of mass m,

$$\frac{1}{R} \cdot m v^2. = e v H, \quad \therefore \frac{e}{m} = \frac{v}{R H}.$$

If A B (Fig. 359) represents the line of moving particles before the field is on, and A C the line after the field is on,

Fig. 359.

$$A B^2 = B C (2 R - B C),$$

from which, after measuring AB and BC, R can be found. If H is known, and if v can also be found, it is a matter of easy calculation to find $\frac{e}{m}$.

If the particle is projected obliquely to the magnetic field, it will move along a helix whose axis is parallel to the field.

236. Deflection of a stream of Electrified Particles shot across an Electrostatic Field. If the field is uniform and of strength X the force on each particle throughout its motion is X e, and as before, if the field is perpendicular to the initial direction of motion, we get

$$\frac{1}{R'} m v^2 = X e,$$

or

$$\frac{e}{m} = \frac{v^2}{X R'},$$

where R′ is the radius of curvature of the path. The deflection in this case is perpendicular to the initial direction of propagation, and in the direction of the electrostatic field.

237. Case when both Fields are in action. If the electrostatic field is perpendicular to the magnetic field the sign of the difference of potential may be altered so that the separate deflections due to the fields are in opposite directions. In this case, if the strengths of the field are so adjusted as not to deviate the stream of particles, we must have

$$e v H = X e,$$

or

$$\frac{X}{H} = v,$$

so that by an experiment of this nature v can be found. If now either of the two foregoing experiments, Arts. 235-236, are performed the value of $\frac{e}{m}$ may be found.

If the particles have different velocities, or there are different values of $\frac{e}{m}$, a dispersion effect will be observed in both the magnetic and electrostatic experiments.

To study this effect it is best to repeat the experiment with both magnetic and electrostatic fields on, modifying it however so that the deflections are at right angles to each other. Let Fig. 360 represent an end view of the screen on which the particles strike, O the point of impact when both fields are absent, and O S, O M the directions of the deflections due to the electrostatic and magnetic fields respectively.

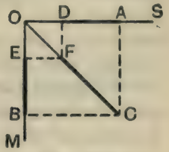


Fig. 360.

Assuming for the moment that $\frac{e}{m}$ and v are constant for all particles, let A be the point of impact when only X is on, and B the point of impact when only H is on. Then when both X and H are in action the point of impact is C.

If v is constant but $\frac{e}{m}$ varies, then instead of getting points of impact in the cases of the separate fields we get lines A D, B E, and when both fields are on, a line F C which will be straight and whose prolongation will pass through O, the end F belonging to those particles which have the least value of $\frac{e}{m}$. If now the velocity of the particles also vary we shall not get a straight line F C, but we may get a confused region, depending exactly how the values of $\frac{e}{m}$ change with variations in v. Let us, however, take the simple case which occurs when $\frac{e}{m}$ decreases as the velocity increases.

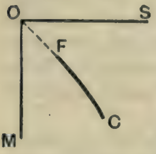


Fig. 361.

From the formulae proved above the deflection due to the magnetic field is shown to be inversely as v, while the deflection due to the

electrostatic field is inversely as v^2. Therefore for faster velocities the ratio $\frac{\text{electrostatic deflection}}{\text{magnetic deflection}}$ decreases and the line F C will bend towards the axis O M as indicated in Fig. 361.

238. Cathode Rays. The phenomenon known as Cathode Rays has been dealt with in Art. 59. We proceed now to show how the nature of these rays was elucidated. They were discovered by Plücker in 1859. He observed the phosphorescence of the walls of the tube and that the patches of phosphorescence shifted when a magnet was brought near. He ascribed the phenomenon to currents of electricity leaving the cathode. Hittorf (1869) first noticed that these rays would cast shadows, and Goldstein (1876) made the further observation that the rays left the cathode in a definite direction. He regarded the phenomenon as waves in the ether, a view long upheld in Germany, though in England Varley (1871) and later on Crookes (1876) advanced the theory that the phenomenon consisted of a stream of charged particles shot out normally from the cathode, producing by their impact with bodies phosphorescence, heat, and mechanical effects and deflectable by a magnetic field by reason of the charge they carried. Recent experiments by Perrin, Lenard, J. J. Thomson, and others have shown that the rays consist of negatively charged particles having the mass of about $\frac{1}{1900}$ of that of the hydrogen atom and projected from the cathode with a velocity comparable with that of the velocity of light. The particles are always shot out normally from the cathode; their direction is in no way connected with the position of the anode. These particles have been called corpuscles by J. J. Thomson and electrons by Johnstone Stoney (see Art. 160). The latter name seems to have met with the more general approval.

239. Lenard Rays. Using the piece of apparatus shown in Fig. 89 it is evident that cathode rays are stopped either totally or very nearly so by thin slices of solid matter, and it was long thought that even the thinnest slice of matter would absorb these rays, but in 1892 Hertz obtained phosphorescence behind a thin piece of metallic foil and altered the position of the phosphorescent patch

by a magnet. In 1894 Lenard made the piece of apparatus shown in Fig. 362. The cathode K was of aluminium, and the anode a brass tube A lining the tube behind it. The

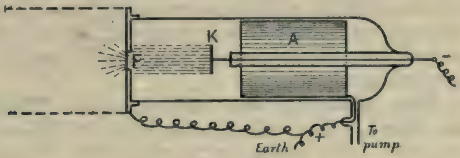


Fig. 362.

anode was earthed. The end of the tube was closed by an earthed metal plate, out of which a central hole had been cut. This hole was closed by a thin bit of aluminium foil F, $\frac{1}{400}$ mm. thick. The tube was exhausted and a strong cathode stream obtained. The room was then darkened, when the air outside F was observed to be glowing and bodies placed just beyond F phosphoresced. The issuing rays were found to be deflectable, and by putting on another vacuum tube in the position indicated by the dotted lines Lenard showed, by methods similar to those described later for the rays inside the tube, that these rays were identically the same as cathode rays. The rays before reaching the foil are called cathode rays, but to these rays which have penetrated the foil the name Lenard Rays is usually given.

240. Positive or Canal Rays. If the anode is placed facing the cathode and the latter is perforated streamers may be observed *behind* the cathode if the pressure in the tube is within certain limits. These rays also produce phosphorescence and have been shown to consist of a stream of positively charged particles of mass about the same as that of an ordinary gas atom. (See Art. 248.) Like cathode rays, these rays have also been found to penetrate thin aluminium foil.

241. Phosphorescence produced by Cathode Rays. The colour produced on the walls of the tube where struck by these rays depends on the chemical nature of the glass.

Lead glass phosphoresces blue, soda glass yellowish green. Phosphorescence is also produced in many bodies, *e.g.* barium platino-cyanide, the rare earths (cerium, lanthanum, etc.). Some bodies change colour, *e.g.* rock salt, which becomes violet, while in some cases chemical changes occur, though very likely this is partly due to the heating effect as well as to the phosphorescence. The cathode rays have a reducing effect.

242. The Heating and Mechanical Effects of Cathode Rays. By making the cathode spherical the rays projected from it all go to the centre of curvature and bodies placed there get raised to incandescence.

An apparent mechanical effect is shown by the apparatus of Fig. 90, but it has been proved that it is not the mere impact which moves the vanes of the wheel. The momentum carried by the rays is too small for this. The motion is probably a radiometer effect caused by the heating of the surface on which the rays fall.

243. The Cathode Rays are Negatively Charged. This was first shown by an experiment performed by

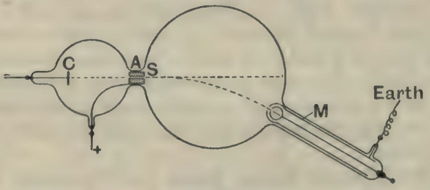


Fig. 363.

Perrin in 1895. His apparatus as modified by Prof. J. J. Thomson is shown in Fig. 363. It consists of a vacuum tube provided with two bulbs. C is the cathode and some of the rays leaving it pass through the slit S in a brass plug A, which is used as the anode, and strike the opposite wall of the larger bulb, giving a phosphorescent patch. Attached to this bulb is a side tube containing an earthed cylinder M, and within that but carefully insulated from

it, another small cylinder* connected by a wire passing through the end of the tube to an electroscope or electrometer.

The cylinders are out of the direct line of fire, and when the rays are undeviated the electrometer shows no increase of charge. A magnet is now brought up to the bulb and the rays are deviated until—as shown by their phosphorescence†—they pass through the slit of M into the insulated cylinder. The electrometer at once shows that the inner cylinder is being negatively charged, thus proving that the rays carry a negative charge.

The charge on the inner cylinder will, however, not go on increasing indefinitely, for cathode rays render the gas through which they pass conducting, and so, as the potential of the insulated cylinder rises, more and more charge passes from it across the space to M and so to earth.

244. Deflection of Cathode Rays by Magnetic and Electrostatic Fields. Ratio of the Charge of an Electron to its Mass. It has been mentioned before that a magnet deflects a stream of cathode rays in the same way as it would deflect a wire carrying a negative current. J. J. Thomson, who made quantitative experiments on this subject, used the apparatus of Fig. 364 similar in some respects to that described above. The apparatus was placed between the poles of a large magnet so that an approximately uniform field could be exerted within the dotted oval area perpendicular to the line of fire of the rays. When no field was on phosphorescence appeared at *a*. When the field was on one way the phosphorescent

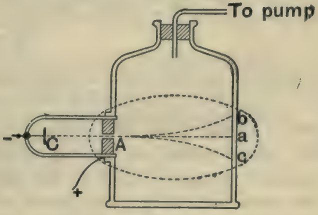


Fig. 364.

* This combination of cylinders constitutes what is often called a Faraday cylinder. (See Art. 14.)

† The patch besides being deviated is enlarged, showing a dispersion effect due to the differing velocities of the particles.

patch was sent up to b, and when the field was reversed the patch went down to c. The deflection of the rays, ab, is equal to half bc, and from this the radius of curvature of the path can be found.

Then by applying the equation of Art. 235, viz. $\frac{e}{m} = \frac{v}{\mathrm{R\,H}}$, we can calculate $\frac{e}{m}$ if H and v are also known.

To eliminate v, J. J. Thomson devised an experiment in which the deflection due to an electrostatic field was made to neutralise that due to the magnetic field. His apparatus is shown in Fig. 365.

The rays leaving the cathode C are reduced to a narrow pencil by passing through horizontal slits in the anode A

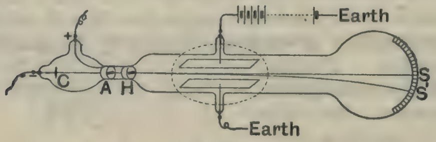


Fig. 365.

and a metal plug H. They produce phosphorescence on the glass walls at S. A strong magnetic field is then applied perpendicular to the plane of the paper and within the area shown by the dotted oval. The phosphorescence now appears at S′. (Beyond the dotted oval the path of the rays is straight and not curved as in Fig. 365.) By means of the horizontal metal plates a strong electrostatic field is applied in the right direction and the phosphorescence brought back to S. By Art. 237 we have

$$v = \frac{\mathrm{X}}{\mathrm{H}}.$$

From these two experiments Thomson calculated values of e/m and v. In order to get accurate results the pressure of the gas must be low, so that little current passes between the charged plates, and adjusted so that the potential difference between C and A is the same for all gases. Also the two fields must act over identically the same

length of path. With these adjustments it was found that the deflection in the magnetic experiment was the same for all gases and also independent of the metal of the cathode.

The value of e/m was found to be $7{\cdot}7 \times 10^6$ electromagnetic C.G.S. units per gramme, v was found to vary between $2{\cdot}2 \times 10^9$ and $3{\cdot}6 \times 10^9$ cms. per sec. (Note.—The velocity of light in a vacuum $= 3{\cdot}00 \times 10^{10}$ cms. per sec.)

Simon and Kaufmann working on much the same lines have obtained $e/m = 1{\cdot}86 \times 10^7$ *e. m.* units per gm. This result is more accurate than the early value of Thomson's.

245. Another method for determining e/m and v. In a repetition of the experiment described in Art. 243 J. J. Thomson measured the actual charge carried in unit time at the beginning of the experiment into the cylinder joined to the electrometer. He also fixed a blackened thermo-electric junction behind the slit, which, as it was bombarded by the rays, got hot and thus a current was sent through the galvanometer. From the magnitude of this current and a previous calibration of the junction the energy received per second by the junction could be deduced.

If N = number of particles passing through the slit per second,

$\frac{1}{2} N m v^2 = W$, the energy received by the thermo-pile on the assumption that all the energy of the particles is turned into heat........ (1)

Also if Q = the charge received by the electrometer per second

$$Q = N e \qquad \text{(2}$$

From 1 and 2 we get $$\frac{1}{2} \cdot \frac{m}{e} \cdot v^2 = \frac{W}{Q},$$

but $$\frac{e}{m} \cdot \frac{1}{v} = \frac{1}{RH}.$$

$$v = \frac{2W}{QRH}, \text{ and } \frac{e}{m} = \frac{2W}{QR^2H^2}.$$

This method gave $\frac{e}{m} = 1{\cdot}2 \times 10^7$ electromagnetic C.G.S. units per gramme and $v = 2{\cdot}4$ to $3{\cdot}2 \times 10^9$ cms. per sec., but it is hardly so accurate as that given in the last article owing to the greater difficulty in measuring W and Q.

246. Ubiquity of the Corpuscle. Becquerel found that radio-active substances gave out penetrating rays which consist of negatively charged corpuscles. The value of $\frac{e}{m}$ for these rays and also for the corpuscles of Lenard

rays has been found to be the same as for Cathode rays. Röntgen, Cathode, Lenard, and Becquerel rays all ionise the gas through which they pass (see also Art. 257), and this has been shown to be due to the corpuscles of these rays liberating in the gas other corpuscles, and the value of $\frac{e}{m}$ for these corpuscles has also been found to be the same as for Cathode rays. Incandescent metallic, basic, or carbon filaments also emit exactly the same corpuscles in gases at low pressure and ultra-violet light also liberates them from zinc plates.

Again, an effect called the Zeeman effect, observed in the spectrum of an incandescent vapour when the vapour is placed in a strong magnetic field, can also be explained on the supposition that corpuscles having the same value for $\frac{e}{m}$ as the Cathode rays are vibrating in the atoms of the vapour. Besides having the same value for $\frac{e}{m}$ all corpuscles, however produced, have the same value for e and for m, the m being about $\frac{1}{1900}$ of that of the hydrogen atom. Thus there are bodies smaller than atoms, and, as will be explained later, the atoms are looked upon as consisting of systems of corpuscles. This idea, which has been worked out by Lorentz, Larmor, and J. J. Thomson, is known as the Corpuscular or Electronic Theory of Matter.

247. Condensation Experiments. It is well known that if air saturated with water-vapour is subjected to a sudden (*i.e.* adiabatic) expansion condensation occurs and the space is filled with a cloud. Aitken and Kelvin have shown* that the formation of such a cloud is impossible unless nuclei are provided on which the water may condense. In ordinary air dust particles provide most of the nuclei, but C. T. R. Wilson showed that condensation occurred in dust-free gas provided that a stream of cathode, Röntgen, or ultra-violet rays, or a stream of rays from radioactive matter, was allowed to pass through the gas. With a

* See *Properties of Matter*, Art. 286.

certain expansion the condensation is only formed on the negative ions;* with a larger expansion, however, condensation occurs on both negative and positive ions. When such minute spheres fall through a gas they quickly attain a steady velocity, due to the upward force of viscosity balancing the downward force of gravitation, and Sir G. G. Stokes has calculated that the steady velocity of a sphere of substance of density ρ and radius a falling in a medium of viscosity μ is given by

$$v = \tfrac{2}{9} \cdot \frac{\rho g a^2}{\mu}.$$

J. J. Thomson in 1898 and H. A. Wilson in 1903 used this property to find the charge on the negative ion. The latter experimenter placed two horizontal metal plates C and D (Fig. 366), about 4 cms. in diameter and ·5 cm. apart, in a vessel AB 10 cms. long connected to an expansion chamber and a manometer by the tube E. The lower half of AB contains water so that the space CD is saturated. A Röntgen ray bulb is placed at the side so as to send rays down between, and parallel to, C and D. The method of experiment is as follows:—After a few preliminary expansions to get rid of all the dust present, the expansion apparatus is arranged to give an expansion in which condensation will occur only on negative ions. C and D are then joined by a wire to equalise their potentials and the rays are turned on for a short time to ionise the gas. The rays are now shut off and the expansion immediately performed producing a cloud throughout AB. The space between C and D is then watched and the velocity, v_1,

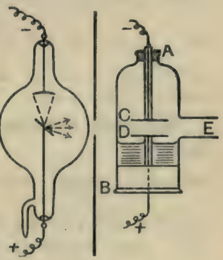


Fig. 366.

* A negative ion is an electron loaded up by having attached to it one or more neutral atoms. A positive ion is an atom which has lost one electron, either alone or loaded up by having attached to it one or more neutral atoms. The charge carried by a negative ion is equal and opposite to the charge carried by a positive ion.

with which the cloud settles down (the top of the cloud is quite definite) is observed. The field is now put on between C and D. As the cloud tends to evaporate C is connected to the negative end of the battery and D to the positive end to hasten the motion of the cloud. The velocity, v_2, with which the cloud now descends is observed. In the first experiment the downward force on any one particle is

$$\tfrac{4}{3}\pi a^3 . \rho . g;$$

in the second experiment, on the assumption that each drop has for its nucleus one negative ion, it is

$$\mathrm{X}e + \tfrac{4}{3}\pi a^3 \rho . g.$$

We have therefore

$$\frac{\tfrac{4}{3}\pi a^3 \rho g}{\mathrm{X}e + \tfrac{4}{3}\pi a^3 \rho g} = \frac{v_1}{v_2},$$

or

$$\mathrm{X}e = \tfrac{4}{3}\pi a^3 \rho g \frac{v_2 - v_1}{v_1}.$$

We must eliminate a from this equation. By Stokes' formula

$$a = \sqrt{\frac{9\mu v_1}{2\rho g}},$$

$$\therefore \mathrm{X}e = \tfrac{4}{3}\pi \frac{9\mu v_1}{2\rho g} . \frac{3\sqrt{\mu v_1}}{\sqrt{2\rho g}} . \rho g \frac{v_2 - v_1}{v_1}$$

$$= 9\pi\sqrt{\frac{2\mu^3}{\rho g}} . \frac{v_2^{\frac{1}{2}}(v_2 - v_1)}{v}.$$

Now μ for air $= 1{\cdot}8 \times 10^{-4}$ C.G.S. unit, $g = 981$ cms./sec.^{2}, and ρ for water $= 1$ gramme/c.c.

$$\therefore \mathrm{X}e = 3{\cdot}08 \times 10^{-6} . v_1^{\frac{1}{2}}(v_2 - v_1).$$

In one experiment d was ·40 cm., and the potential difference between C and D 2000 volts. X was therefore equal to $\dfrac{2 \times 10^{11}}{3 \times 10^{10}} \div \dfrac{4}{10}$ or 16·7 electrostatic units.

Time of fall of cloud when no field was on = 20·7 secs., $\therefore v_1 =$ ·0193 cm./sec.; time for fall of cloud when field was on = 12·9 secs., $\therefore v_2 =$ ·0310 cm./sec.

This gives $$e = \frac{3{\cdot}08 \times 10^{-6}}{16{\cdot}7} \sqrt{{\cdot}0193}\,({\cdot}0310 - {\cdot}0193)$$
$$= 3{\cdot}0 \times 10^{-10} \text{ electrostatic C.G.S. unit.}$$

Superposed on the chief cloud he found two minor clouds from whose velocity of descent he deduced that some drops carried a charge $2e$ and others a charge $3e$.

Wilson's mean value of e was $3{\cdot}1 \times 10^{-10}$ *e.s.u.* Thomson obtained $3{\cdot}4 \times 10^{-10}$ *e.s.u.*, and more recently Millikan and Begeman obtained $4{\cdot}1 \times 10^{-10}$ *e.s.u.* It is, however, highly probable that the values of e obtained by condensation methods are too low. Planck, by his optical theory of temperature-radiation, found $e = 4{\cdot}69 \times 10^{-10}$ *e.s.u.*, and Rutherford, working on the α-particles from radium deduced $e = 4{\cdot}65 \times 10^{-10}$ *e.s.u.* Taking the latter results as more correct we get

$$e = 4{\cdot}65 \times 10^{-10} \text{ e.s.u.} = 1{\cdot}55 \times 10^{-20} \text{ e.m.u.}$$

Combining this value of e with the mean value of e/m found by Simon and Kaufmann, viz. $e/m = 1{\cdot}86 \times 10^7$ electromagnetic C.G.S. units per gramme, we get $m = 8{\cdot}4 \times 10^{-28}$ gramme as the mass of an electron.

248. Value of $\frac{e}{m}$ for the Positive Rays. Similar experiments to those described in Art. 244 for the cathode rays have been made by Wien on the positive rays. For these rays $e/m = 10^4$ electromagnetic C.G.S. units per gramme and $v = 10^8$ cms. per sec. approximately. More recent experiments by J. J. Thomson show that the positive rays consist of two sets of rays, e/m for one set of rays having the value given above, and e/m for the other set having the value 5×10^3 electromagnetic C.G.S. units per gramme. This value, which is half the former value, is the same as the value of e/m for the α-particles (see Art. 264) emitted by radio-active bodies. The first value (10^4) is numerically very nearly the same as that for the hydrogen ion in electrolysis (see Arts. 158 and 249). The positive electron is therefore probably very similar in size to an atom of ordinary matter, and it is hoped that a further study of the positive rays will throw much light on the nature of positive electricity. (See also *Nature*, Aug. 26, 1909.)

249. Comparison of results obtained in Discharge through Gases, Electrolysis, and the Kinetic Theory of Gases. Let D be the coefficient of **diffusion** of ions through a gas, n the number of ions per c.c., and p the partial pressure they exert at any given temperature. The number of ions per sec. crossing unit area taken perpendicular to the x axis $= \mathrm{D}\dfrac{dn}{dx}$. The velocity of an ion parallel to the x axis is therefore $\dfrac{1}{n}\mathrm{D}\dfrac{dn}{dx}$. Since n is proportional to p, this may be written $\dfrac{1}{p}\mathrm{D}\dfrac{dp}{dx}$. Let $\mathrm{N} =$ number of molecules in a gas at the same temperature as that taken and at pressure H, then on the assumption that ions behave like gas molecules we have $\dfrac{n}{p} = \dfrac{\mathrm{N}}{\mathrm{H}}$, or $\dfrac{1}{p} = \dfrac{\mathrm{N}}{n\mathrm{H}}$.
The velocity is therefore $\dfrac{\mathrm{N}}{n\mathrm{H}} \cdot \mathrm{D} \cdot \dfrac{dp}{dx}$. Now $\dfrac{dp}{dx}$ is the force acting parallel to the axis of x on the ions in a c.c. Let the motion of the ions be due to an electric field of unit strength acting parallel to the axis of x. Denote the ionic charge by e, then the force acting on the number of ions in a c.c. is equal to ne, and if u is the ionic velocity parallel to the axis of x we have

$$u = \frac{\mathrm{N}}{n\mathrm{H}}\mathrm{D} \cdot ne = \frac{\mathrm{N}e}{\mathrm{H}}\mathrm{D},$$

or

$$\frac{u}{\mathrm{D}} = \frac{\mathrm{N}e}{\mathrm{H}}.$$

Townsend has measured D and Rutherford and Zeleny have measured u. From their results

$$\mathrm{N}e = 1{\cdot}24 \times 10^{10} \text{ in electrostatic units.}$$

Now experiments in **electrolysis** show that the passage of one electromagnetic unit or 3×10^{10} electrostatic units through an acid solution of water liberates 1·16 c.c. of hydrogen at 0° C. and 760 mms. of mercury.

The mass of this hydrogen

$$= 1{\cdot}16 \times 9 \times 10^{-5} \text{ grammes}$$
$$= 1{\cdot}04 \times 10^{-4} \text{ grammes,}$$

so that e/m for the hydrogen in electrolysis $= 9{\cdot}6 \times 10^3$ electromagnetic C.G.S. units per gramme very nearly.]

If N = number of molecules of hydrogen per c.c. of gas at N.T.P.

2N = number of atoms per c.c. of gas at N.T.P.;

$\therefore$ there are 2·32 N atoms of hydrogen in 1·16 c.c.

But 2·32 N atoms of hydrogen carry 3×10^{10} electrostatic units of charge; therefore if E is the charge in electrostatic units carried by one atom

$$2{\cdot}32.NE = 3 \times 10^{10},$$

whence $NE = 1{\cdot}29 \times 10^{10}$ where E is in electrostatic units.

Now in the **Kinetic Theory of Gases,** on the assumptions that the molecules are in rapid motion, that their bombardments of the sides of the vessel produce the pressure, that their motion is linear until a collision occurs, that no energy is lost in collision, etc., we can deduce a formula which will give N.* The more accurate our assumptions the better will be our value of N, and the latest value deduced by Jeans from the experimental results on viscosity of gases and the deviations of gases from Boyle's Law is $N = 4{\cdot}6 \times 10^{19}$. Substituting this in the relation above we get

$$E = 2{\cdot}6 \times 10^{-10} \text{ electrostatic C.G.S. unit,}$$

which is of the same order of magnitude as the value for e, viz. $e = 4{\cdot}65 \times 10^{-10}$ electrostatic C.G.S. unit.

These results also go to show that the electronic or corpuscular theory of matter must fairly well represent the facts and that the charge carried by an electron or a gaseous ion is of very nearly the same magnitude as that carried by a hydrogen ion in electrolysis.†

From the above figures it follows that the ratio of the

* See Poynting and Thomson's *Properties of Matter*, p. 149.

† Assuming that E is exactly equal to e, *i.e.* to $4{\cdot}65 \times 10^{-10}$ *e.s.u.*, $N = 2{\cdot}77 \times 10^{19}$, and therefore the mass of a hydrogen atom $= 1{\cdot}62 \times 10^{-24}$ gm. (also the mass of a helium atom (at. wt. 4) $= 6{\cdot}4 \times 10^{-24}$ gm).

mass of the hydrogen atom in electrolysis to the mass of a carrier in the cathode stream

$$= \frac{\frac{e}{m} \text{ for an electron}}{\frac{\text{E}}{m} \text{ for a hydrogen ion}}$$

$$= \frac{1 \cdot 86 \times 10^7}{9 \cdot 6 \times 10^3} = 1900,$$

so that the mass of an electron is only $\frac{1}{1900}$ of that of a hydrogen atom.

250. J. J. Thomson has thus summed up the foregoing results :—

1. The value of *e*, the charge on a negative ion, in all gases is the same and numerically equal to the value of E, the charge on the hydrogen ion in electrolysis.

2. When the pressure of the gas is so low that the motion of the negative ion is not interfered with by the presence of the gas molecules it is found that the mass of the negative ion is a constant and equal to about $\frac{1}{1900}$th of that of an atom of hydrogen.

3. In gases at low pressures when very little ordinary matter is present negative electrification merely means the presence of some of these negative ions. Positive electricity is, however, always associated with ordinary gaseous atoms.

4. We have therefore a revival of the one fluid theory of electricity. The electric fluid now consists of negative corpuscles. When a body has an excess of corpuscles it is negatively charged, when a defect it is positively charged. Thus positive electricity is always associated with ordinary matter, while negative electricity may or may not be. When we electrify a metal by rubbing it with india-rubber, all that we do is to remove some of the negative corpuscles from the metal and give them to the rubber, thus giving the metal a positive charge and the india-rubber a negative charge. A current of electricity consists merely of the transference of corpuscles, while the difference between a conductor and an insulator is that a conductor allows this transference while an insulator forbids it.

SECTION II.

ELECTRICITY AND THE ETHER. THE STRUCTURE OF THE ATOMS.

251. Dielectric Currents. The inferences from the results of nearly all electrical experiments go to show that the seat of all electric effects is in the medium. Let A and B (Fig. 367) be two insulated parallel plates forming a condenser. A is charged positively to a uniform surface density σ, and B negatively to a uniform surface density $-\sigma$. There is at present only electrostatic force between the plates, and Faraday tubes stretch from plate to plate; let N be their number per unit area. Now let A and B be joined by a wire of high resistance. At once the Faraday tubes joining A and B move towards the wire, their ends meeting in the wire, and besides this drifting there is a current down one plate and up the other. Hence a magnetic field is created, *i.e.* a field is created due to the motion of Faraday tubes. This field is perpendicular to the direction of the tubes and to the direction of their motion, *i.e.* perpendicular to the plane of the paper.

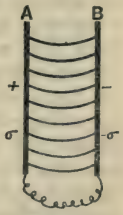


Fig. 367.

Let i equal the current per unit length of A and B measured perpendicular to the paper. Then if H is the magnetic field between A and B we have by Ampère's Theorem (Art. 138)

$$H = 4\pi i.$$

If v is the velocity of the ends of the Faraday tubes we have (by Art. 234)

$$i = \sigma v$$
$$= N v,$$
$$\therefore H = 4\pi v N.$$

Since there is a magnetic field between A and B there

is also magnetic energy stored between A and B. This energy (by Art. 107)

$$= \frac{\mu \mathrm{H}^2}{8\pi} \text{ per unit volume}$$

$$= 2\pi\mu v^2 \mathrm{N}^2 \text{ per unit volume}$$

$$= \tfrac{1}{2}\{4\pi\mu \mathrm{N}^2\} v^2 \text{ per unit volume,}$$

which may be written as $\frac{1}{2}\mathrm{M}v^2$, *i.e.* the tubes may be supposed to drag up through the space between A and B something whose mass per unit volume $= 4\pi\mu\mathrm{N}^2$.

As an analogy we may consider the hydrodynamical cases of spheres and cylinders moving through liquids. With a moving sphere is associated a volume of liquid equal to half the volume of the sphere, while with a cylinder moving perpendicular to its length is associated a volume of liquid equal to the volume of the cylinder. Therefore, regarding these tubes as cylinders moving through the ether in a direction perpendicular to their length, we can say that $4\pi\mu\mathrm{N}^2$ is the mass of the ether per unit volume carried up by the N tubes passing through unit area. The mass of the bound ether per unit length of a single tube is therefore $4\pi\mu\mathrm{N}$.

The pull in a tube is, by Arts. 34 and 46, equal to $2\pi\mathrm{N}/\mathrm{K}$, so that the pull bears a constant ratio to the mass per unit length. Faraday tubes may, therefore, be regarded as stretched strings of variable tension and mass per unit length, the ratio between pull and mass per unit length being constant at all points along the tube.

It is obvious that if the direction of motion is inclined at an angle θ to the length of the tubes the magnetic intensity is not $4\pi\mathrm{N}v$ but $4\pi\mathrm{N}v\sin\theta$, and the bound mass per unit length $4\pi\mu\mathrm{N}\sin^2\theta$.

252. Motion of an Electrified Sphere. In free space the Faraday tubes of an electrified sphere at rest are uniformly radially distributed. If the sphere begins to move uniformly along a straight line its tubes of force are distorted if the velocity is large, but not so if the velocity is very small.

Take first the latter case—the sphere carries its lines of force (undistorted) along with it. Let a be the radius of the sphere, e be the charge (considered positive), then e Faraday tubes leave it. Take unit area A at a point (Fig. 368) along a line in the plane of the paper inclined at an angle θ to the direction of motion. The number of tubes through this area $= \dfrac{e}{4\,\pi\,r^2}$, and therefore the magnetic force due to these tubes

$$= 4\,\pi \,.\, \frac{e}{4\,\pi\,r^2} \,.\, v \sin\theta = \frac{e}{r^2} \,.\, v \sin\theta$$

and acts perpendicularly to the paper. Draw a circle A B C through A perpendicular to the line of motion. The magnetic force due to the motion of the Faraday tubes is uniform around this circle, and it is obvious that the lines of force are systems of circles, the distribution being the same as if an element of current were moving along the path of the sphere.

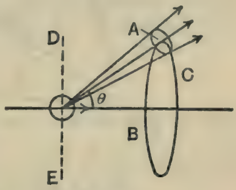


Fig. 368.

Now the energy in a magnetic field $= \dfrac{\mu\,\mathrm{H}^2}{8\,\pi}$ per c.c.,

$\therefore$ in the field around the sphere it

$$= \frac{\mu}{8\,\pi} \,.\, \frac{e^2\,v^2 \sin^2\theta}{r^4} \,.\ \text{per c.c.}$$

The volume of a little element defined by the coordinates $r,\ \theta,\ \phi,\ r + d\,r,\ \theta + d\,\theta,\ \phi + d\,\phi, =$ volume of the little block $abcdefgh$ (Fig. 369).

Now $a\,d = r\,\theta$, $d\,c = r \sin\theta\,d\,\phi$, $a\,e = d\,r$, $\therefore$ the volume is $d\,r \,.\, r \sin\theta\,d\,\phi$, $2\,d\,\theta$. Hence the energy in this volume

$$= \frac{\mu}{8\pi} \,.\, \frac{e^2\,v^2 \sin^2\theta}{r^4} \,.\, (d\,r)\,(r \sin\theta\,d\,\phi)\,(r\,d\,\theta),$$

and therefore the whole amount of the magnetic energy due to the moving sphere

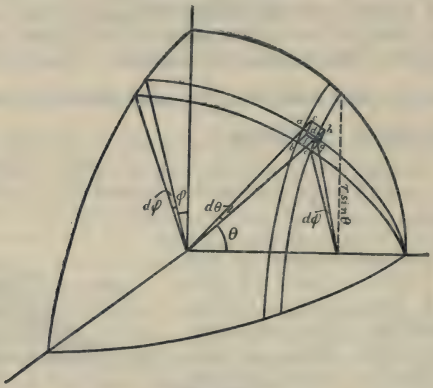


Fig. 369.

$$= \frac{\mu^2 e^2 v^2}{8\pi} \int_a^\infty \int_0^\pi \int_0^{2\pi} \frac{\sin^2\theta}{r^4} \cdot r^2 \, dr \cdot d\theta \cdot \sin\theta \cdot d\phi$$

$$= \frac{\mu^2 e^2 v^2}{4} \int_a^\infty \int_0^\pi \frac{\sin^2\theta}{r^2} \cdot dr \cdot d\theta . \sin\theta$$

$$= \frac{\mu^2 e^2 v^2}{4} \int_a^\infty \int_1^{-1} \frac{\cos^2\theta - 1}{r^2} \, dr \cdot d(\cos\theta)$$

$$= \frac{\mu^2 e^2 v^2}{4} \int_a^\infty \frac{dr}{r^2} \left[\frac{\cos^3\theta}{3} - \cos\theta \right]_{\cos\theta = 1}^{\cos\theta = -1}$$

$$= \frac{\mu^2 e^2 v^2}{3} \int_a^\infty \frac{dr}{r^2} = \frac{\mu^2 e^2 v^2}{3} \left[-\frac{1}{r} \right]_a^\infty$$

$$= \frac{\mu e^2 v^2}{3a}.$$

Now if m = mass of the sphere, its kinetic energy

$$= \tfrac{1}{2} m v^2,$$

hence whole amount of energy possessed by the sphere

$$= \tfrac{1}{2} m v^2 + \frac{\mu e^2}{3a} v^2,$$

or it is the same as if the mass m were virtually increased by $\frac{2}{3}\frac{\mu e^2}{a}$. Part of the mass of the sphere is therefore due to its charge. It has been shown that the maximum value of $\frac{e}{a^2}$ is 100. From this it follows that the density of the sphere is virtually increased by 10^{-16} of itself.

Professor J. J. Thomson has shown* that it is not impossible that the whole mass of the sphere may rise in this way. Assuming therefore that the whole mass of the sphere is equal to the electrical mass m, we get for such a particle

$$m = \tfrac{2}{3}\frac{\mu e^2}{a} \quad \text{or} \quad \frac{e}{m} = \frac{3a}{2\mu e}.$$

Now $\frac{e}{m}$ has been measured for negative corpuscles (see Art. 244) and found to be $1{\cdot}86 \times 10^7$ electromagnetic C.G.S. units, therefore taking μ as unity, we get

$$1{\cdot}86 \times 10^7 = \frac{3a}{2e},$$

from which

$$a = \tfrac{2}{3} \times e \times 1{\cdot}86 \times 10^7,$$

and since (see Art. 247)

$$e = 1{\cdot}55 \times 10^{-20}$$

electromagnetic C.G.S. units,

$$\therefore\ a = 1{\cdot}9 \times 10^{-13} \text{ cm.}$$

The radius of this electrical atom is therefore about 10^{-13} cm., which is very small compared with 10^{-8} cm., the radius of an ordinary gaseous atom.†

* See *Electricity and Matter*, by Prof. J. J. Thomson (Constable & Co.); *Conduction of Electricity through Gases*, by Prof. J. J. Thomson (Camb. Univ. Press).

† See Art. 249 and Wagstaff's *Properties of Matter*, Arts. 183, 192.

Assuming that all the mass is electrical and due to the tubes carrying forward some of the medium, it is evident that since the whole mass m is all in the ether the body is therefore everywhere, and every body occupies space occupied by every other body.

Mathematics also shows that the mass, though everywhere, is intensely localised, for only $\frac{1}{100000}$ of the mass is outside a sphere of the same size as an ordinary atom.

253. The Mass of a Body is entirely Electrical. So far we have assumed no distortion of the tubes of force, but an elongated body always tends to set broadside on when falling or flowing through a medium, as compare falling leaves or bits of wood floating in a stream. If this hydrodynamical force were the sole force, all the tubes would crowd into the equatorial plane DE (Fig. 368). But the tubes repel each other, hence the actual configuration is a compromise.

When v is small the electrical repulsions greatly exceed the equatorial condensations.

Mathematics shows that if v is comparable with V the velocity or light, the electrical mass

$$I = \frac{2}{3}\frac{\mu e^2}{a} v \left\{ 1 + \frac{2}{5}\frac{v^2}{V^2} \right\}, *$$

from which we get the following table:—

v in cms. per sec.	Mass of particle at velocity given by first column / Mass of particle moving slowly.
$2{\cdot}85 \times 10^{10}$	3·09
$2{\cdot}59 \times 10^{10}$	2·04
$2{\cdot}36 \times 10^{10}$	1·65
$\cdot 3 \times 10^{10}$	1·005

Kaufmann, working on the negative corpuscles given out

* This formula is not quite accurate. Righi has shown that the electrostatic energy must also be taken into account.

by radium, has obtained experimental results which very nearly agree with these, and enlarging on this J. J. Thomson has postulated that the *whole mass is electrical, i.e.* all mass is mass of the ether, all momentum is momentum of the ether, and all kinetic energy kinetic energy of the ether. One chief objection to this view is that it requires the ether near a corpuscle to be of great density (5×10^{10} gms. per c.c.).

254. Velocity of a Transverse Pulse along a Moving Faraday Tube. We showed in Art. 250 that a Faraday tube bears a considerable resemblance to a stretched string. It is well known that the velocity of propagation of a transverse wave along a stretched string

$$= \sqrt{\frac{\text{pull in the string}}{\text{mass of unit length of string}}},$$

but before we can apply this formula to find the velocity of a transverse displacement occurring in a Faraday tube we must remember that, as the Faraday tube is bounded by other tubes, the problem is not exactly analogous to that of finding the velocity of a transverse wave in a stretched string.

In Arts. 34 and 46 it is shown that the tension (*i.e.* pull per unit area) in the Faraday tube system $= \dfrac{\text{K F}^2}{8\pi}$, while the transverse pressure (*i.e.* force per unit area) on a tube is also $\dfrac{\text{K F}^2}{8\pi}$.

Now $\text{F} = \dfrac{4\pi\text{N}}{\text{K}}$, therefore the tension and lateral pressures are each equal to $\dfrac{2\pi\text{N}^2}{\text{K}}$ per unit area at points where N is the number of Faraday tubes per unit area taken perpendicular to their length.

To find what the tension would be if there were no side pressures, let us imagine a portion of the tube placed in an envelope (Fig. 370) and a negative pressure equal to $\dfrac{2\pi\text{N}^2}{\text{K}}$ applied to the whole of the contained ether. This neutral-

ises the lateral pressures and makes the tension along the tube equal to $\frac{4\pi N^2}{K}$ at points where there are N tubes per unit area, *i.e.* the pull in a single tube $= \frac{4\pi N}{K}$.

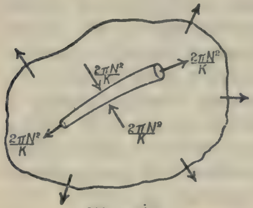


Fig. 370.

The velocity of the transverse disturbance is therefore equal to

$$\sqrt{\frac{4\pi N/K}{4\pi\mu N}} = \frac{1}{\sqrt{\mu K}}.$$

But $\frac{1}{\sqrt{\mu K}} = 3 \times 10^{10}$ cms. per second, the velocity of light, therefore the disturbance travels along the Faraday tube with the velocity of light.

This result is in one sense obvious, for light is a species of transverse wave-motion in the ether, and it is therefore highly probable that other transverse motions would travel with the same velocity.

The above formula for the velocity of a transverse displacement also holds for the case of a tube moving at any angle θ to its length; for though the bound mass per unit length of such a tube is $4\pi\mu N \sin^2\theta$, yet in a transverse displacement a portion of the tube is moving at right angles to its length, and therefore the bound mass of ether for such a portion is equal to $4\pi\mu N$.

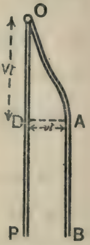


Fig. 371.

255. Röntgen Rays. Let us now consider what happens in the ether when a negative corpuscle meets an obstacle. As a preliminary we may study the question of a heavy rope hanging on a support O (Fig. 371) which is moving forward with a velocity v. Suppose at a certain instant the support is suddenly brought to rest. The uppermost part of the rope is also brought to rest, but the lower parts of the rope continue to move onward. At a certain instant t secs. after the stoppage the rope hangs in the shape

O A B, A B being vertical and still moving forward with velocity v. Draw A D perpendicular to O P the vertical through O, then if V is the mean velocity of propagation of the nick A downwards, O D = Vt. If the rope is so shaped that the mass per unit length at any point is proportional to the stretching force at that point V is uniform, and hence the nick A travels with a uniform velocity down the rope. If v is very small compared with V the shape of the rope at successive instants will not be very different from that of Fig. 372, *a. b. c.*

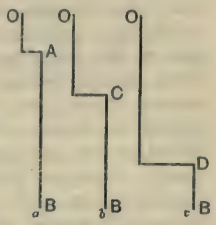


Fig. 372.

Consider now a negative corpuscle moving uniformly forward along A B (Fig. 373) with a small velocity v. The Faraday tubes are distributed uniformly all around it. Suppose that when the corpuscle reaches O a force acts upon it which quickly brings it to rest in a small time δt, the final position of the particle being not sensibly different from O. To find the position of the Faraday tubes a time t after

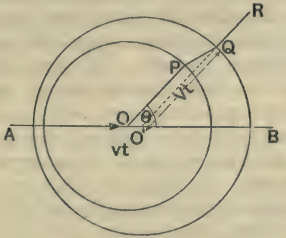


Fig. 373.

the first application of the force measure along O B a distance O O′ equal to $v\,t$, with O as centre describe a sphere with radius $V(t-\delta t)$ and with O′ as centre describe a sphere of radius V t, where V is the velocity of light (see Art. 254). Then if no force had acted on the

corpuscle the tubes would be all radiating from O′. As it is, however, only the tubes outside the larger sphere are radiating from O′; the disturbance having passed over the tubes inside the inner sphere they radiate from O, while the portions of the tubes within the spherical shell are in the transition state, the disturbance shifting them from the radiating centre O′ to that of O. They, of course, must join up the interior tubes to the exterior tubes, so that a tube inclined at an angle θ to the direction of motion appears as in Fig. 373, P Q being nearly straight if δt is very small. P Q has a tangential component, thus within the shell there is a tangential electric force, and J. J. Thomson has calculated that the electric and magnetic forces brought into existence by this tangential shearing of the Faraday tubes are greater than the forces due to the radial tubes simply moving forward. The pulse due to this tangential shearing travels outwards along the tube with a velocity V (V being equal to the velocity of light), and this, in J. J. Thomson's opinion, constitutes the well-known Röntgen rays produced when the negative carriers of the cathode stream strike a solid obstacle.

We may therefore look upon Röntgen rays as transverse pulses in the ether of very short wave-length. The energy in the pulse depends upon the suddenness with which the stoppage occurs. If the stoppage is very sudden the shell is thin, the output of energy large, and the Röntgen rays produced are very penetrating, or in technical language "hard," while if the stoppage is relatively slow the rays carry very little energy, and are easily absorbed by matter. In this case they are termed "soft" rays. It was shown theoretically in Art. 254 that the velocity of Röntgen rays is the same as that of light. This has been experimentally confirmed by Marx.

Another theory, proposed by Professor Bragg, suggests that the constitution of a Röntgen ray is that of a closely associated pair of electrically charged particles moving with great velocity, one particle being a corpuscle or negative electron, the other a positive electron. It is not easy to see how such a neutral pair can be ejected from an anticathode struck by a negative electron.

256. The Structure of the Atom. The periodic law points to the fact that the elements have something in common. This is also borne out by the study of spectra and of radio-activity. In the most modern theory of atomic structure—due to Prof. J. J. Thomson—the active part of the atom consists of a number of rapidly revolving negative corpuscles, the rest of the atom consisting of positive electricity, though as to how the positive electricity is arranged we have as yet little or no idea, though J. J. Thomson assumes it equally distributed throughout the sphere. The total negative charge must, however, equal the positive charge. However, the rotating corpuscles are the main things to consider. They give rise to the whole of the mass of the atom, so that the atomic weight is proportional to the number of corpuscles contained by the atom, and any energy which the atom gives out is due to changes in their configuration. At present J. J. Thomson has not been able to work out in detail the three-dimensional problem of the distribution of the corpuscles within a spherical atom, but he has solved the problem in two dimensions, and he has shown that when the number of corpuscles is large they are arranged in concentric rings. Treating the problem as a statical one, the system of corpuscles will be stable if their mutual forces of repulsion balance the attraction of the positive sphere. The simplest atom would consist of a single corpuscle at the centre of the sphere, the next form of atom would contain 2 corpuscles situated on a diameter, the distance of each from the centre being half the radius of the sphere; the next 3 corpuscles at the corners of an equilateral triangle, the distance of the corpuscles from the centre being ·57 the radius of the sphere. The next forms would contain 4 and 5 corpuscles arranged equidistantly. The configuration of 6 corpuscles in a ring is unstable, but if we take one of them and put it in the centre the configuration is stable. Also 7 in a ring is unstable, but 6 in a ring plus 1 in the centre is stable. Similarly 8 in a ring is unstable, but 7 in a ring plus 1 in the centre is stable. To accommodate 11 corpuscles we must put 2 in the centre and 9 in the ring. For very

large numbers we have to greatly increase the central cluster as shown in the table :—

Number of corpuscles in the ring	1	2	3	4	5	6	7	8	9	10	15	20	30	40
Number of corpuscles to be put in the centre..	..	..	..	..	..	1	1	1	2	3	13	39	101	242

An increasing number in the centre means increasing stability, thus the configuration of 10 in the ring and 5 in the centre is more stable than that of 10 in the centre and 3 in the ring. When a large number are in the centre they tend to form another ring, and when the number of corpuscles is large we get several concentric rings, the number in the outermost circle being only a small fraction of the total number. Thus, for example, in the following table the top line gives the total number of corpuscles, and the lower lines the numbers in the successive rings :—

60	55	50	45	40	35	30	25	20	15	10	5
20	19	18	17	16	16	15	13	12	10	8	5
16	16	15	14	13	12	10	9	7	5	2	
13	12	11	10	8	6	5	3	1			
8	7	5	4	3	1						
3	1	1									
5 rings.			4 rings.			3 rings.			2 rings.		1 ring.

All the possible arrangements for an outer ring of 20 are given in the following table, the top line representing the total number of corpuscles in the sphere :—

59	60	61	62	63	64	65	66	67
20	20	20	20	20	20	20	20	20
16	16	16	17	17	17	17	17	17
13	13	13	13	13	13	14	14	15
8	8	9	9	10	10	10	10	10
2	3	3	3	3	4	4	5	5

Until we get 5 in a ring we can always put another one there. In the table above the first column is one of minimum stability, the next arrangement can spare one corpuscle and still be stable, and so on, the last only being able to spare 8 corpuscles. For 68 corpuscles we must put 21 in the outer ring, and this is the minimum stable arrangement for 21 in the outer ring. There is thus a kind of discontinuity at this point just as there is in the *periodic table* when one passes from one row to the next. The facility with which a system can lose corpuscles is a measure of its electro-positive character.

An arrangement containing 67, distributed as in the table, is a stable electro-negative system, 59 and 68 are unstable electro-positive systems. Suppose the 59-system lost a corpuscle from its 20-ring. It now becomes very stable and electro-negative. If the 60-system lost one corpuscle it would become more electro-positive. It could not lose 2 corpuscles without changing its character: it is therefore a monovalent atom. The 61-system is more stable. It could lose 2 corpuscles without losing its character. It is therefore divalent. The 62-system is trivalent. The 66-system is very stable. It can stand the addition of 1 corpuscle, but not very well that of 2. It is a monovalent electro-negative system. The 65-system is divalent and electro-negative.

We have so far supposed in our inquiries into stability that the corpuscles are at rest. In this case no energy will be radiated from the atom. But to explain many phenomena—radio-activity in particular—it is necessary to assume that the corpuscles are in rapid rotation. This will drive the corpuscles further away from the centre of the atom than deduced by the statical method, but it will not in general alter the general configuration of the system. The velocities are thus constantly changing, the acceleration being directed towards the centre, and in this case Larmor has calculated that energy will be slowly radiated from the atom. The internal supply of energy thus becomes less, the stability less, and at a certain stage the configuration becomes unstable and breaks down. In the breaking-down the total potential energy decreases and the kinetic energy

of the corpuscles increases, and perhaps increases so much for some corpuscles that one or more of them are expelled from the atom and the system settles down into a different configuration involving less corpuscles. When the explosion occurs a large amount of energy is radiated, and this is what is supposed to be occurring pretty frequently in the case of the strongly radio-active elements, and more slowly in all bodies.

In a more recent communication Professor Thomson has remarked that the amount of ether carried by the small negative end of the system composed of an electron and its Faraday tubes is small compared to that carried by the Faraday tubes, while the amount of the ether carried by the large positive end of a system of a positively charged atom exceeds that carried by the tubes; hence the mass of the corpuscle seems wholly electrical, while that of the atom does not. Both masses are probably identical in origin, being masses of portions of ether.

SECTION III.

CONDUCTION OF ELECTRICITY THROUGH GASES.

257. Ionisation Currents. A gas in its normal state conducts electricity very poorly. Thus if a gold-leaf electroscope is charged and left to itself the leaves remain diverged for some time. Still, the fact that they do fall gradually together shows that they are losing their charge. This loss may occur either along the insulation or through the gas. Experiments with quartz and sulphur as insulating agents, and different lengths of these have shown that, in any case, there is a leak, though usually very small, through the gas, and other experiments have shown that this leak increases as the potential of the gold-leaf system increases. Research on this subject has shown that the leak from a charged electroscope set up in the open can be greatly increased by bringing near it a vacuum tube giving cathode, Lenard, or X-rays, or a source of ultra-violet light, or a flame, or quantities of uranium, radium, thorium, etc.

The question as to how this conductivity is caused may be partly solved by the following simple experiment.

E (Fig. 374) is an electroscope containing a gold-leaf fastened to a narrow brass plate held up by a sulphur plug S. Two tubes lead into E, one of which, a long

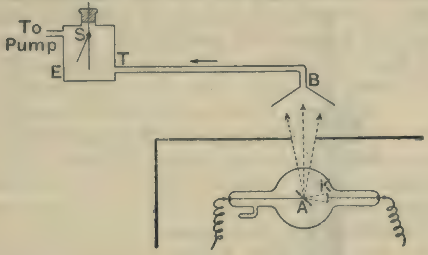


Fig. 374.

one T B, is bent down to face a window in a lead box, below which is situated the anti-cathode of a Röntgen ray bulb. The use of the lead box is to screen E from the direct effect of the bulb. The short tube connects to a water-pump, by means of which air can be slowly dragged through T B and E. It is found that the leak of the electroscope is much faster when the bulb is on than when off. If, however, the tube T B is plugged with cotton-wool so that the air is filtered the leak is unaltered by running the bulb. Or if T B is made of metal and an insulated wire is stretched down the middle of the tube, the leak is not increased when the bulb is running if the insulated wire is charged and the tube earthed. These experiments point to the existence of charged particles in the gas, which if allowed to come into the electroscope neutralise the charge on the gold-leaf. Since the gas on the whole is neutral, it is evident that the amount of charge held by the positively electrified particles is equal to the amount of charge held by the negatively electrified particles. These charged particles—some are positively charged and others negatively—are called **ions,** and the process of producing them in a gas is called the **ionisation of the gas.** The rate of production of ions

is proportional to the strength of the radiation from the ionising agent, but the number of ions in existence does not increase indefinitely with time, as some of the positive and negative ions recombine, the rate of recombination being

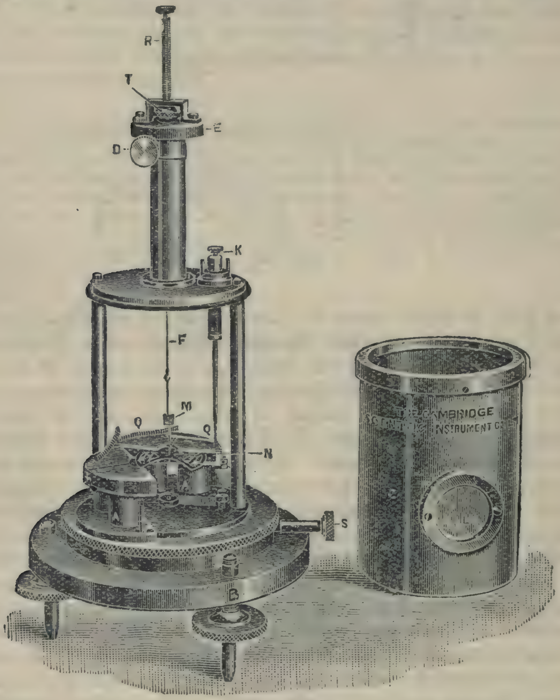


Fig. 375.

proportional to the square of the number present. In an electric field the positive ions move in the direction of the field, and the negative ions in the opposite way. When the ions meet a metal plate they give up their charge to it, thus their motion through the gas constitutes a current through the gas. The ionisation of a gas by most agents—

Röntgen rays is an exception—falls off very rapidly as the distance from the agent increases. Ionisation proceeds in gases, though very slowly, even when no known external ionising agent is at hand. This is called **spontaneous ionisation** and may be due to an unknown radiation of great penetration. In finding the current through a gas due to any agent the current due to this natural ionisation must be first measured and then deducted from the current observed when the agent is in action.

258. Methods of Measurement. The current through a gas, unless strongly ionised by radium salts, is too weak to be measured by a galvanometer. A quadrant electrometer or gold-leaf electroscope must be used instead. If C is the capacity of the electrometer or electroscope and V the rise of potential per second, C V is the current flowing into it.

The form of quadrant electrometer usually employed is that known as the Dolezalek instrument (Fig. 375). It consists simply of a needle of silvered paper hanging by a fine fibre (quartz, phosphor bronze, or platinum) between four quadrants well insulated by amber supports. The resistance offered by the air to a needle of such light construction is sufficient to render the motion nearly dead beat, without any further damping devices being required. The instrument can be turned around on the base and clamped in any position. The torsion head from which the needle is suspended can also be rotated and clamped, while the cover (shown in the figure), which serves as a protection from draughts and from stray electrostatic fields, can be independently turned so as to make its glass window face in any desired direction. Access to the needle is rendered easy by mounting two adjacent quadrants on a brass piece so that they can be swung to one side and returned again to the working position as desired. The needle is usually kept charged to about 80 volts, and the spot of light will then move about a metre on a scale at a metre distance for a potential difference between the quadrants of 1 volt. There is no Leyden jar as in the forms represented in Figs. 56 and 59. The working capacity of the electrometer is about 50 electrostatic units.

The electroscope usually employed was invented by C.T.R. Wilson. It consists of a small, nearly cubical brass box provided with holes at C and D. Through C there projects an insulated rod carrying a brass plate P which nearly fills up the end of the electroscope, while through D projects a well-insulated metal rod carrying a very narrow gold leaf L. The leaf is about a millimetre wide and should be just long enough not to reach across to P. W is a window through which its motion is observed. The insulation at C may be ebonite, that at D must be sulphur or quartz. When in use P is kept charged to, say, 160 volts, and the instrument is placed in an oblique position (30°, about), to increase the sensitiveness. The case A is earthed and L is joined to the insulated system into which the current is flowing. As

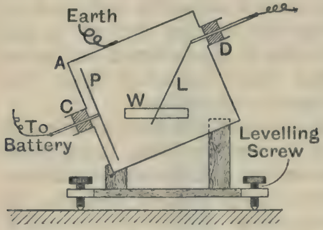


Fig. 376.

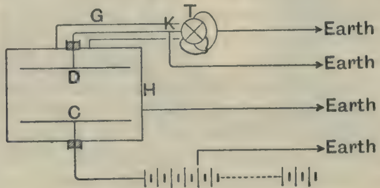


Fig. 377.

the voltage of this system rises L is attracted towards P and its movement is measured by a microscope directed to look through the window. The capacity is about 1 electrostatic unit, and in a sensitive position the leaf will move about 1 mm. for one-tenth of a volt, and a small fraction of this can be easily read with a good microscope furnished with an eye-piece scale.

259. Saturation Current. Let C and D (Fig. 377) represent two insulated parallel discs placed inside an earthed metal vessel H. C is attached to a battery B of many cells. By means of an earthed wire making contact with the plates of the cells the voltage on C can be varied within wide limits. D is well insulated and attached to one pair of quadrants of an electrometer or electroscope T, the other pair of quadrants being earthed. K is a key by which D may readily be earthed. G is an earthed tube-guard to protect the wire from D to T from external induction effects. The air in H being ionised by some ionising agent, C is charged successively to different voltages and the current flowing from C to D observed. If

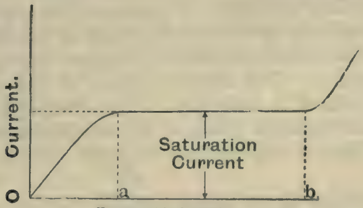


Fig. 378.

the plate C is connected to the negative pole of the battery it repels the negative ions to D, which thus receives a negative charge. If C is positive it receives a positive charge. The results being plotted, a curve like Fig. 378 is obtained. It shows that starting from small voltages the current is at first nearly proportional to the voltage. As the voltage increases the rate of increase of current diminishes, gradually becomes zero, and then for a long range of voltage the current through the gas remains practically constant. With very large voltages, however, the current rapidly increases. The value of the practically constant current is called the *saturation current.* When the potential is small (*i.e.* $< \mathrm{O}\,a$) only a few of the ions produced by the ionising agent are able to reach the plates

before recombination sets in, and the current is therefore small. As the voltage is gradually increased up to O *a* more and more ions reach the plates, and the current increases. When the P.D. is anything between O *a* and O *c* all the available carriers are employed, and the current is steady, *i.e.* independent of the voltage. The last part of the curve shows that the number of ions in the gas is rapidly increasing. This is probably due to the negative ions getting up sufficient velocity—due to the strong field between C and D—to ionise fresh molecules by **colliding** with them and detaching corpuscles. If the ionisation present is that due to the spontaneous ionisation of the air, a potential gradient of about 10 volts per cm. is sufficient to produce saturation, but if ionising agents such as ultra-violet light, Röntgen rays, or radio-active bodies are used, the ionisation is so great that the rate of recombination is very large and therefore also the voltage required to produce saturation. The same happens if the distance the ion has to travel is increased. The student should note the difference between the laws of the conduction through gases and that of conduction through metals, viz. Ohm's Law.

If the ionising agent is removed the current drops down to the normal air current in a few seconds. This is due to recombination occurring between the positive and negative ions.

260. Constitution of the Positive and Negative Ions. Experiments have shown that the negative ion consists of a negative corpuscle either alone or loaded up by attracted gas-molecules, while the positive ion consists of an atom of gas which has lost a corpuscle also either alone or loaded up by attendant molecules. The masses of positive and negative ions are thus comparable with the masses of the atoms of the gas, but while the mass of the positive ion is approximately the same at all pressures, that of the negative ion decreases greatly as the pressure is decreased, showing that at low pressures the negative corpuscle throws off its attendant molecules. As a rule the velocity of the negative ion is greater than that of the positive ion, the velocities in dry air at 12° C. being

respectively 1·8 and 1·4 cms. per sec. for a potential gradient of one volt per cm.

Condensation experiments (Art. 247) have shown that if the expansion is sufficient to bring down all the positive and negative ions, the number of positive ions is equal to the number of negative ions. From this it follows that since the gas was electrically neutral to begin with, the charges carried by positive and negative ions are numerically equal. The charge carried by an ion is independent of the nature of the gas, and the gas ion thus differs very much from the ion of electrolysis (see also Art. 248). Townsend has shown that even with the strong ionisations produced by radium only one molecule of gas out of a hundred millions is ionised per second. Seeing also each corpuscle projected from a radio-active body is capable of creating many thousands of ions before all its energy is used, it follows that the number of corpuscles projected by the active body per second need not be very large. In weak electric fields only the negative ion produces ions by collision, in strong fields the positive ion is also able to produce ions by collision; this explains the rise of the current curve for large voltages (Fig. 378).

SECTION IV.

RADIUM AND RADIO-ACTIVITY.

261. Radio-active Elements. In 1896 Becquerel discovered that uranium and its compounds gave out rays which affected a photographic plate and ionise a gas just like Röntgen and Cathode rays do. In 1898 Schmidt discovered the same thing for thorium. Substances which act like uranium and thorium are said to be **radio-active.** Just after these discoveries M. and Mme. Curie made a systematic study of the radio-activity of many minerals, and having discovered that radio-activity is an atomic property, being in no wise dependent on the chemical combination entered in by the element, found that some of these minerals, pitch-

blende and chalcolite in particular, were more active than the amounts of uranium and thorium they contained would justify. From this they concluded that these extra-active minerals must contain another active element, and from a ton of uranium residues from pitchblende M. and Mme. Curie and M. Bemont, after a most laborious chemical separation, isolated ·3 gramme of the very active element to which they gave the fitting name of radium. Incidentally a less active element, which they called polonium, was also discovered.

The method of separation of the radium relies on the fact that radium behaves very much like barium, and can be precipitated with it. Finally the radium bromide is separated from the associated barium bromide by its greater insolubility in water. Demarçay examined the spectrum of radium and found it gave many lines not given by any other element, thus justifying the assertion made by the Curies that radium was a new element.

Mme. Curie determined the atomic weight of radium by the addition of silver nitrate to a solution of radium chloride. Silver chloride was precipitated, and from the weight of this the chlorine in the radium chloride could be estimated. Her final result, on the assumption that radium is divalent like barium, is 226, which places radium in the periodic table in the same column as the metals of the alkaline earths, and in the same row as uranium and thorium. Polonium has been isolated by similar methods to that of radium. It bears the same relation to bismuth that radium does to barium.

Another third strongly radio-active body has been identified in pitchblende by M. Debierne, who has named it actinium. In its chemical characteristics it is very similar to thorium. Radium, polonium, and actinium have as yet been only obtained in small quantities. There has not been a gramme of pure radium bromide manufactured yet. The amounts of polonium and actinium yet obtained are indefinitely small.

262. Measurement of the Radio-activity of Bodies. Three methods are in use, depending on

(1) The action of the rays on a photographic plate.

(2) The fluorescence produced by the rays on a screen of zinc sulphide, barium platinocyanide, willemite, etc.

(3) The ionising action of the rays on the surrounding gas.

(1) is slow and open to objections, though it has done considerable service. It does not readily give quantitative effects, but is useful in measuring the deviations of the rays in a magnetic field when the rays are very weak. Some rays which are weak ionisers have a considerable photographic action, hence photography proves an efficient agent in their measurement.

(2) can only be used for very active bodies. It is, however, useful as a detector of such bodies. In observing the deviation of the cathode rays in a magnetic field the phosphorescence of the glass is used to indicate the point where the rays strike the glass. In the case of the canal rays the glass does not phosphoresce sufficiently to enable the observer to detect the position of the point of impact. If, however, the glass is covered with a sprinkling of willemite (done by first wetting the glass with a weak solution of sodium silicate) the fluorescence produced in the willemite by the canal rays easily marks the point of impact.

(3) depends on the fact that the radiations ionise the air or other gas through which they pass and so increase its electrical conductivity. It is the most accurate method of increasing the intensity of the rays in all cases where ionisation occurs. Very slowly moving particles, however, do not ionise the gas; special means therefore must be adopted to detect such particles. The electrical methods admit of infinite variation, so that methods most suitable for the purpose in hand can be devised. More will be given on this method in the next article.

263. Three Types of Radiation from the Radioactive Substances. It has already been mentioned that Becquerel found, in 1896, a compound of radium affects a photographic plate. To obtain this effect, place at the bottom of a shallow pill-box a compound of radium, thorium,* or uranium, cover the open end of the box with

* An incandescent gas mantle consists of nearly pure thoria (ThO_2).

black paper, place the photographic plate film downwards on the box and exclude all light. After 48 hours develop the plate. A dark patch the shape and size of the open end of the box will result.

In 1899 Rutherford made a systematic examination of the radiation from uranium by an electrical method.

The apparatus adopted is similar to that shown in Fig. 377. The uranium compound is placed in a recess in the plate C. The space C D was electrically saturated and the saturation current obtained first with the uranium covered and then with the uranium covered with different thickness of thin foil. From the variation of current with the number of layers of foil he deduced that the radiation from uranium must consist of three kinds:—(1) A part easily absorbed by thin foil, (2) a part more difficultly absorbed by thin foil, (3) a part hardly absorbed at all by thin foil. To distinguish these rays from each other he called them α, β, and γ rays respectively. From his results he calculated that the α rays are reduced to half their initial value by passing through ·0005 cm. of aluminium foil,* the β rays to half value by ·05 cm. of aluminium foil, and the γ rays to half value by at least 8 cm. of foil, so that the relative powers of penetration of the α, β, and γ rays are as 1 : 100 : 10000. He also showed that for a thin layer of unscreened radio-active matter the relative ionising-powers are as 10000 : 100 : 1.

More recent work has elucidated the following information about the rays :—

264. The α Rays. These are the least penetrating rays from radio-active bodies, being completely absorbed by ·01 cm. of aluminium foil. Owing to this great absorption they are not easily detected by photography except in the case of the strong α rays from radium and polonium. They are only slightly deviated in a strong magnetic field. The deviation is, however, enough to show that the α rays consists of a stream of positively charged particles, projected from the active matter with a velocity of from

* The absorption of α rays by different elements is very nearly proportional to the thickness of the plate and the square root of the atomic weight of the element. In the case of β rays the absorption coefficient is a periodic function of the atomic weight. For elements in the same groups it increases as the atomic weight increases.

$1{\cdot}55 \times 10^9$ to $2{\cdot}25 \times 10^9$ cms. per sec. Their value of e/m is $5{\cdot}1 \times 10^3$ electromagnetic C.G.S. units per gm. By direct counting, electrically and optically, Rutherford and Geiger found that one gm. of radium itself (*i.e.* the products excluded) gives off $3{\cdot}4 \times 10^{10}$ α-particles per second. They also measured the charge carried off by the α-particles, and from the knowledge of the number deduced that the charge borne by each α-particle is $9{\cdot}3 \times 10^{-10}$ *e.s.u.*, *i.e.* $3{\cdot}1 \times 10^{-20}$ *e.m.u.* Combining this with the value of e/m they found that the mass of an α-particle is $6{\cdot}2 \times 10^{-24}$ gms. This is equal to the mass of a helium atom (p. 681). The charge on an α-particle is twice the electronic charge (Art. 247). Since helium is always found in company with radio-active bodies Rutherford concludes that the α-particle is an atom of helium, or rather that the α-particle, after it has lost its positive charge, is an atom of helium. He has confirmed this with the spectroscope. Most of the energy emitted by radio-active bodies is carried by the α rays, and, as will be shown later, they play a far more important part in the phenomena of radio-activity than either β and γ rays. Fig. 383 shows that in the disintegration process which radium undergoes most of the changes are accompanied by the expulsion of α-particles. During the flight of an α-particle through a gas it knocks off negative electrons from many of the atoms and is thus an effective ioniser (200,000 ions per α-particle), rapidly spending its energy in the process (see Art. 278).

265. The β Rays. These are more penetrating than the α rays. A thickness of aluminium foil equal to ·05 cm. is required to reduce their intensity to half-value, while for complete absorption ·5 cm. of aluminium is required. Owing to their penetrating power they are easily detected by photography, and in 1900 Becquerel by photographic methods measured their deflection in a magnetic field and found that e/m was very nearly 10^7 electromagnetic C.G.S. units per gm., and that v was very nearly $1{\cdot}6 \times 10^{10}$ cms. per second, which is nearly equal to the velocity of light. They are therefore identical with the electrons of the cathode stream. The β-particles from radium have, on the whole, higher velocities than those from uranium, but these latter are more uniform in velocity. In an experi-

ment to show that the β-particles are negatively charged M. and Mme. Curie embedded a thick lead plate in a block of paraffin enclosed in an earthed metal casing. The casing over one face of the block was very thin. The lead plate was connected to an electroscope and the thin part of the casing was held over some barium-radium chloride. The lead became increasingly charged with negative electricity. Since all the α rays are absorbed by the thin casing the negative charge must be due to the absorption of the β rays. Since radium is originally electrically neutral, if we enclose it in a glass vessel of sufficient thinness to let the β rays escape but to absorb the α rays, the inside of this vessel should become positively charged. This has been experimentally confirmed by Mme. Curie.

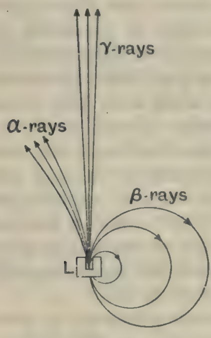


Fig. 379.

Wien found that 1 gramme of radium in radio-active equilibrium with its products gives out $6{\cdot}5 \times 10^9$ β-particles per second, and Rutherford has found that the same mass of radium would emit $13{\cdot}6 \times 10^{10}$ α-particles in the same time.

266. The γ Rays. These are extremely penetrating rays; Rutherford has detected them after they have passed through a foot of iron. They readily excite luminosity in various platino-cyanides and in willemite (zinc silicate) after passing through half an inch of lead.

Only the γ rays from radium have been detected by photography. Many experiments have been made to catch their charge and to deflect them in a magnetic field, but with no success, so that it seems that they are not a stream of electrified particles. From the similarity of their action to penetrating Röntgen rays it has been suggested that they consist of pulses in the ether of very short wave-length generated when an electron is expelled from an atom. This view is borne out by the fact that γ rays always accompany β rays and are proportional to them.

267. Magnetic Deviation of the Rays. The behaviour of the rays in a magnetic field is well illustrated by the following diagram (Fig. 379), due to Mme. Curie:—L is a lead block containing a narrow hole at the bottom of which is placed some radium. The α, β, γ rays from the radium emerge from the hole as a narrow vertical pencil. On the application of a strong magnetic field the α rays behave like positively charged bodies and are slightly deviated to one side. The β rays, acting like negatively charged particles, are deviated to a much greater extent on the other side of the vertical, and if the field is strong the β particles describe circles of variable sizes, depending on the velocity with which they leave the radium and the strength of the field. The γ rays are not deviated.

268. Radio-Active Emanations. Radium, actinium, and thorium differ from uranium and polonium in that each constantly gives off a heavy gas which is strongly radio-active. This gas is called an *emanation*, and it has many of the properties of an ordinary gas. It can be carried along pipes by a current of air, it diffuses through porous bodies, it can be condensed, etc. The emanations from radium, actinium, and thorium are very similar in their general properties. They differ, however, greatly in their rates of production and decay, and in some other properties.

The first emanation discovered was that of thorium. In 1899 Owens, who was measuring a current between a horizontal plate on which some thoria was spread and another horizontal plate some distance above, noticed that the current was very unsteady, especially in an open room subject to draughts. On repeating the experiment in a closed vessel he found that the current at any instant was steady, although it gradually varied as time went on. He also showed that the emanation could diffuse through one or two thicknesses of paper. The emanation, or rather the radiations from the emanation, have strong ionising and photographic effects. Rutherford made a further study of thorium emanation. He showed that no emanation could escape from a sealed vessel containing thorium, but that if air passed over the thorium the emanation could be carried

along with the air from one place to another, filtered unaltered through closely packed cotton-wool, bubbled unaltered through solutions, or passed unaltered through red-hot tubes. These experiments showed that the emanation could not be suspended dust particles nor gaseous ions like the α and β particles, and Rutherford rightly concluded that it was a gas. The quantity of emanation is almost indefinitely small compared with the air which carries it along, in fact if thorium is placed in a vacuum tube the emanation is not generated in sufficient quantity to affect the pressure. Although existing in such minute quantities the emanations from radium and thorium are so strongly radio-active that they can be easily detected.

Radium exists very widely distributed throughout the earth's crust, and therefore radium emanation exists (to a very small extent) in the atmosphere. It may be absorbed from atmospheric air by drawing the air through a tube packed with coco-nut charcoal. If the charcoal is afterwards heated to redness the emanation is driven off and may be collected and tested.

269. Study of Thorium Emanation. To study the **life** of thorium emanation the apparatus of Fig. 380 is suitable. A water-pump draws air through cotton-wool (to filter out dust particles) in a tube D, and then over a layer of thorium oxide or hydroxide in a tube T. Charged with the

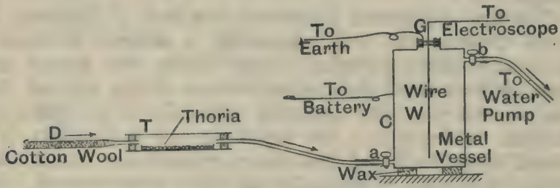


Fig. 380.

emanation the air current enters an insulated metal can C through a tap at a, and after passing through the can and mixing with the contents leaves it by a tap at b. The mouth of the can is sealed by a sulphur plug in a short tube G, G being fitted into an ebonite plug which fits the mouth of C. A wire penetrates the sulphur plug and

hangs axially within the can; its upper end is connected to an electroscope or electrometer. The can is connected to a battery of sufficient voltage to saturate the space between the wire and the walls of the can, the other terminal of the battery being earthed. The guard ring G is earthed, thus all possibility of a leak from the can to the wire is checked, and the good insulation of the sulphur combined with the low potential difference which usually exists between the wire and G during an experiment stops any leakage from the wire.

The air leak being first taken the air current is started. It is observed that the leak in C gradually increases owing to the flow of emanation into it. The leak, however, is not entirely due to the emanation, for, as will be explained later, the emanation causes the deposition of active matter on the walls of C and on the wire. The leak due to this active matter gradually increases, but in about ten minutes attains a maximum which is not very large compared with the emanation leak, and keeps practically constant during the rest of the experiment. As soon as the leak is practically steady, the taps are turned off, and the pump stopped. The emanation in C is thus isolated, and it will be found that the current through C gradually decreases. The readings usually taken in such an experiment are the deflection of a gold-leaf of an electroscope, or of a needle of an electrometer. The former deflection must be translated into voltage, and the latter one is already proportional to voltage. The voltage-time curve must be first plotted, and then by the method of differences $\left(\dfrac{d\ (\text{voltage})}{d\ (\text{time})} \propto \text{current}\right)$ a current-time curve constructed.* If accurate measurements are taken the current-time curve will be very similar to Fig. 381. The ordinates of A B represent the total current. After ten minutes or so the leak will be steady at the value given by O C. O C will be greater than the normal air leak O E owing to the active deposit mentioned above. Through C draw a line C D

* This method assumes that the capacity of the insulated system is not altered appreciably by the motion of the gold-leaf or needle.

parallel to the axis of time. The ordinates of A B measured from C D will give the current due to the emanation which is proportional to the amount of emanation present. A casual glance at the curve shows that the intensity of the

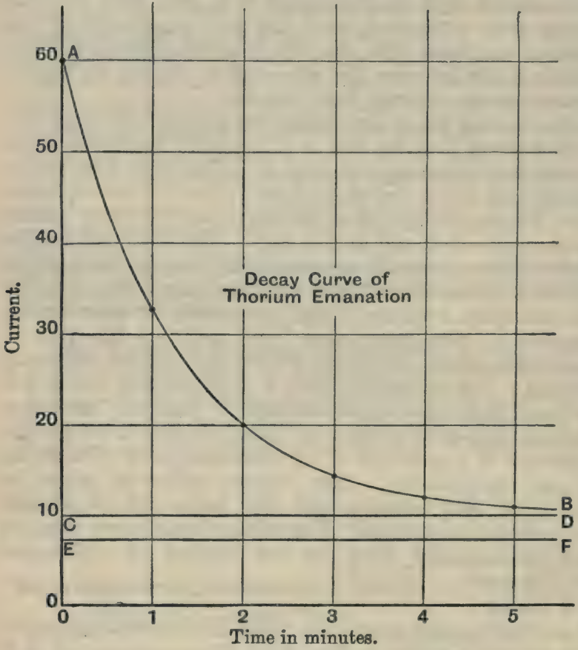


Fig. 381.

ionisation due to the emanation decays to half value in about a minute, *i.e.* any ordinate reckoned from C D is half the value of the ordinate drawn about a minute earlier. A curve of this nature, *i.e.* one in which as the abscissae increase in arithmetical progression the

ordinates decrease in geometrical progression, is called exponential, and its equation may be written

$$I_t = I_0 e^{-\lambda t},$$

where I_0 is the value of the ordinate when the time is begun, and I_t is the value of the ordinate t seconds afterwards.

In the case under consideration, I_0 and I_t are the intensities of the ionisation due to the emanation and are proportional to the amount of emanation, λ is called the constant of radio-active change of the emanation. In radio-activity it is often useful to know how long a substance takes to decay to half its value. Calling this time T we have on substitution in the equation above

$$\tfrac{1}{2} = e^{-\lambda T}$$

or $$\lambda T = \log_2 e = \cdot 693.$$

Studying closely the curve of Fig. 381 we see that T for thorium emanation is about 53 seconds, from which it follows that

$$\lambda = \cdot 013 \text{ or } 1/76.$$

The curve of decay of thorium emanation is therefore

$$I_t = I_0 e^{-\frac{t}{76}}.$$

Differentiating the equation

$$I_t = I_0 e^{-\lambda t}$$

we get $$\frac{dI_t}{dt} = -\lambda I_0 e^{-\lambda t} = -\lambda I_t,$$

or in general $$\frac{dI}{dt} = \lambda I \text{ (numerically)},$$

which may be expressed in words by saying "the rate of decay of emanation is proportional to the quantity of emanation left unchanged, and is numerically equal to the product of this quantity and the radio-active constant of the emanation."

For thorium emanation

$$\frac{dI}{dt} = \frac{I}{76},$$

which means that in one second $\frac{1}{76}$ of the amount of emanation in existence at the beginning of that second has decayed into other matter. We have fully treated this case of decay because all radio-active bodies decay according to an exponential law, and the period of decay to half value is one of the most important radio-active constants. The properties of thorium emanation have been carefully studied by Rutherford and Soddy, and others. By passing air first over thoria and then through a spiral in liquid air the emanation has been condensed. Its volatilisation point is about $-120°$ C. Its molecular weight has been found by diffusion to be about 220, and on the assumption that it is monatomic this is also its atomic weight. Its rapid rate of decay, however, prevents many of its properties being carefully studied.

270. Radium Emanation is a much more stable gas. Its decay curve may be studied by the same apparatus as that used for thorium emanation. It decays much more slowly than thorium emanation, its time for half value being equal to 3·86 days. To find λ we have

$$\lambda \times 3{\cdot}86 \times 24 \times 60 \times 60 = {\cdot}693,$$

from which $$\lambda = 1/480{,}000,$$

or only 1/480,000 of the number of existing atoms of radium emanation disintegrate per second. Radium emanation is a heavy inert gas of molecular (and atomic) weight 222. It condenses at $-150°$ C. Owing to its slow rate of decay it may be kept in vessels for some time.

271. Excited Radio-activity. It was mentioned above that thorium emanation deposited a radio-active substance upon the walls of the can. To study the rate of change of this product it is best to cause the deposit to be made on a wire by inserting the wire* over some thoria in

* To get most of the active deposit on the wire the wire should be charged negatively to a high voltage (300 volts, say) and the tube earthed.

a metal tube. After a time the wire is removed and inserted in a testing vessel; the rate of change of current in the vessel with time then gives the rate of change of the deposit. Although nothing can be seen on the wire the deposit is intensely active, more so, weight for weight, than radium itself is. The deposit may be removed from the wire by rubbing it with glass-paper; in this case the glass-paper is now active. The changes occurring in the active deposits from radium, thorium, and actinium emanations are very complex; for the theory of the changes and the experiments by which they may be studied a book such as Rutherford's *Radio-activity* should be consulted.

It may be here remarked that all rates of decay in radio-activity have so far been found unalterable by any means that man can devise.

272. Disintegration of Radio-active Matter. It has been conclusively shown by Rutherford and others that the radio-active elements are elements in a more or less rapid state of disintegration. Assuming that the structure of the atom is like that given in Art. 256, and we have every justification for this, it follows that when a corpuscle, *i.e.* a β particle, gets loose and escapes the remainder become a new form of atom. Sometimes the expulsion of a corpuscle is accompanied by an expulsion of an α particle, and sometimes an α particle is expelled by itself. The reasons of this we cannot at present discover. It has been found that γ rays are always expelled with β particles, the intensity of each being always very nearly in the same ratio, from which it follows that it is very probable that the expulsions of the β particles cause the pulses in the ether which constitute the γ rays. (See Art. 255.)

The rate of disintegration always obeys the same laws as a mono-molecular change in chemistry, *i.e.* the rate of disintegration of a collection of atoms of the same nature is proportional to the number of the unchanged atoms. From this it follows that the curve expressing the relation between the number of atoms and the time is an exponential. If in any radio-active process the products of disintegration could be removed from the testing vessel as

fast as they are formed so that the ionisation is always proportional to the amount of unchanged matter, the curve between current and time would be as shown in Fig. 382. Its equation is

$$I_t = I_o e^{-\lambda t},$$

where I_o is the initial current, *i.e.* the current when $t = o$, and I_t is the current at time t.

It is because this curve between current and time is found to be an exponential that the law of disintegration is as stated before.

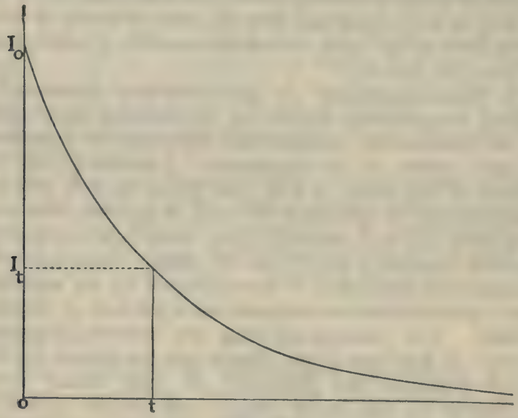


Fig. 382.

From the curve T and λ can be found, and in tables the values of T and λ are given, so that one can form an idea of the rapidity of disintegration of the various products.

If N_o and N_t are the number of atoms initially and at time t, then, since the ionisation current is proportional to the number of atoms present,

$$N_t = N_o e^{-\lambda t},$$

or

$$\frac{dN_t}{dt} = -\lambda N_t,$$

i.e. λ is equal to the ratio of the rate at which the atoms disintegrate to the number still unchanged.

As soon as an atom of one kind disintegrates into an atom of another kind, this new atom also starts disintegrating, so that if we keep a quantity of the primitive substance and allow nothing to escape, we shall have in it a collection of atoms of all the products of disintegration, and if we keep it long enough a steady state will be obtained in which the number of atoms of any one product changing per second into the next product will be equal to the number of atoms of the product formed by disintegration of the previous product. The products are then said to be in state of *radio-active equilibrium.*

Sometimes an atom of one product will change into an atom of another product without the expulsion of any rays. This change—called a rayless change—is most likely caused by a redistribution of the corpuscles within the atom.*

Figs. 383, 384, 385 represent the successive disintegration products or "metabolons," as they are called, which occur in the disintegration of uranium, radium, and polonium, of thorium and of actinium.

273. Uranium. (See Fig. 383.) The change from Uranium to Uranium X was first detected by Crookes and Becquerel. Crookes in 1900 added ammonium carbonate to a solution of uranium nitrate and dissolved the precipitate by excess of the reagent. He found that the precipitate did not totally redissolve, and that the slight precipitate left was photographically as active as the original uranium, while the uranium in the filtrate was photographically inactive. This precipitate he called Uranium X. The two were kept apart for some time, and it was found that the solution wholly regained its activity, while the precipitate completely lost its.

More recent experiments have shown that the decay of

* Recent work shows that the "rayless" changes are not absolutely rayless. The "rayless" products give off β particles of such small velocity that they are absorbed by the air before they travel a sensible distance and are then not easily detected.

photographic activity of the Ur. X follows the exponential law

$$I_t = I_o e^{-\lambda t},$$

and that the rate of recovery of the photographic activity of Uranium follows the complementary law

$$I_t = I_o - I_o e^{-\lambda t},$$

where λ is the radio-active constant of Ur. X.

The Ur. X falls to half value in 22 days, hence expressing this in seconds we get

$$\lambda \times (22 \times 24 \times 60 \times 60) = \cdot 693,$$

or $$\lambda = 3\cdot 6 \times 10^{-7} \text{ (sec)}$$

Electrical measurements showed that Uranium lost little or none of its activity on the precipitation of the Ur. X, and that the Ur. X was almost inactive.

Rutherford has completely explained the above experiments on the assumption that Uranium itself only gives off α rays, while the Ur. X into which it disintegrates only gives out β rays. The α ray activity, and therefore nearly all the ionisation activity, is thus unaltered by the removal of the Ur. X, while this removal takes all the β ray activity, and therefore nearly all the photographic activity, with it.

The rate of disintegration of Ur. into Ur. X is much slower than the rate of disintegration into the next element (which so far has not been detected), but the mass of the Uranium is immense compared with the mass of the Ur. X, hence in the short time of 22 days the Uranium has generated half its equilibrium quantity of the Ur. X, while the Ur. X has half-disintegrated into the next product. When Uranium is kept by itself the rate of production of Ur. X is exactly equal to the rate of disintegration of Ur. X, hence the amount of Ur. X present remains constant.

Further work on the expulsion of the α particle has shown that probably one atom of Uranium gives off only one

α-particle in becoming one atom of Uranium X. The uncharged α-particle is a helium atom (of atomic weight 4), so that the atomic weight of Ur. X is probably 238·5 − 4 or 234·5.

274. Uranium and Radium. It was found by Boltwood that in all old rocks the proportion between the amounts of Radium and Uranium they contain is constant, hence it seemed probable that Radium was a disintegration product of Uranium. Recently the intermediate member of the series has been detected by Boltwood. It has been named Ionium.

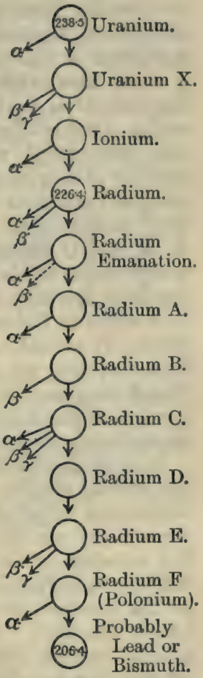


Fig. 383.

275. Radium. The process of disintegration of a radium atom consists of the expulsion of α and β particles, the residue becoming a gaseous atom —the emanation (see Fig. 383). The change from radium to radium emanation is comparatively slow, a quantity of radium decaying to half value in about 2000 years. The emanation atoms change by the expulsion of α particles and slowly moving β particles into Radium A, which constitutes the first product of the active deposit. Only one α particle but more than one β particle is expelled from the emanation atom in changing into Radium A. The emanation being previously neutral, this leaves the Radium A atom positively charged, and hence in an electric field it at once goes to the cathode, as mentioned in Art. 271. The whole of the remaining changes now occur in the deposit on the wire. Radium A is very short-lived. In three minutes from the deposition of a number of Radium A atoms on the wire only half are left, the remainder having changed by the expulsion of an α particle per atom into Radium B.

Radium B has been called a "rayless" change, but it is not really so. Slow moving β particles are expelled, and Radium C results. Radium B decays to half value in 27 minutes, Radium C in 19 minutes; these, together with Radium A, constitute the rapidly changing active deposit. An atom of Radium C gives out α and β particles and γ rays, meanwhile becoming an atom of Radium D. Radium D is rayless, it slowly changes into Radium E, which quickly changes on the expulsion of β and γ rays into Radium F, which is identical with Polonium. The activity of a wire due to the A, B, and C deposits is much greater than that due to the D, E, F deposits into which they disintegrate. This is due to the fact that the A, B, and C deposits are rapidly changing, and give out α rays twice, while the D, E, and F, especially the D and F deposits, are very slowly changing. The properties of the next atom to Polonium have not been worked out, but it is extremely probable that this atom, or perhaps the next to this, will turn out to be lead or bismuth. Lead is always found associated with Uranium in Uranium ores, the proportion of the amount of Lead to the amount of Uranium, though extremely small, is very nearly constant.

It is an interesting speculation to find a relation between the atomic weights of the metabolons and the number of α-particles expelled. The charges carried by the α and β particles are very nearly equal in magnitude, and also the number of α-particles expelled from a quantity of radium in radio-active equilibrium is approximately equal to the number of β-particles expelled in the same time (Arts. 264, 265). Fig. 383 is in accordance with this if we assume that an atom of the "rayless" change D also gives off one slow-moving β-particle. A uranium atom (At. wt. 238·5) loses 3 helium atoms (At. wt. 4) before becoming a radium atom (At. wt. 226·4), for $238{\cdot}5 - 4 \times 3 = 226{\cdot}5$. A radium atom gives out 4 α-particles before it becomes an atom of polonium, hence the atomic weight of polonium should be about $226{\cdot}4 - 4 \times 4$ or 210·4, which suits the position of polonium in the periodic table very well. The expulsion of an α-particle from polonium would leave a remainder of atomic weight 206·4, which is very nearly

the atomic weight of lead (207). Compared with the radio-active elements lead has been found to be practically inactive, but Campbell has found that nearly all ordinary metals are more or less active, and that lead is one of the most active, so that most probably lead is in the series of radium (or rather uranium) disintegration products.

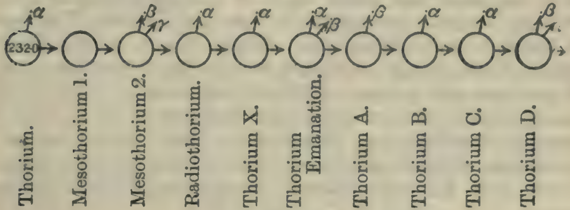


Fig. 384.

276. Thorium. The Thorium changes are as given in Fig. 384. The relation between Thorium X and Thorium (or perhaps Radiothorium) is very similar to that between Uranium X and Uranium. It has been found that Thorium emanation gives off slow-moving β particles as well as a particles, and possibly the "rayless" deposit Thorium A also emits these slow β particles.

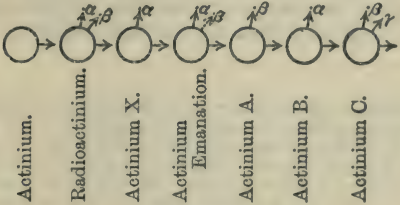


Fig. 385.

277. Actinium. The changes occurring with Actinium are given in Fig. 385. They are very similar to those of Thorium. Actinium emanation is remarkable for its very short life ("half value" in 4 secs., *i.e.* $\lambda = \cdot 17$).

278. Energy in the Atom. The atom is a huge storehouse of energy. The corpuscles arranged within the atom are in extremely rapid rotation. When a corpuscle (*i.e.* a β particle) gets loose and escapes some of the energy is made manifest. When an α particle becomes detached much more energy is lost by the atom. With most bodies it is very rare that either an α or a β particle gets loose, but it is common enough with Uranium, Radium, Thorium, Actinium, and Polonium. Hence their radio-activity. Most of the energy set free is due to the α particles; the β particles and γ rays contribute only a small share to the energy total. The escaping α particles from the interior of a mass of radium are absorbed by the outer layers with evolution of heat and hence the radium becomes warm. Curie and Laborde showed experimentally that radium keeps itself about 2° C. hotter than its surroundings, energy being emitted at the rate of 118 gm.-calories per hour per gm. of radium, which rate, like all other radio-active changes, cannot be altered by any physical or chemical change whatever.*

In the process of disintegration one atom of radium gives off 5 α particles. If the radium is new the energy carried by the last particle (the one from polonium) can be neglected in comparison with either of the others. For radium in radio-active equilibrium and sealed up so that no emanation can escape and so that all the metabolons are present, it has been shown that each of the five

* Radium exists very minutely but very widely distributed throughout the crust of the earth. Strutt and Eve have measured the amount of radium in various surface rocks and found it to be $1{\cdot}4 \times 10^{-12}$ gm. of radium per gm. of rock. Strutt has calculated that the heat generated by disintegration of the radium present in 40 miles of crust of this radium content is sufficient to keep up the temperature of the earth to its present value without calling upon any possible source of internal heat to make good the loss by radiation from the surface of the earth. Unless the amount of heat generated by radium when finely distributed is much less than that generated when in bulk, it follows that the interior of the earth (*i.e.* below 40 miles) contains no radium. Incidentally all former calculations on the age of the earth have been falsified.

products which yield α rays give up approximately equal amounts of energy. The amount of energy given up by one product will, however, be proportional to the square of the velocity with which the α particle is emitted. The velocity of an α particle can be calculated from the distance it will travel in air before its ionisation and photographic powers have dwindled to zero. This has been found by Bragg and Kleeman to be different for the α particles from different sources, but approximately the same for the α particles from any one source. The following table gives in the first column the greatest distance the α particles from the metabolons enumerated travel in air, in the second column the initial velocity of these same particles, and in the third column the relative amounts of energy these same particles carry :—

	cms.	cms. per sec.	
Radium	3·5 ...	$1{\cdot}56 \times 10^9$ ...	24
Radium Emanation ...	4·33 ...	$1{\cdot}70 \times 10^9$ ...	29
Radium A	4·83 ...	$1{\cdot}76 \times 10^9$ ...	31
Radium C	7·08 ...	$2{\cdot}06 \times 10^9$ ...	42
Radium F	3·86 ...	$1{\cdot}62 \times 10^9$ ...	26

From considerations similar to the above it has been calculated that the total heat given out by one gramme of radium and its various products is between 2 and 4 times 10^9 gramme-calories. The heat given out by thorium and uranium is much smaller than that given out by radium.

The ionising-power of an α particle depends on its velocity, and it has been found that the α particles cease ionising even when their velocity is a large fraction of the initial velocity. Thus in the case of the α particle from Radium C its initial velocity is $2{\cdot}06 \times 10^9$ cms. per sec., and it ceases to ionise when its velocity has dropped to $\cdot 82 \times 10^9$ cms. per sec. At present an α-particle having a velocity less than $\cdot 82 \times 10^9$ cms. per sec. cannot be detected. There may be in existence bodies emitting streams of α-particles having velocities less than this lower limit, and we have no cognisance of it.

The theories of the conduction of electricity through gases and radio-activity given in the above pages have been

well established by thousands of experiments, but the subject is by no means fully worked out or the theories universally accepted and many of the explanations given above may have to be modified in the light of future experiments.

Examples V. (on Chap. XXXIV.)

1. An electrified particle of mass m carrying a charge e is projected horizontally with a velocity v through a vertical electric field of intensity f. Show that the distance between the original point of impact on a screen placed normal to the direction of projection at a distance l from the origin and the point of impact after deflection is $efl^2/2mv^2$.

2. An electrified particle traverses an electric field, the intensity of the field being normal to the original direction of motion of the particle. Find an expression for the deflection of the particle. What other experiments must be made in order to determine the ratio of the mass of the particle to its electric charge?

3. Explain how a magnetic force may be represented as due to the motion of tubes of electrostatic induction.
Show that in unit volume of an electromagnetic field an amount of momentum exists equal to μ H N, where μ denotes the magnetic permeability, H the magnetic intensity, and N the number of tubes of electrostatic induction crossing unit area at right angles to their length.

4. How may the electrical conductivity of an ionised gas be determined? What is meant by the *saturation current*?

5. Describe a modern form of quadrant electrometer. A is a gold-leaf electroscope, B a quadrant electrometer. For measuring small potential differences B is found to be more sensitive than A; does it necessarily follow that it will be more sensitive for the measurement of the small charges which occur in the experiments on the conduction of electricity through gases? Give reasons for your answer.

6. Show that the stoppage of a moving electrified particle will produce an electromagnetic pulse spreading out from the particle.

7. Describe the arrangements for generating Röntgen Rays, and discuss briefly their character and the question of how they are produced.

8. What do you know of the properties of the three classes of rays from Radium?

9. Give some account of the radio-activity of Radium.

ADDITIONAL EXAMPLES.

ELECTROSTATICS.

Chapters I.-VII.

1. A small pith ball weighing one decigram suspended by a silk fibre and charged with positive electricity is repelled when a charged glass rod is brought near it. If the direction of the electric field of the glass rod near the ball is horizontal and its magnitude is equal to 20 C.G.S. electrostatic units when the deflection of the fibre is 45° what is the charge on the ball?

2. An insulated soap bubble 10 cm. in radius is charged with 20 C.G.S. electrostatic units. Taking the atmospheric pressure as 10^6 dynes per sq. cm., find the increase in radius due to the charge.

3. What is meant by an electric image? A charge of electricity q is situated at a distance l from a large earthed plane conducting sheet. Find the distribution of the induced charge in terms of the distance from the point.

4. A circular metal plate A of radius 10 cms. is earthed. At a distance of 1 mm. from it is placed another plate B of the same size, which is insulated and charged with 100 units. Find approximately the charges on the four surfaces and the force on the plate A. [The capacity of a charged circular disc of radius a at a large distance from all other conductors is $\frac{2a}{\pi}$.]

5. A brass ball 7 cms. in radius is suspended concentrically inside a spherical brass vessel of internal radius 9 cms. and external radius 10 cms. If the charge on the ball is 56 units, and the potential of the outer vessel is 5, what is the potential of the ball?

6. A metal ball of mass one gramme suspended by a dry silk fibre forms a simple pendulum whose period of vibration is two seconds. The ball is now charged with 100 units of electricity, and a large earthed metal plane is held 2 cms. below it. The pendulum being now set swinging find its period. (Take $g = 1000$ cms./sec.^{2})

Chapters VIII.-XII.

1. What is "electrostatic measure"? Describe the attracted disc guard-ring electrometer, name the uses of its parts, and show how to use it to measure differences of potential in electrostatic measure.

2. Describe Faraday's concentric sphere condenser. If the radii of the spheres are 5 and 10 cms. and the permittivity of the dielectric is 3, find the capacity (1) of the inner plate when the outer plate is earthed, (2) of the outer plate when the inner plate is earthed.

3. A metal sphere of radius a is surrounded by a concentric dielectric shell of permittivity k whose internal and external radii are b and c. Find the capacity of the sphere and the apparent charges on the shell if the sphere receive a charge Q.

4. A disc of ebonite (permittivity = 4) 1 cm. thick is laid on a table and its upper surface rubbed till it has a charge Q. An insulated brass plate of the same diameter as the disc is laid on the disc and separated from it by a layer of air 0·1 mm. thick. The plate is then earthed. Find the charges on the table and on the plate before and after earthing.

5. Describe the Wimshurst machine and explain how it acts. What limits the length of the spark?

6. Describe and explain the action of the water-dropping machine.

7. What arrangements would you make if you were required to show the variation in the potential of the air outside a window?

MAGNETISM.

Chapters XIII.-XVI.

1. A and B are two small magnets of equal moment M placed at right angles to each other so that the axis of A passes through the middle of B. Calculate the couple due to A acting on B and that due to B acting on A. These are not equal and opposite. Would you expect that, if the magnets were fixed to a freely floating board, the board would be set in rotation?

2. Find from the definitions the dimensions of magnetic pole strength, magnetic moment, magnetic field, and employ the last to determine the C.G.S. value of the earth's magnetic field at a place where it is 4 on the foot-grain-second system. [Take 1 grain = ·0648 gramme.]

3. Explain what is meant by a magnetic shell, and show that the potential at any point near a uniform magnetic shell is equal to the strength of the shell multiplied by the solid angle subtended by the shell at the point.

4. Two exactly equal magnets are attached together at their mid points so that the axes of the magnets are at right angles, and the combination is pivoted so that the axes of the magnets are horizontal and they can turn freely about a vertical axis. How can the system set itself under the influence of the horizontal component of the earth's field? If the moment of each magnet is M, and the moment of inertia about the axis round which it can turn is K, what will be the period of vibration of the system?

5. A thin uniform magnet 1 metre long is suspended from the north end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle θ ($\sin\theta = \cdot 1$, $\cos\theta = \cdot 995$). If the mass of the magnet is 10 grammes, the horizontal component of the earth's field is ·2 C.G.S. units, and the vertical component ·4 C.G.S. units, find the moment of the magnet.

6. In an experiment to find H the following observations were taken:—Deflection experiment:—End on position. Distance from centre to centre = 40 cms. Mean deflection = 8° 36′. Oscillation experiment:—6 complete vibrations took 140 seconds, mass of magnet 100·5 grammes, length 15·2 cms., breadth ·65 cms., depth 1·2 cms. Find the values of M and H.

CHAPTERS XVII., XVIII.

1. Define magnetic force H and magnetic induction B. Show that the energy per unit volume of the magnetic field between two plane poles is given by $BH/8\pi$.

2. What is meant by magnetic permeability, and show how its value may be deduced from observations on the magnetic field produced by a long rod placed in a solenoid carrying a current. Why need the rod be long in proportion to its diameter?

3. What is meant by magnetic hysteresis. A watch has had its rate spoiled by being brought near a dynamo. It is hung by a piece of string between the poles of a large steel magnet and made to spin around. While spinning it is slowly removed from the magnet and is then found to be nearly cured. How is this explained?

4. Define Magnetomotive Force. An iron ring composed of a wire of diameter 1 cm. and length 50 cms. is wound with 50 turns of wire. Calculate the intensity of magnetisation produced by a current of one ampère [$\mu = 2000$]. What would be the effect on the magnetisation if the ends of the iron wire were not truly welded together? Illustrate this by discussing the effect of a slit across the wire $\frac{1}{10}$ mm. long.

5. In a magnetometer experiment to find the permeability of the iron in a long wire it was found that when the field was equal to 5, 11·4, 31·4, 43·6, 53·3 C.G.S. units the respective values of the intensities of magnetisation of the wire were respectively 527, 699, 787, 819, 834 C.G.S units. In each case find the value of the susceptibility, permeability, and induction, and plot curves between field and permeability, field and induction.

6. Show that if a sphere of crystal always sets itself with its direction of greatest permeability axial, it will set itself with its axis of greatest intensity of magnetisation axial or equatorial according as it is of paramagnetic or diamagnetic.

CURRENT ELECTRICITY.

Chapters XIX.-XXV.

1. What is meant by contact electricity and how have measurements been made on this subject?

2. Discuss critically Ampère's formula, $l = \frac{C\,a\,b\cos a}{r^2}$ (p. 370) for the magnetic intensity due to an element of current. Find the value of the intensity at a point on the axis of a circular coil of 10 turns and 5 cms. mean radius. The point is 20 cms. from the plane of the coil and the current through the coil is 0·5 ampère.

3. What are the special characteristics of a ballistic galvanometer? Show that the sine of half the angle of throw of the needle is equal to $Q\,G\,\pi/H\,T$, where G is the "galvanometer constant," H the horizontal component of the earth's magnetic field, T the time of one complete oscillation of the needle, and Q the total electric transfer through the instrument.

4. If we define a current in a circuit on the strength of a uniform magnetic shell having the same boundary as the current which would have the same magnetic field as the current, show that the field at the centre of a circular coil of radius r conveying a current C is $\frac{2\pi C}{r}$ and that the magnetic moment of the coil is $\pi r^2 C$.

5. What is the magnitude and direction of the force acting on a straight conductor, 10 cms. long, placed at right angles to a magnetic field of 50 lines per sq. cm., the current through the conductor being 5 ampères?

6. A wire of resistance r connects A and B, two points in a circuit, the resistance of the remainder of which is R. If without any other change being made A and B are also connected by $n-1$ other wires, the resistance of each of which is r, show that the heat produced in the n wires together will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$.

7. Twelve straight equal wires of resistance 1 ohm each are connected up to form a skeleton cube. A current of 1 ampère is led in at one corner and out at the opposite corner. Find the difference of potential between the two corners.

8. The positive terminals of two batteries, one composed of 5 equal cells and the other of 4 equal cells, are connected to one end of a wire of 10 ohms resistance, and the negative terminals are connected to the other end. Each cell has an E.M.F. of one volt and a resistance of one ohm. Find the currents in the different parts of the circuit.

9. Four points, A, B, C, D, are connected together as follows:—A to B, B to C, C to D, D to A, each by a wire of 1 ohm resistance; A to C, B to D, each by a cell of 1 volt E.M.F. and 2 ohms resistance. Determine the current flowing through each of the cells.

10. A current is sent from a Daniell's cell (E.M.F. 1·1 volts) through a galvanometer of resistance 8400 ohms shunted by a wire of resistance 8·4 ohms. The deflection read on a scale a metre away from the mirror is 13·36 cms. Find the Figure of Merit of the galvanometer (*i.e.* the current requisite to give a deflection of 1 mm. on a scale at a distance of 1 metre).

11. The resistance of a certain column of copper sulphate was measured by a Kohlrausch bridge and telephone at different temperatures and the following results were obtained:—At 59·5° C. the resistance was 15 ohms, at 52°, $15{\cdot}5^{\omega}$; at 48°, 16^{ω}; at 44°, $16{\cdot}5^{\omega}$; at 38°, $\cdot 18^{\omega}$; at 31·5°, $20{\cdot}1^{\omega}$; at 20·5°, $25{\cdot}2^{\omega}$; at 20·5°, $25{\cdot}9^{\omega}$. Plot a curve between resistance and temperature and find from it the temperature coefficient of the change of resistance at 41° C. and 26° C.

12. How may the capacity of a condenser be determined absolutely by experiment? In such an experiment a Daniell cell gave a steady deflection of 203 mms. when 1200 ohms was interposed in the circuit of the cell and galvanometer. The galvanometer resistance was 30,000 ohms, and it was shunted with 100 ohms. The condenser was charged from the cell and discharged through the galvanometer (now unshunted). The kick was 263·6 mms. (corrected for damping). The time of swing of the needle 1·900 seconds. Find the capacity of the condenser.

13. Give the leading features of the phenomenon of electrolysis, and give some account of the methods which have been employed for observing the velocity of ionic migration.

14. Explain in what way the E.M.F. of a galvanic cell depends on the character of the chemical action taking place in it.

15. An aqueous solution of copper sulphate is placed in a vertical cylindrical tube, in the top and bottom of which are placed horizontal copper discs, the upper one being just immersed in the liquid. Describe carefully what happens when an electric current is sent *upwards*

through the liquid from one disc to the other and explain that which is observed.

16. Give an account of the theory of electric heating as a method of applying a measurable amount of heat to a system, and point out what are the standard determinations upon the accuracy of which the accuracy of the method depends.

17. Assuming that the rate at which a wire loses heat at a given temperature is proportional to the area of its surface, show that the currents required to keep wires of the same material at the same temperature are proportional to the cubes of the square roots of the diameter.

CHAPTERS XXVI.-XXX.

1. Write a historical account of the development of our knowledge of electromagnetic induction.

2. Explain how the current in a vertical ring of wire rotating about a vertical axis in the earth's field varies with the orientation of the coil in the two cases of (1) resistance of wire very large, (2) resistance of wire very small. Explain also in each case how the value of the current depends on the velocity of rotation v, the resistance R, the inductance L, and the area of the coil a.

3. A copper ring is hung by a torsionless thread between the poles of an electromagnet. The plane of the ring is vertical and is inclined at 45° to that of either pole face. If a current is started round the magnet the ring turns through a moderate angle, but quickly comes to rest. If it is replaced in its former position and the current is stopped it starts twisting in the opposite direction and keeps on twisting. Account for this.

4. A small accumulator sends a current of ·497 ampères through the primary of a standard inductor. This primary is 64 cms. long, of mean radius 2·34 cms., and contains 18·72 turns per cm. Its secondary consists of 450 turns of wire wound around the middle of the primary. In circuit with the secondary is an earth-inductor and a ballistic galvanometer. The earth-inductor consists of a rectangular coil of effective area 106,000 sq. cms. which can be rapidly rotated through two right angles. Three experiments were made—

(1) The inductor was placed with its axis vertical and its plane magnetic east and west, and rapidly rotated through two right angles. The kick of the galvanometer needle was observed to be 5·16 cms. on a scale.

(2) The inductor was placed with its axis horizontal, and starting with its plane horizontal rapidly rotated through two right angles. The kick was 12·13 cms.

(3) The current through the primary was now broken. The kick was observed to be 12·05 cms.

From these readings find the horizontal and vertical components of the earth's magnetic field and the angle of dip.

5. A copper disc having a diameter of 40 cms. is rotated about a horizontal axis perpendicular to the disc and parallel to the magnetic meridian. Two brushes make contact with the disc, one at the centre and the other at the edge. If the value of the horizontal component of the earth's field is 0·2 C.G.S. units, find the potential difference in volts between the two brushes when the disc makes 3000 revolutions per minute.

6. A solid flywheel, one metre in diameter, spins on an axis which is directed to the north at a place where $H = 0{\cdot}18$. It makes 250 revolutions per minute. Calculate the potential difference between the axle and the circumference.

7. A circuit contains a battery of E.M.F. 10 volts and 2 glow-lamps in series, one of which has its terminals connected by a copper wire. Each lamp has a resistance of 10 ohms, it glows red with 0·5 ampère, is well lit with 1 ampère, and would be destroyed by 1·5 ampères. The rest of the circuit has large self-inductance and negligible resistance. State and explain what happens if the copper is suddenly cut.

8. A brass rod 50 cms. long and 1 cm. in diameter is uniformly wound with 5000 turns of wire, and then the bar is bent till its ends touch, thus forming a circular ring. Calculate the self-inductance of the coil so formed.

9. A constant electromotive force E is suddenly applied at the time $t = 0$ to a circuit which has resistance R and self-induction L. Write down and integrate the differential equation connecting the current with the time. Show by a sketch how the current increases.

10. A cell of E.M.F. 4 volts sends a current through a resistance-box and a coil of negligible resistance, but of self-inductance 3 henries. At first the resistance unplugged is 2 ohms. The resistance is suddenly increased by unplugging the 2000 ohms. Express the current now in terms of the time, and show that the current will drop to within an infinitesimal amount of its final value by the end of one second.

11. The mutual inductance between two coils was measured as in Art. 180. Balance of kicks occurred when the values of Q, S, and R were 10, 1, and 40 ohms respectively. C was 20 microfarads. Find M.

12. An alternating electromotive force $E \sin pt$ is applied to a circuit containing resistance and self-induction. Write down the differential equation for the current, and obtain an expression for the current when the permanent state is reached.

13. A circuit which has resistance R, self-induction L, and capacity F has at time $t = 0$ an electromotive force $E \cos \omega t$ impressed on it. (*a*) Obtain the differential equation for the current (and integrate it to obtain current) when the permanent state has been reached. (*b*) Find the value of ω in order that the current may have the same value as in a circuit of resistance R free from inductance and capacity.

14. In some large transformers which have been recently constructed it is found that if a steady (continuous) electromotive force is applied to the primary winding it takes a long time for the current to rise to the value $C = E/R$. How do you account for this? Ought it to make any difference if the secondary circuit is short-circuited instead of being opened?

15. Give a general explanation of the oscillatory nature of the spark discharge of a Leyden jar, and describe any experiment which exhibits or illustrates these oscillations.

16. A condenser of capacity C, charged to potential V, is discharged through a circuit of resistance R and self-induction L. Obtain and solve the differential equation connecting the potential with the time.

What is the condition that the discharge may be oscillatory?

Find the frequency of oscillation if $C = \cdot 5$ microfarads, $R = 8$ $L = \cdot 02$ henries.

17. A microfarad condenser is discharged through a coil whose self-inductance is 100 henries and whose resistance is negligible. Find the frequency of the musical note given by the spark.

18. Give a description of the production and detection of electromagnetic waves.

19. Give some account of Hertz's experiments on electromagnetic waves. What are the principal changes in Hertz's apparatus necessary to produce a practical method of wireless telegraphy?

Chapters XXXI.-XXXIII.

1. Write a clear account of the C.G.S. system of units of electric current, electromotive force and resistance (1) as defined theoretically, (2) as determined experimentally.

2. Define electrical resistance, and explain how it happens that when electrical resistance is expressed on the electromagnetic or on the electrostatic system of units, it is in one case of the dimensions of a velocity, and in the other of a slowness. Explain how Wheatstone's light can be conveniently used for measuring temperature by the change in the resistance of a wire.

3. Find the number of C.G.S. electromagnetic units of resistance in 20 C.G.S. electrostatic units.

4. If the unit charge were defined to be such that two unit charges at a distance of one metre, in a medium of which the specific inductive capacity was twice that of air, repelled each other with a force in which in one second would produce a velocity of one metre per second in a centigramme, find the number of ordinary electrostatic units to which it is equivalent.

5. Show from the formula of Art. 191 that the frequency of the oscillatory discharge of a condenser of capacity F farads through an inductance of L henries and a resistance of R ohms

$$= \frac{1}{2\pi}\sqrt{\frac{1}{LF} - \frac{R^2}{4L^2}}.$$

6. A parallel plate condenser is made up of two equal discs of tinfoil pasted on glass plates placed horizontally and separated by ebonite slabs. The radii of the tinfoil discs are 9·98 cms. and the width of the intervening air space is ·182 cms. Find the capacity of the condenser in electrostatic units.

The terminals of a condenser of capacity one-twentieth of a microfarad is connected to the pairs of quadrants of a quadrant electrometer of negligible capacity. The deflection observed is 14·05 cms. By means of a tapping key the charge in this condenser is shared with the air condenser above; the air condenser is then discharged. It is again charged and again discharged. The operation is repeated 100 times. The deflection is now 10·74 cms. Find the capacity of the air condenser in electromagnetic units and hence deduce the velocity of light.

7. Give an account of the Peltier effect and of the Thomson effect. What is meant by Thermo-electric Power, and how can the data for a diagram representing it be found?

8. The thermo-electric height of iron is 1734 at 0° C., 1247 at 0° C.; of copper 136 at 0° C., 231 at 100° C., all in electromagnetic C.G.S. units. Construct a thermo-electric diagram for lead, iron, and copper, and state how the amounts of heat absorbed and given out in different parts of a copper iron circuit with its junctions at 0° C. and 100° C. when there is a current of 1 ampère in the circuit are shown in the diagram, and also the E.M.F. in volts.

9. Show that the electromotive force in a circuit of three metals A, B, C, the AB, BC, CA junctions being at t_1° C., t_2° C., t_3° C. respectively, is equal to the sum of the E.M.F.'s in two circuits, one of which consists of A and B with junctions at t_1° C. and t_3° C., and the the other of B and C with junctions at t_2° C. and t_3° C.

10. Describe and explain the working of Bell's telephone. Would a soft iron core to the electromagnet be as effective as a steel one?

11. Explain how telephone currents may be measured experimentally. What function of the current is it that is directly measured?

12. Describe the means by which coils rotating in a magnetic field may be arranged to furnish (1) alternating currents, (2) continuous currents.

13. Give a short account of any form of dynamo or electromagnetic generator; explain the production of the current.

14. A dynamo feeds 1000 sixteen-candle glow lamps. What current must the dynamo supply if the Potential Difference of its terminals is 200 volts and each lamp absorbs 3·6 watts per candle-power?

15. Current is supplied to a series motor at 100 volts, the resistance of the circuit being ·5 ohm. Determine the power expended in turning the armature when the current is 10 ampères. Determine also the current when the power thus expended is a maximum. Compare the values of the electrical efficiencies in the two cases.

16. (*a*) Show that when an electromotor is working most efficiently it is working very slowly. (*b*) Is it then running very slowly? (*c*) Is it true that when it is working very slowly it is always working very efficiently? (*d*) How must a motor be connected to a pump in order to pump the greatest possible quantity of water for the electrical energy used? (*e*) How must the connection be altered to make it pump as much as possible in a given time?

17. A continuous-current motor, which is to work at the rate of 100 horse-power, is to be driven by energy conveyed by means of a conductor, whose resistance is 100 ohms, with a waste of not more than 10 % of the energy supplied by the dynamo. What must be the E.M.F. in the dynamo and the back E.M.F. in the motor? How many calories of heat are produced per hour?

18. A shunt-wound electromotor is connected to the town mains through a certain considerable resistance. It is then found that the motor remains at rest, but if a starting resistance is interposed in the armature branch it immediately starts, or if without introducing the starting resistance it is started by hand, it will increase in speed to a small extent and continue to rotate. Explain this.

ANSWERS.

Examples I. (pages 28-31).

6. 1 dyne. **7.** 1 dyne parallel to AB. **8.** ·00173 dyne.
9. ·283 dyne. **10.** $2\sqrt{2} : 3\sqrt{3}$. **11.** 14·4°.

12. $3\sqrt{3}\frac{q}{4a^2}$ dynes, where q denotes the magnitude of each of the equal charges, and a the side of the hexagonal base in cms.

Examples II. (pages 119-123).

8. $8\sqrt{2}$; $8 + \frac{8}{5}\sqrt{5}$. **9.** 5. **10.** $2a/(a+b)$, $2b/(a+b)$.
11. $\frac{1}{4}Q^2/C$. **12.** 10 cm. and 12 cm.
13. 25 : 189. **14.** 25 : 24. **18.** $\frac{1}{16}$, $\frac{1}{4}$, $\frac{1}{16}$.

20. The jar retains $\frac{1}{3}$ of its charge, and the final potential of both jars is $\frac{1}{3}$ of the initial potential of the first charged jar.

22. 1 dyne and $1\frac{9}{16}$ dynes for charges of the same sign, 1 dyne and $\frac{9}{16}$ dyne for charges of opposite sign.

23. 108 : 36.

Examples III. (pages 282-284).

7. 1·09 : 1. **8.** 1080 C.G.S. units.
9. $(7{\cdot}5)^2 : (8{\cdot}3)^2 : (10{\cdot}4)^2$. **10.** 9·31.

Examples IV. (pages 503-511).

11. 1·72 ampere; 5·16 volts. **12** 5·04 ohms.
13. Deflection is practically unchanged. See Art. 148.
14. $\frac{12}{13}$ ampere, $\frac{9}{13}$ ampere, $\frac{3}{13}$ ampere. **15.** $166\frac{2}{3}$ divisions.
16. ·5 ampere, ·1585 ampere. **17.** 39 : 20.
18. 37 ohms. **19.** $\frac{2}{3}$ ampere.
20. (*a*) $1\frac{5}{6}$ amperes, (*b*) $1\frac{19}{69}$ amperes, (*c*) $\frac{4}{5}$ ampere.
21. 2·1. **22.** $\frac{84}{239}$ ampere, $\frac{21}{239}$ ampere, $\frac{63}{239}$ ampere.

24. H = 0·0154 gram.; H_2SO_4 = ·75 gram.; Pb = 1·59 grams.; Cu = ·49 gram.; Ag = 1·66 grams. Current = ·83 ampere.

25. 1395 units of heat.

27. ·317 gram. Cu, 1·08 grams. Ag; ·634 gram. Cu, 2·16 grams. Ag.

28. 225 : 4 (inside); 25 : 12 (outside).

29. The current in each case is ·5 ampere. One-half as much zinc is consumed in case (*b*) as in (*a*) in the same time.

30. $\frac{e^2}{R+r}$ units of work; $\frac{e^2 r}{J(R+r)^2}$ units of heat.

31. 2·02 grams. **32.** 20 units of heat.

EXAMPLES V. (page 725).

2. See Art. 236.

5. A has small capacity; B a relatively large capacity. The capacity of a C. T. R. Wilson electroscope is about 1 cm., that of a Dolezalek electrometer about 50 cms.

ANSWERS

TO THE ADDITIONAL EXAMPLES.

ELECTROSTATICS.

CHAPTERS I.-VII.

1. 4·91 C.G.S. electrostatic units. 2. $\dfrac{1}{6\pi \times 10^7}$ cm.

4. Charges on A, 0 and $98\frac{2}{3}$. Charges on B, $98\frac{2}{3}$ and $1\frac{1}{3}$. Force on A = 197 dynes. 5. 6·78.

6. The attraction between sphere and plane $= \dfrac{100 \times 100}{\cdot 4^2} = 625$ dynes,

$$\therefore \ t_2 = 2\sqrt{\frac{1000}{1000 + 625}} = 1{\cdot}57 \text{ secs.}$$

CHAPTERS VIII.-XII.

2. 30, 40.

3. Capacity $= 1\Big/\left[\dfrac{1}{a} - \dfrac{1}{b}\left(1 - \dfrac{1}{K}\right) + \dfrac{1}{c}\left(1 - \dfrac{1}{K}\right)\right]$. Charge on inner surface $= -\,Q\dfrac{K-1}{K}$, on outer surface $+\,Q\dfrac{K-1}{K}$.

4. Before earthing, $\frac{100}{103}$Q on table, $\frac{3}{103}$Q on each side of plate After earthing, $\frac{100}{2600}$ Q on table, $\frac{2500}{2600}$ Q on lower side of plate, nothing on top of plate.

MAGNETISM.

Chapters XIII.-XVI.

1. $\frac{2M}{r^3}$, $\frac{M}{r^3}$, No.

2. Pole strength $M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$, moment $M^{\frac{1}{2}} L^{\frac{5}{2}} T^{-1}$, field $M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$. ·184 C.G.S. units.

4. $T = 2\pi\sqrt{\frac{K\sqrt{2}}{MH}}$.

5. 205,000 C.G.S. units.

6. $\frac{M}{H} = 4500$, $MH = 141$, $\therefore M = 795$ and $H = \cdot 177$ C.G.S. units.

Chapters XVII.-XVIII.

2. To make the demagnetising force small.

3. The hysteresis cycle is gradually reduced to zero.

4. 199·9 C.G.S. units. Reduced. Intensity reduced to 142·7 C.G.S. units.

5.

K.	μ.	B.
10·5	1320	6620
61·5	773	8790
25·1	316	9930
18·8	237	10330
15·7	198	10530.

CURRENT ELECTRICITY.

Chapters XIX.-XXV.

2. ·009 C.C.S. units (approximately). 5. 250 dynes. 7. $\frac{5}{6}$ volt.

8. $\frac{2}{11}$, $\frac{1}{11}$, $\frac{3}{11}$ ampères through the 5-cell battery, the 4-cell battery and the wire respectively.

9. $\frac{1}{3}$ ampère.

10. 10^{-9} ampère.

11. $-\cdot 014$, $-\cdot 020$.

12. 1·003 microfarads.

15. Velocity of anion > velocity of cathion, hence liquid near the cathode becomes nearly colourless.

Chapters XXVI.-XXX.

2. Case I.—Current a maximum when coil is in the meridian, and is $\propto \frac{v\,a}{R}$.

Case II.—Current a maximum when coil is perpendicular to meridian. It is $\propto \frac{a}{L}$ and independent of v and R.

4. H = ·18 and V = ·43 dynes per unit pole. Dip = 66·9°.

5. $1{\cdot}25 \times 10^{-4}$ volts.

6. $5{\cdot}89 \times 10^{-5}$ volts.

7. Before: the unshunted lamp is well lit. On cutting both lamps are well lit for a time, then both become dull red.

8. $4{\cdot}93 \times 10^{6}$ cms. or $4{\cdot}93 \times 10^{-3}$ henries.

9. See § 172.

10. $C = {\cdot}002 + 1{\cdot}998\, e^{-\frac{2002}{3}t}$.

11. $8{\cdot}8 \times 10^{6}$ cms. or $8{\cdot}8 \times 10^{-3}$ henries.

12. See § 182.

13. (*a*)

$$\text{Current} = \frac{E_0 \cos(\omega t - a)}{\left\{ \left(L - \frac{1}{F\omega^2}\right)^2 \omega^2 + R^2 \right\}^{\frac{1}{2}}}$$

$$\text{where } \tan a = \frac{\left(L - \frac{1}{F\omega^2}\right)\omega}{R}.$$

(*b*)

$$\omega^2 = \frac{1}{LF}.$$

14. See Art. 189. Closing of the secondary circuit at first assists and afterwards hinders the rise of the primary. On the whole the rise is more rapid when the secondary is closed.

15. See Art. 190.

16. See Art. 191. 1591

17. Frequency of note = twice the frequency of spark = 32.

Chapters XXXI.-XXXIII.

3. 60×10^{20}.

4. $100\sqrt{2}$.

6. 137 C.G.S. electrostatic units, $14{\cdot}5 \times 10^{-20}$ C.G.S. electromagnetic units. $V = 3{\cdot}07 \times 10^{10} \frac{\text{cms.}}{\text{sec.}}$.

8. E.M.F. = ·00131 volts.

9. $$\mathrm{E} = {}_1\Pi_{AB} + {}_2\Pi_{BC} + {}_3\Pi_{CA} + \int_{t_3}^{t_1} \sigma_A dt + \int_{t_1}^{t_2} \sigma_B dt + \int_{t_2}^{t_3} \sigma_C dt$$

$$= {}_1\Pi_{AB} + {}_2\Pi_{BC} + {}_3\Pi_{CA} + \int_{t_3}^{t_1} (\sigma_A - \sigma_B)\, dt + \int_{t_3}^{t_2} (\sigma_B - \sigma_C) dt$$

$$= ({}_1\Pi_{AB} - {}_3\Pi_{AB}) + ({}_2\Pi_{BC} - {}_3\Pi_{BC}) + ({}_3\Pi_{AB} + {}_3\Pi_{BC} + {}_3\Pi_{CA})$$

$$+ \int_{t_3}^{t_1} (\sigma_A - \sigma_B) dt + \int_{t_3}^{t_2} (\sigma_B - \sigma_C) dt$$

$$= {}_{13}\mathrm{E}_{AB} + {}_{23}\mathrm{E}_{BC}.$$

11. See § 230. The square root of mean squares, *i.e.* $\frac{1}{\sqrt{2}}$ the maximum current (on the assumption that the current follows the sine law).

14. 288 ampères.

15. $47\frac{1}{2}$ watts, 100 ampères, $\frac{18}{19}$.

16. (*a*) Most efficient when e nearly equals E, work done is $\frac{(\mathrm{E}-e)}{\mathrm{R}}$ and is therefore small. (*b*) No, e increases as speed increases. (*c*) No. (*d*) Motor must move fast and pump slow. Gear accordingly. (*e*) Alter gearing till $e = \mathrm{E}/2$.

17. $\mathrm{E} = 9110$ volts. $e = 8200$ volts. $7{\cdot}1 \times 10^6$.

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